# Cost-Constrained Selection of Strand Diameter and Number in a Litz-Wire Transformer Winding

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***Abstract*—Design of litz-wire windings subject to cost constraints is analyzed. An approximation of normalized cost is combined with analysis of proximity effect losses to find combinations of strand number and diameter that optimally trade off cost and loss. The relationship between wire size, normalized cost, and normalized loss is shown to have a general form that applies to a wide range of designs. A practical design procedure is provided. Applied to an example design, it leads to less than half the original loss at lower than the original cost, or, alternatively, under one fifth the original cost with the same loss as the original design.**

***Index Terms*—Eddy currents, inductors, litz wire, magnetic devices, optimization costs, power conversion, power transformers, proximity effect, skin effect.**

## I. Introduction

LITZ WIRE[1] can be used to reduce the severe eddy-current losses that otherwise limit the performance of high-frequency magnetic components. But litz wire is often avoided by designers because it can be very expensive. In this paper, we develop a design methodology considering cost. This approach enables significant cost reduction with no increase in loss, or more generally, enables a designer to select the minimum loss design at any given cost. In a design example, the cost is reduced by better than a factor of five with no increase in loss, compared to a design based on a conventional rule of thumb.

Losses in litz-wire transformer windings have been calculated by many authors [1]–[6], but relatively little work addresses the design problem: how to choose the number and diameter of strands for a particular application. In [7], the optimal stranding giving minimum loss is calculated. However, this can result in a very expensive solution with only slightly lower loss than is possible at considerably lower cost. Although [7] also addresses the choice of stranding under constraints of minimum strand diameter or maximum number of strands, the real constraint is more likely to be cost rather than one of these factors.

Analysis of cost is performed at two levels in this paper. First, a general form for functions describing the cost of litz wire is hypothesized. This leads to general analytical results describing the best choice of litz wire for a given transformer winding, in terms of a cost function. At the second level, results that are less general but are more explicit are obtained through making the cost function explicit with a polynomial curve fit to manufacturers' price quotations. A design methodology, applicable to the general case, but fleshed out in terms of the specific cost function, is outlined and illustrated with a design example.

Many analyses of winding loss address only sinusoidal current waveforms, but magnetics in high-frequency power converters rarely have waveforms that approximate sinusoids. A number of authors have developed methods of extending the analysis of winding loss to nonsinusoidal waveforms [7]–[13]. Of particular interest is the use of an "effective frequency" [7], [10], [11], [13] because that approach allows the use without modification of optimizations based on sinusoidal waveforms (including the optimization described here), as explained in the appendix of [7]. Particularly useful is [13] for a thorough discussion and a compilation of the relevant data for a large number of common waveforms.

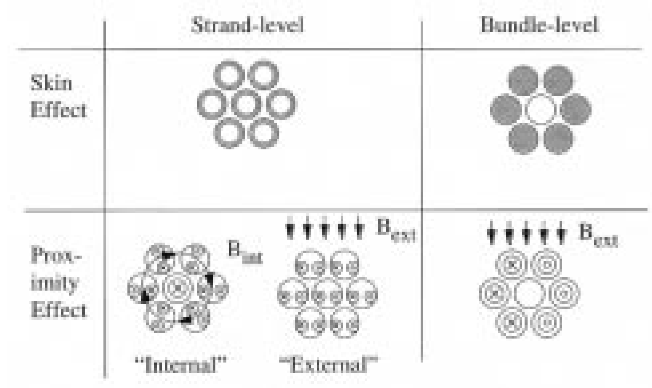


Fig. 1. Types of eddy-current effects in litz wire.

## II. Loss Model

Skin effect and proximity effect in litz-wire windings may be divided into bundle-level and strand-level effects, as illustrated in Fig. 1. With properly chosen construction, strand-level proximity effect is the dominant effect that needs to be considered for choosing the number of strands [7].

We represent winding losses by

$$P_{\text{loss}} = F_r I_{\text{ac}}^2 R_{\text{dc}} \tag{1}$$

where $F_r$ is a factor relating dc resistance to an ac resistance which accounts for all winding losses, given a sinusoidal current with rms amplitude $I_{\text{ac}}$. As discussed in Appendix A, internal

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[1]Sometimes the term *litz-wire* is reserved for conductors constructed according to a carefully prescribed pattern, and strands simply twisted together are called bunched wire. We will use the term *litz-wire* for any insulated grouped strands.

and external strand-level proximity effect loss can be accounted for with the approximate expression

$$F_r = 1 + \frac{\pi^2 \omega^2 \mu_0^2 N^2 n^2 d_c^6 k}{768 \rho_c^2 b_c^2} \tag{2}$$

where

- $\omega$ radian frequency of a sinusoidal current;
- $n$ number of strands;
- $N$ number of turns;
- $d_c$ diameter of the copper in each strand;
- $\rho_c$ resistivity of the copper conductor;
- $b_c$ breadth of the window area of the core;
- $k$ factor defined in Appendix A, accounting for field distribution in multiwinding transformers, normally equal to one.

For waveforms with a dc component, and for some nonsinusoidal waveforms, it is possible to derive a single equivalent frequency that may be used in this analysis [7]. In an inductor, the field in the winding area depends on the gapping configuration, and this analysis is not directly applicable [14].

## III. Cost Analysis

Attempting to quantify cost for academic analysis is problematic; prices change with volume, manufacturer, time, and negotiation. However, many important results depend only on the general form of the cost function. In particular, the general solutions derived in the Appendix for optimal cost/loss tradeoff designs depend only only the assumption that the cost of a length of litz wire can be approximately described by

$$\text{Cost} = \left(C_0 + C_m(d_c) d_c^2 n\right) \ell \tag{3}$$

where

- $C_0$ base cost per unit length associated with the bundling and serving operations;
- $C_m(d_c)$ cost basis function proportional to the additional cost per unit mass for a given strand diameter $d_c$;
- $n$ number of strands;
- $\ell$ length of the wire.

Since we have not specified a form for $C_m(d_c)$, the only loss of generality in assuming this form (3) is in the assumption that $C_m$ depends only on $d_c$, and not on $n$. Examination of pricing from litz-wire manufactures indicates that this assumption is a valid approximation. Note that for the purpose of optimization with a fixed winding length, we can ignore $C_0$, and consider only the cost variation which is proportional to $C_m(d_c) d_c^2 n$.

In order to gain intuition about the variation of cost, and to provide specific numerical results, it is useful to find an approximate expression for $C_m(d_c)$. From manufacturers' pricing, we find that the following function, normalized to a value of one for large-diameter wire, is a good approximation for a wide range of values of $n$ and $d_c$:

$$C_m(d_c) = 1 + \frac{k_1}{d_c^6} + \frac{k_2}{d_c^2} \tag{4}$$

where $d_c$ is in meters, $k_1 = 1.1 \times 10^{-26}$ m$^6$, and $k_2 = 2 \times 10^{-9}$ m$^2$. This function, proportional to cost per unit mass, is shown in Fig. 2, along with the normalized cost per unit length, $C_m(d_c) d_c^2$. $C_m$ is approximately constant for large diameters, but by around 40 AWG it has started rising significantly. 44 AWG is notable as the size at which the cost per unit length is a minimum. At 48 AWG, cost per unit length has increased significantly and cost per unit mass has increased dramatically. Few manufacturers will provide constructions using finer strands than this, and though (4) is not based on data beyond this point, it does appropriately rise very rapidly. Although (4) represents a smooth function, wire based on standard sizes is cheaper than arbitrary choices, and the actual cost function has significant ripples because of this. In particular, even-numbered sizes are generally cheaper and more readily available than odd-numbered sizes. The extent of this variation is highly sensitive to volume—at sufficiently high volumes, there would be no penalty for using odd, or even custom sizes. Thus, such variations are omitted from this analysis; we assume the cost is described by the smooth function shown.

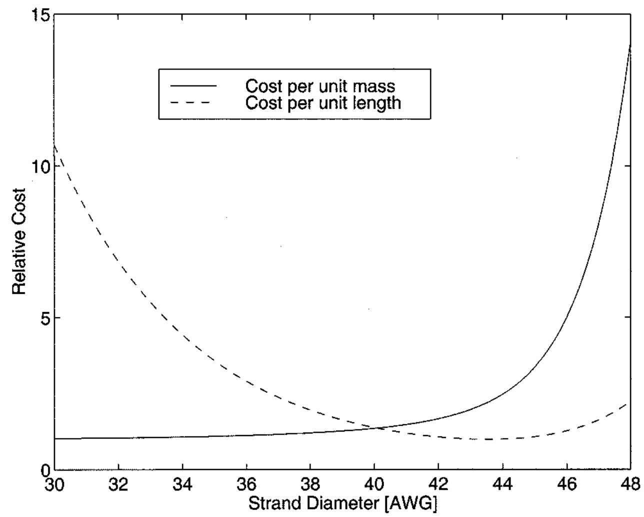


Fig. 2. Normalized cost per unit mass and normalized cost per unit length, as modeled by (4). Both are normalized such that the minimum values are one, for the purpose of display in this graph. The cost per unit mass increases monotonically, reflecting the cost of drawing a given quantity of copper into finer and finer strands. The cost per unit length is found by multiplying cost per unit mass by mass per unit length, as described in the text. Below 44 AWG, the decreasing mass dominates the trend, making the cost per unit length decrease as the wire gets smaller. Above 44 AWG the cost per unit mass increases rapidly enough that the increased manufacturing cost dominates the decreased material cost, and cost per unit length increases. Both 38 AWG and 48 AWG cost about twice as much as 44 AWG. For 38 AWG, this cost increase is a result of the larger mass of copper required. For 48 AWG, the cost increase is due to the expense of forming the wire into very fine strands.

## IV. Choosing Number and Diameter of Strands

The design choice of number and diameter of strands can be conceptualized and illustrated as a two-dimensional (2-D) space. In the case of a full bobbin, the choices in this space form a line, and the tradeoff between cost and loss becomes a simple matter of evaluating both cost and loss along this line, which can be described by using calculations in [7]. However, with cost constraints, a full bobbin often is not optimal, and we must choose a point in 2-D space rather than simply a point on a line.

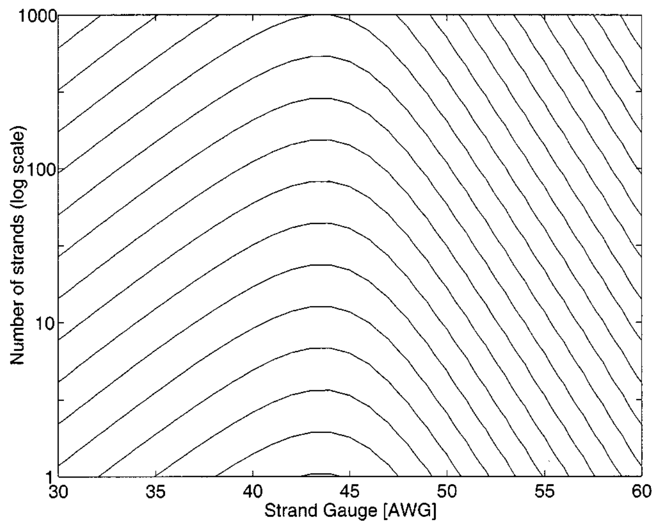


Fig. 3. Equal-cost contour lines.

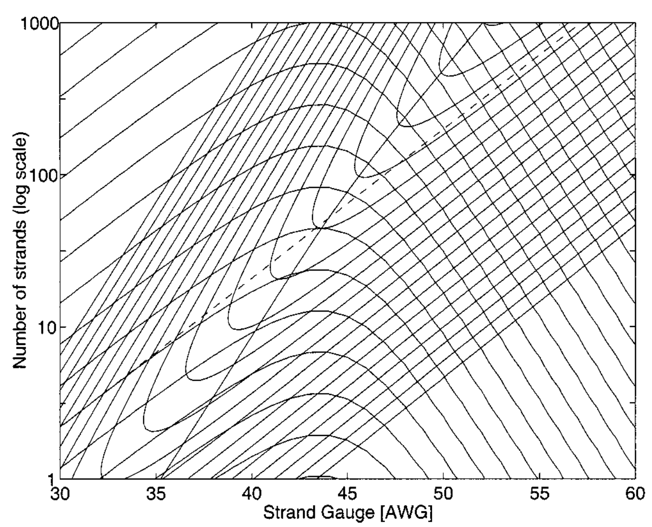


Fig. 4. Equal-cost contours shown with equal-loss contours. Designs with an optimal cost/loss tradeoff are found at points where lines from these two sets are tangent. The diagonal solid line curving up from the lower left indicates these points. The dotted line indicates a full-bobbin constraint.

In this section, we explore this strand diameter/number space graphically, using the approximate curve-fit cost function (4). An algebraic derivation of equivalent but more general results, independent of the particular cost function (4), is provided in the Appendix.

We can represent the total cost, given by (3) and (4), as a set of contour lines in the size-of- and number-of-strands space (Fig. 3). These are curves of constant cost, having shapes that can be understood by considering the shape of the dashed cost-per-unit-length curve in Fig. 2. As the size approaches 44 AWG, the cost of the wire per unit length decreases, so the number of strands that can be bought for the same price increases. Thus, the curves in Fig. 3 go to a maximum number

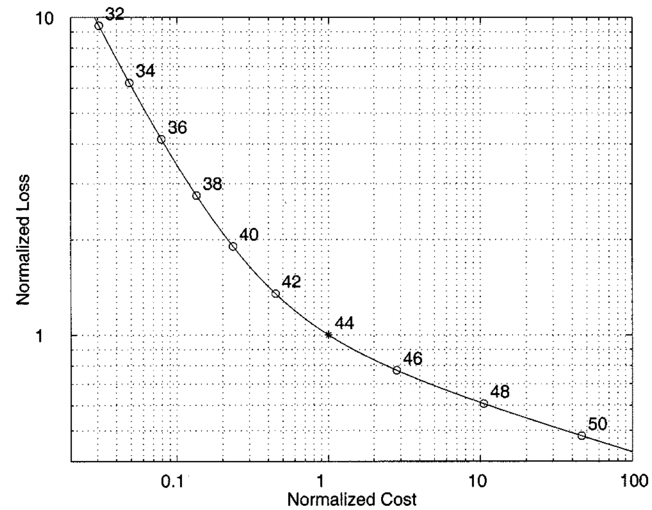


Fig. 5. Cost and loss, normalized to an optimal cost/loss design using 44 AWG strands. This graph applies to any design in which the bobbin is not full, given the cost function (4). Points are indexed with the AWG strand size used. Note that a point on this graph does not represent the minimum-loss design for that strand gauge; rather, it represents the minimum-loss design at a given cost; the strand size used to achieve this is indicated.

of strands at the wire gauge where cost per unit length is a minimum.

Along any given constant-cost curve, the best design choice is the point giving minimum loss. In Fig. 4, contour lines for loss are shown with the cost contours from Fig. 3. These are based on an example design of a 14-turn winding on an RM5 size ferrite core, with 1 MHz current in the winding. The breadth of the bobbin is 4.93 mm, and the breadth of the core window 6.3 mm. The loss is proportional to $F_r R_{\rm dc}$, and so the loss contours can be computed from (2) and a simple dc resistance calculation. On each cost contour, the tangent point to the set of loss contours is the minimum loss point. This set of points is also the set of minimum cost points for any given loss constraint. The set of these points is also shown in Fig. 4. The same set of points can be plotted on axes of cost and loss, from which a designer may chose the appropriate tradeoff (Fig. 5).

The cost/loss tradeoff curves, such as in Fig. 5, have the same shape regardless of design parameters. Thus, normalized to the loss and cost for the same reference strand diameter, they are identical to the curve in Fig. 5, where cost and loss are normalized to that for 44 AWG strands. This curve can be used to evaluate the cost/loss tradeoffs in any design as long as the bobbin is not full. Note that a point on this graph does not represent the minimum-loss design for that strand gauge; rather, it represents the minimum-loss design at a given cost; the strand size used to achieve this is indicated.

The remaining information needed to realize a design for any given point chosen on Fig. 5 can be provided in the form of a plot of $F_r$ values for optimal cost/loss designs (Fig. 6). Like Fig. 5 (but unlike Fig. 4), Fig. 6 shows general results that apply to any transformer design, in the region where the bobbin is underfilled. The results depend only on the cost function, (4).

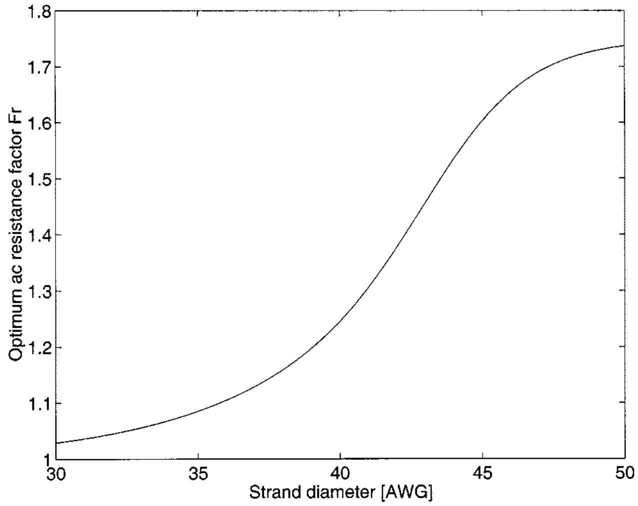


Fig. 6. AC-resistance factor $F_r$ for optimal cost/loss tradeoff designs as a function of strand diameter. These data are valid for any geometry or frequency, given the cost function modeled by (4).

TABLE I
PARAMETERS FOUND FOR OPTIMAL COST/LOSS DESIGNS USING STANDARD STRAND SIZES

| strand gauge (AWG) | relative cost | relative loss | $F_r$ for optimal cost/loss tradeoff |
|---|---|---|---|
| 32 | 0.031 | 9.4 | 1.045 |
| 34 | 0.049 | 6.22 | 1.068 |
| 36 | 0.079 | 4.14 | 1.104 |
| 38 | 0.131 | 2.80 | 1.161 |
| 40 | 0.234 | 1.90 | 1.246 |
| 42 | 0.45 | 1.35 | 1.376 |
| 44 | 1 | 1 | 1.535 |
| 46 | 2.83 | 0.77 | 1.655 |
| 48 | 10.5 | 0.61 | 1.715 |
| 50 | 46 | 0.48 | 1.737 |

In the Appendix, the results shown in Figs. 5 and 6 are derived analytically. To plot the equivalent of Fig. 6, we can use

$$F_{r,\mathrm{CL}}(d_c) = 1 + \frac{1}{1 - \frac{2C_m(d_c)}{C_m'(d_c)d_c}} \tag{5}$$

with any given cost function $C_m(d_c)$.

Also from the Appendix

$$C_1 = \frac{1}{\sqrt{\zeta}} \frac{C_m(d_c)}{d_c} \sqrt{F_{r,\mathrm{CL}}(d_c) - 1} \tag{6}$$

where $C_1$ is the cost with the constant term $C_0$ subtracted, and

$$\text{total loss } \alpha\, d_c \frac{F_{r,\mathrm{CL}}(d_c)}{\sqrt{F_{r,\mathrm{CL}}(d_c) - 1}}. \tag{7}$$

Equations (6) and (7) can be used with $d_c$ as a parameter to generate plots such as Fig. 5 for any cost function $C_m(d_c)$.

## V. DESIGN EXAMPLE

In this section, we illustrate the use of the above results with a design example; a general method will be outlined in Section VI.

The design example is a 30-turn to 30-turn transformer on a EC-70 ferrite core with a 150 kHz, 8 A rms sine-wave current in both windings. While for present purposes it is not necessary to know the voltage, we can, for the sake of concreteness, assume a 300 V square-wave voltage (600 V p-p), as would occur in a parallel-loaded resonant converter. This would lead to a flux amplitude of 60 mT, a core loss around 1.4 W in a typical power ferrite material, and a power output of 2160 W. The breadth of the core window is $b_c = 44.6$ mm; the bobbin allows a winding area of $b_w = 41.5$ mm by 24 mm high; each of the two windings may then take up a height of 12 mm.

A standard design procedure might be to start with a manufacturer's catalog, which recommends 40 AWG strand litz wire for the 100 to 200 kHz range. Fitting 30 turns in the allotted window area, we find the largest permissible standard bundle of 40 AWG strands has 1100 strands. An analysis of internal proximity effect losses [15], as outlined in wire manufacturers' application notes predicts a mild ac resistance factor of 1.19 for this construction, seemingly confirming the catalog recommendation. However, this is only correct for an isolated litz bundle, and it does not take into account the external proximity effect that dominates ac resistance in a typical transformer. Using (2) to accurately predict the ac resistance of this bundle, we obtain an ac resistance factor of $F_r = 9.2$. This leads to 5.6 W of loss in each winding, and a total temperature rise of 87 °C including both windings and the core loss, based on an empirical thermal resistance of 7°C/W [16].

The calculation used here (2) is not valid for strands much larger than a skin depth. Strands will very rarely be that large in a *good* litz design, but the design calculated above is far enough from a good design that checking is wise. The skin depth in copper at 150 kHz is about 0.17 mm—the diameter of 33 or 34 AWG wire—and so (2) is valid in the range of interest. Note that even for this poorly chosen design, the ac resistance is lower than it would be for any single-strand design; the optimum single-strand design in this case is a single-layer winding that would have almost triple the ac resistance of the first design.

We now apply the results obtained in Section IV to this transformer. First, we assume 44 AWG wire, and calculate the number of strands to obtain the corresponding ac resistance factor shown in Fig. 6 (also shown in Table I). We find $F_r = 1.535$ with 1131 strands of 44 AWG. Although this has higher dc resistance than the first design (1100 of #40), its overall ac resistance is 59% lower, and furthermore, the predicted relative cost is 25% lower.

Table II collects data on these and further designs. The cost and loss figures are shown normalized to both the original design based on manufacturers' data, and to this new optimal cost/lost design using 44 AWG wire. With this latter normalization, the cost/loss possibilities are mapped out by Fig. 5. One can now select, on this plot, the desired cost/loss tradeoff. For example, one could chose to keep the loss constant at the level in the original design, or could optimize for minimum total cost including the cost of the energy dissipated over the life of the equipment, and other costs that indirectly result from lower efficiency and higher heat production.

TABLE II
STRANDING OPTIONS FOR EXAMPLE DESIGN

| Design | Number of Strands | Strand Gauge | Loss, per Winding (watts) | Loss Normalized to: orig. des. | Loss Normalized to: 44 AWG des. | Predicted Cost Normalized to: orig. des. | Predicted Cost Normalized to: 44 AWG des. | Actual Cost Norm. to Orig. Des. Mfr. A | Actual Cost Norm. to Orig. Des. Mfr. B |
|---|---|---|---|---|---|---|---|---|---|
| Based on catalog rule of thumb | 1100 | 40 | 5.55 | 1 | 2.43 | 1 | 1.35 | 1 | 1 |
| Optimum cost/loss with #44 | 1131 | 44 | 2.28 | 0.41 | 1 | 0.74 | 1 | | |
| Closest catalog size | 1050 | 44 | 2.34 | 0.42 | 1.025 | 0.69 | 0.93 | 0.75 | 0.98 |
| Min. cost with original loss | 100 | 38 | 5.32 | 0.96 | 2.33 | 0.129 | 0.17 | 0.119 | 0.171 |
| Min. loss at any cost (theoretical) | 220,000 | 63 | 0.65 | 0.117 | 0.285 | 268,000* | 361,000* | | |
| An expensive but plausible low-loss design | 5200 | 48 | 1.39 | 0.25 | 0.61 | 7.7 | 10.3 | | |
| Single-layer single-strand | 1 | 16 | 15.1 | 2.72 | 6.62 | < 0.15* | < 0.2* | | |

* Indicates extrapolated values that are not expected to be accurate.

The designs in Table II include 100 strands of 38 AWG wire, for about the same loss as the original design at about 13% of the cost, and 1050 strands of 44 AWG, a standard catalog construction close to the calculated choice of 1131 strands for this size, and providing similar cost and loss reductions. With this design, the temperature rise would be reduced from the original 87 °C to 42.5 °C with no increase in cost.

For comparison, the minimum loss design calculated using the methods of [7] is also included in Table II; for this transformer, that method indicates that 220 000 strands of 63 AWG would produce the minimum loss. The cost estimate produced by (4) for this strand size is not expected to be at all accurate, but it is certain that the cost would be extreme if it were possible to produce such litz wire. However, if it were possible, the 63 AWG construction would allow reducing the loss to about one quarter the loss obtained with 1050 strands of 44 AWG wire. A 48 AWG design is included to illustrate a more practical high-cost, low-loss construction.

Also included on Table II are the predicted relative cost based on (4) and the actual relative cost based on quotes from two manufacturers. These quotes were obtained separately from the quotes used to generate the curve fit in (4), and so provide an opportunity to independently assess the accuracy that can typically be expected from (4). For the 38 AWG design, the two manufacturers' normalized costs, at 0.17 and 0.12, differ by 35%, and the estimate of 0.13 falls between them. For the 1050-strand 44 AWG design, the two manufacturers' normalized costs, at 0.98 and 0.75, differ by 27%, and the estimate of 0.69 is below either actual cost, off by 35% or 8%. Overall, one should not count on (4) to give cost predictions accurate to better than 35%. Rather it should be used as a guide to general trends. As described in Section III, the smooth curve of (4) is an idealization that does not include the many quirks that one can find in a particular supplier's pricing. A wise designer will explore these issues with a supplier. If a slightly smaller number of strands allow the use of a different machine, or a slightly higher number of strands would be a standard product produced and stocked in large volumes, there could be cost savings opportunities that are not captured in this analysis. Looking into possible adjustments like this after finding the theoretical optimum design as described here is recommended.

Another limitation on the accuracy of the cost predictions is that in (3), we dropped the constant portion of the cost. This does not affect the loci of the optima, but it does affect the accuracy of the predicted prices, and determining this constant for the quoted prices could improve the accuracy our predictions. But given that the maximum error in our predictions, 35%, is equal to the maximum difference between the normalized pricing from the two manufacturers, a great improvement in cost accuracy could not be expected. In any case, we have confirmed the usefulness of the model and methodology to reduce cost, loss, or both. In particular, the 1050 strand 44 AWG design achieves a 58% loss reduction at less than the original cost, and the 100 strand 38 AWG design achieves under one-fifth the original cost at the same loss.

The total cost including energy was also evaluated for this design, assuming continuous operation and an energy price of US$0.1/kWh. Annual costs and capital costs were compared using at capital recovery rate of 0.15, representing, for example, a 10 year life with a discount rate of 8.5%. This results in the present value of total energy cost being US$5.84 for each watt of dissipation. The AWG 44 design then has the lowest sum of wire and energy cost for this particular example. This analysis ignores other costs associated with the extra dissipation, ranging

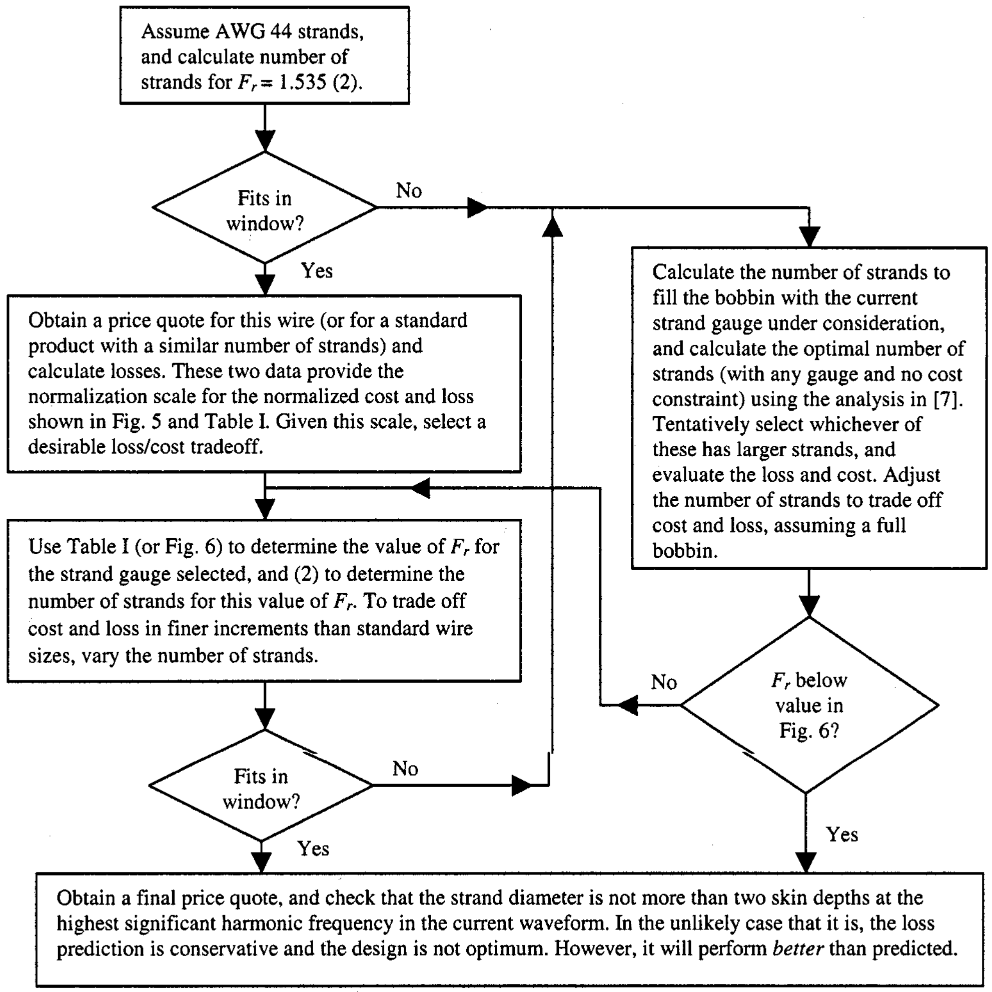


Fig. 7. Design procedure that allows the user to choose cost/loss tradeoff and guarantees minimum loss for the selected cost (and the lowest cost for that loss).

from the heating effect that may degrade reliability or may increase costs by requiring additional heatsinking, fans, etc., to the environmental impacts of the electric power generation. Including these costs could show a lower-loss design with higher wire cost to be worthwhile.

## VI. Design Procedure

A flowchart for a recommended design procedure is shown in Fig. 7. This procedure will provide designs with the minimum loss for any given cost (and the lowest cost for that loss), making use of the data presented in previous plots, and collected for standard strand sizes in Table I. The procedure can be implemented on a computer; however, it cannot be completely automated, as it requires the user to make decisions regarding the cost/loss tradeoff. In addition, consulting a manufacturer to obtain actual current price quotes is valuable, and in cases with a full bobbin, it may be necessary to experimentally measure packing factor.

The choice of construction under the constraint of available wire sizes is explored further in Fig. 8, which includes the ideal cost/loss tradeoff curve of Fig. 5, but also has curves for each wire size. It is apparent that the exact wire size is much less important for smaller gauge numbers (below 40 AWG)—similar cost and loss performance is available with nearby sizes. However, with finer wire, there is more incentive to consider an odd strand size. The actual cost of the wire with an odd strand size may depend on the quantity purchased, and so it is not possible here to determine when it is economically advantageous. But Fig. 8 highlights where it is worth considering.

## VII. Conclusion

Combined analysis of loss and cost of litz-wire windings can lead to substantial improvements in cost, loss, or both. The analysis leads to general expressions describing the relationship between cost and loss in optimal designs, in terms of a cost function. In addition, this cost function can be approximated by a polynomial, leading to numerical data that facilitates a simple design process that leads to minimum loss designs at any given cost, or minimum cost designs for any given loss.

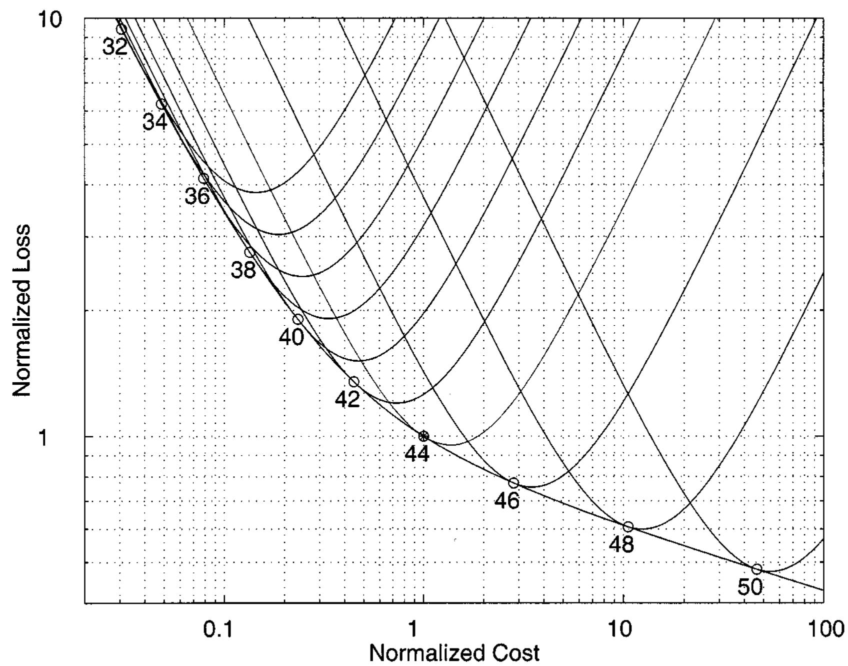


Fig. 8. Cost and loss, normalized to an optimal cost/loss design using 44 AWG strands. The ideal relationship shown as the bottom curve assumes any strand diameter is available. Curves for individual even wire sizes are also plotted to show the penalty for using a standard wire size. For large diameter wire, the curves are close to one another, indicating that the exact choice of diameter is unimportant. However, for fine wire, the choice of a standard even size may entail a significant penalty.

## APPENDIX I
## LOSS CALCULATION

The origin of the expression used for $F_r$, (2), is discussed in [7], and is reviewed here for the reader's convenience. First consider loss in a conducting cylinder in a uniform field perpendicular to the axis of the cylinder, with the assumption that the field remains constant inside the conductor, equivalent to the assumption that the diameter is small compared to a skin depth. This results in power dissipation $P$ in a wire of length $\ell$

$$P = \frac{\pi \omega^2 \ell B^2 d_c^4}{128 \rho_c} \tag{8}$$

where $B$ is the peak flux density (see, for example, [17] for a more detailed derivation of this expression). This is equal to the first term of an expansion of the exact Bessel-function solution [18].

Combining this with the assumption of a trapezoidal field distribution in a winding results in (2). The linear increase of the field across the winding is a result of considering the effect of all the current in the winding; separating the effect of other strands within a particular bundle from the effect of other bundles is not required and would only complicate the calculation [7]. For configurations in which the field is not zero at one edge of the winding, a factor $k = (1-\varphi^3)/(1-\varphi)^3$ is used to account for the resulting change in losses, where $\varphi = B_{\min}/B_{\max}$ [4]. We assume equal current sharing between the strands in the litz wire. This is a good approximation if the construction of the litz wire has been chosen to control bundle-level skin effect and proximity effect [7].

## APPENDIX II
## DERIVATION OF OPTIMAL COST-LOSS CURVE

Lumping constant terms as $\zeta$, we can express (2) as

$$F_r = 1 + \zeta n^2 d_c^6. \tag{9}$$

At a given cost, $C_1$, we wish to find the choice of $n$ and $d_c$ that gives minimum total loss. Total loss is proportional to total resistance factor $F_{rt}$

$$F_{rt} = F_{\mathrm{dc}} F_r = \frac{\text{ac resistance of litz-wire winding}}{\text{dc resistance of single-strand winding}} \tag{10}$$

where $F_{\mathrm{dc}}$ is the ratio of dc resistance of the litz wire to the dc resistance of a single strand winding, using wire with the same diameter as the litz-wire bundle. Based on this definition

$$F_{rt} = \left(1 + \zeta n^2 d_c^6\right) \frac{d_{c\,\mathrm{ss}}^2}{d_c^2 n} \tag{11}$$

where $d_{c\,\mathrm{ss}}$ is the diameter of the largest single-strand wire that would fit. This constant may be dropped for the purposes of optimization; we work with

$$F_{rt} \; \alpha \; \frac{1}{d_c^2 n} + \zeta n d_c^4. \tag{12}$$

To minimize total loss, holding cost constant, we can eliminate $n$ from (12) by using (3), to obtain

$$F_{rt} \; \alpha \; \frac{C_m(d_c)}{C_1} + \frac{\zeta d_c^2 C_1}{C_m(d_c)} \tag{13}$$

where $C_1$ is the specified cost, with the constant term $C_0$ in (3) subtracted. Setting the derivative of this expression with respect to $d_c$ equal to zero, we obtain

$$\frac{C_m'(d_c)C_1^2}{C_m^2(d_c)}\zeta d_c^2 - 2\frac{C_1^2}{C_m(d_c)}\zeta d_c - C_m'(d_c) = 0 \tag{14}$$

where $C_m'(d_c)$ is the derivative of $C_m(d_c)$ with respect to $d_c$.

Given a representation of $C_m(d_c)$ and a cost specification $C_1$, (14) could be numerically solved for a optimum value of $d_c$. However, it is possible to derive several more general results that provide additional insight and lead to Figs. 5 and 6. Solving (14) for $\zeta$, substituting that result into (11), and again eliminating $n$ using (3) leads to

$$F_{r,\mathrm{CL}}(d_c) = 1 + \frac{1}{1 - \frac{2C_m(d_c)}{C_m'(d_c)d_c}}. \tag{15}$$

This expression describes the relationship between wire size and the optimal cost/loss value of $F_r$, denoted $F_{r,\mathrm{CL}}(d_c)$, as shown in Fig. 6. The generality of the result is indicated by the independence of (15) from the design details lumped in the constant $\zeta$.

The generality of the relationship shown in Fig. 5 can be seen as follows. From (14)

$$C_1 = \frac{C_m(d_c)}{\sqrt{\zeta d_c^2\left(1 - \frac{2C_m(d_c)}{d_c C_m'(d_c)}\right)}} \tag{16}$$

or

$$C_1 = \frac{1}{\sqrt{\zeta}}\frac{C_m(d_c)}{d_c}\sqrt{F_{r,\mathrm{CL}}(d_c) - 1}. \tag{17}$$

Using the relationship $(1/d_c^2 n) = (C_m(d_c)/C_1)$, we can write

$$F_{rt,\mathrm{CL}} = F_{r,\mathrm{CL}}(d_c)\frac{C_m(d_c)}{C_1}d_{c\,\mathrm{ss}}^2. \tag{18}$$

Thus, using (17)

$$F_{rt,\mathrm{CL}} = d_{c\,\mathrm{ss}}^2\sqrt{\zeta}\, d_c\frac{F_{r,\mathrm{CL}}(d_c)}{\sqrt{F_{r,\mathrm{CL}}(d_c) - 1}}. \tag{19}$$

If (19) and (17) are normalized as in Fig. 5, the constants specific to a particular design problem, $\zeta$ and $d_{c\,\mathrm{ss}}$, drop out. Thus, with $d_c$ as a parameter, (19) and (17) can be used to plot a curve of normalized cost and loss for any given cost function, as shown in Fig. 5 for (4).

## REFERENCES

[1] J. A. Ferreira, "Improved analytical modeling of conductive losses in magnetic components," *IEEE Trans. Power Electron.*, vol. 9, pp. 127–131, Jan. 1994.

[2] A. W. Lotfi and F. C. Lee, "A high frequency model for litz wire for switch-mode magnetics," in *Proc. Conf. Rec. 1993 IEEE Ind. Applicat. Conf. 28th IAS Annu. Meeting*, vol. 2, Oct. 1993, pp. 1169–1175.

[3] M. Bartoli, N. Noferi, A. Reatti, and M. K. Kazimierczuk, "Modeling litz-wire winding losses in high-frequency power inductors," in *Proc. 27th Annu. IEEE Power Electron. Spec. Conf.*, vol. 2, June 1996, pp. 1690–1696.

[4] E. C. Snelling, *Soft Ferrites, Properties and Applications*, 2nd ed. London, U.K.: Butterworth, 1988.

[5] P. N. Murgatroyd, "Calculation of proximity losses in multistranded conductor bunches," *Proc. Inst. Elect. Eng. A*, vol. 36, no. 3, pp. 115–120, 1989.

[6] B. B. Austin, "The effective resistance of inductance coils at radio frequency," *Wireless Eng.*, vol. 11, pp. 12–16, Jan. 1934.

[7] C. R. Sullivan, "Optimal choice for number of strands in a litz-wire transformer winding," *IEEE Trans. Power Electron.*, vol. 14, pp. 283–291, Mar. 1999.

[8] P. S. Venkatraman, "Winding eddy current losses in switch mode power transformers due to rectangular wave currents," in *Proc. Powercon 11*, 1984, pp. 1–11.

[9] J. P. Vandelac and P. Ziogas, "A novel approach for minimizing high frequency transformer copper losses," in *Proc. IEEE Power Electron. Spec. Conf. Rec.*, 1987, pp. 355–367.

[10] P. N. Murgatroyd, "The torroidal cage coil," *Proc. Inst. Elect. Eng. B*, vol. 127, no. 4, pp. 207–214, 1980.

[11] S. Crepaz, "Eddy-current losses in rectifier transformers," *IEEE Trans. Power Apparat. Syst.*, vol. PAS-89, pp. 1651–1662, July 1970.

[12] J. M. Lopera, M. J. Prieto, F. Nuno, A. M. Pernia, and J. Sebastian, "A quick way to determine the optimum layer size and their disposition in magnetic structures," in *Proc. 28th Annu. IEEE Power Electron. Spec. Conf.*, 1997, pp. 1150–1156.

[13] W. G. Hurley, E. Gath, and J. G. Breslin, "Optimizing the AC resistance of multilayer transformer windings with arbitrary current waveforms," *IEEE Trans. Power Electron.*, vol. 15, pp. 369–376, Mar. 2000.

[14] J. Hu and C. R. Sullivan, "Optimization of shapes for round-wire high-frequency gapped-inductor windings," in *Proc. 1998 IEEE Ind. Applicat. Soc. Annu. Meeting*, 1998, pp. 900–906.

[15] F. E. Terman, *Radio Engineer's Handbook*. New York: McGraw-Hill, 1943.

[16] S. A. Mulder, "Application note on the design of low-profile high-frequency transformers," Tech. Rep., Phillips Components Lab, May 1990.

[17] C. R. Sullivan, "Computationally efficient winding loss calculation with multiple windings, arbitrary waveforms, and two- or three-dimensional field geometry," *IEEE Trans. Power Electron.*, vol. 16, pp. 142–150, Jan. 2001.

[18] J. Lammeraner and M. Stafl, *Eddy Currents*, G. A. Toombs, Ed. Orlando, FL: CRC, 1966.

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