Cross Regulation Mechanisms

in Multiple-Output Forward and Flyback Converters

Bob Erickson and Dragan Maksimovic

Colorado Power Electronics Center (CoPEC)

University of Colorado, Boulder 80309-0425

http://ece-www.colorado.edu/~pwrelect

Cross Regulation Mechanisms

in Multiple-Output Forward and Flyback Converters

Design of transformer in development of a multiple-output supply:

- Typically requires substantial engineering effort
- Can represent the largest risk to success of the project
- There is a need for increased understanding of the mechanisms that govern behavior of multiple-output converters

An old problem that has never been adequately addressed in the literature

- Ideal transformers are typically assumed
- Only conduction losses are modeled
- Reduced-order magnetics models do not predict observed waveforms
- Problem is considered intractable

Objectives of this seminar

Explain the magnetics-associated mechanisms that govern cross-regulation in forward and flyback converters, including

- peak detection
- discontinuous conduction mode
- effects of voltage-clamp snubbers

Describe magnetics models suitable for cross-regulation analysis

Approximate analytical expressions and computer simulation

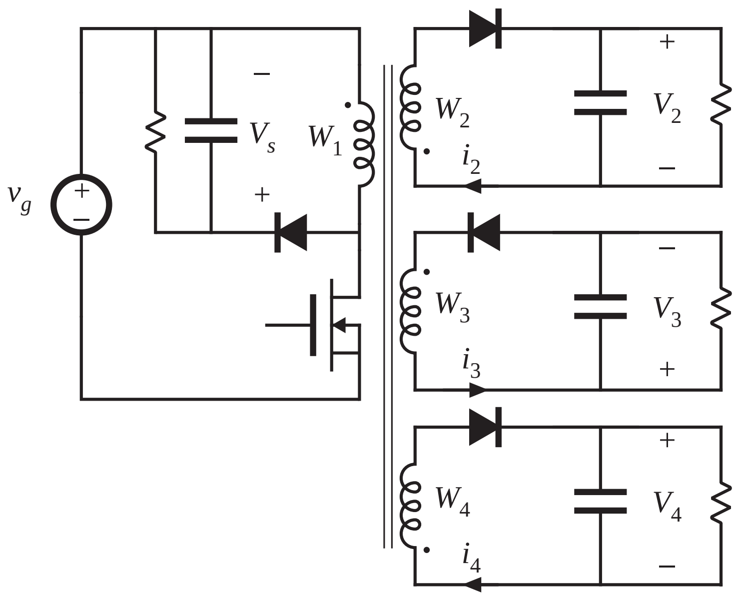
Predict small-signal dynamics

Include laboratory measurement methods and experimental examples

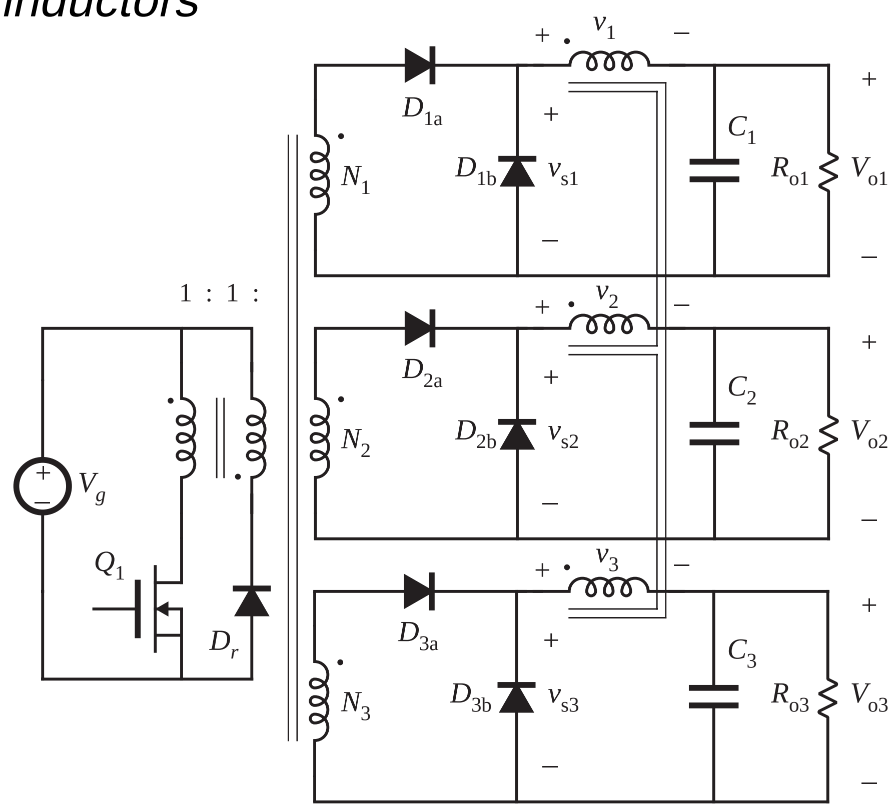
Modeling Multiple-Output Converters

Cross regulation, CCM/DCM boundaries, dynamics

Flyback converter



Forward converter with coupled inductors



Some ways to view multiple-output converters

With ideal transformer

- Does not predict effect of leakage inductances on cross regulation
- Does not predict change of operating mode
- Does not predict observed converter transfer functions
- Predicts that isolated converters behave as their non-isolated parent converter topologies

With actual transformer and its leakage inductances

- Analysis previously viewed as intractable, with no hope of gaining physical insight into cross-regulation mechanisms (not so!)
- Waveforms of isolated converters can differ significantly from their non-isolated parent topologies —new phenomena that are not observed in non-isolated versions
- Some outputs may operate in continuous conduction mode (CCM) while others operate in discontinuous conduction mode (DCM)
- To correctly predict dynamics and conduction losses: must account for leakage inductances

Cross regulation mechanisms

Conduction loss

- Diode forward-voltage drops
- Resistances of windings

Modeling conduction loss

- Existing averaged modeling methods only partially apply

Effect of magnetics

- Leakage inductances control shapes of winding current waveforms, especially slopes of winding currents
- Leakage inductances have a first-order effect on cross regulation, as well as dynamics, operating mode, and conduction losses

What we would like to know
and what kinds of answers to expect

We would like to express the output voltages as functions of the output currents

For example: three-output converter

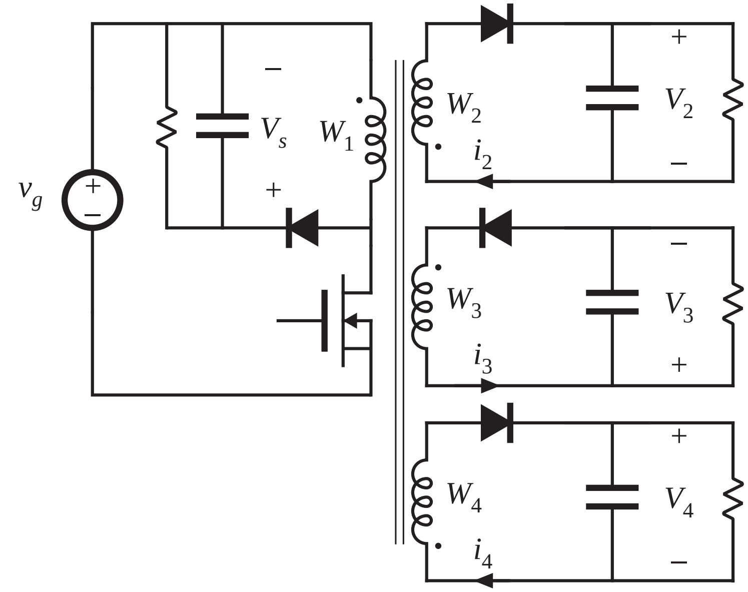
CCM matrix equation, 3x3

$$\begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} R_{22} & R_{23} & R_{24} \\ R_{23} & R_{33} & R_{34} \\ R_{42} & R_{42} & R_{44} \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} V_{o2} \\ V_{o3} \\ V_{o4} \end{bmatrix}$$

Many outputs $\Rightarrow$ many equations

Similar comments for CCM/DCM mode boundaries

Flyback converter example



What we would like to know

and what kinds of answers to expect, p. 2

1. A correct transformer model
   - That is well-suited to analysis of the cross-regulation problem
   - Whose parameters can be directly measured
   - That correctly predicts observed waveforms
2. Equations
   - Of output voltage regulation
   - Of mode boundaries
   - Might be best evaluated by computer when there are many auxiliary outputs
3. Insight
   - Describe fundamental operation of transformer-isolated multiple-output converters
   - Explain the physical mechanisms that lead to poor cross-regulation

Outline of Discussion

1. Transformer Modeling—in context of cross regulation
   - Discussion of transformer models
   - The extended cantilever model
   - The *n*-port model
2. Cross Regulation in Flyback Converters
   - Qualitative behavior
   - Analytical results
   - Laboratory example
   - Discussion of strategies for improvement of cross regulation
   - Dynamic response
3. Cross Regulation in Forward Converters
   - Coupled-inductor approaches, and their qualitative behavior
   - Analytical results
   - Laboratory example
   - Discussion of strategies for improvement of cross regulation
   - Dynamic response

1. Multiple-Winding Transformer Modeling
in the context of the cross regulation problem

Multiple-output converters are more than simple extensions of parent single-output nonisolated converters:

- Imperfect coupling between windings leads to problems in cross regulation, small-signal dynamics, and multiple operating modes, which have not been fully explored in the literature
- These phenomena are governed primarily by the transformer leakage inductance parameters

Need a suitable multiple-winding transformer model

- that predicts observed waveforms
- that yields insight into converter cross regulation, CCM/DCM boundaries, and dynamics
- that explains how converter performance depends on winding geometry
- that is useful in computer simulation

Approaches to Multiple-Winding Transformer Modeling

Inductance matrix

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{12} & L_{22} & L_{23} & L_{24} \\ L_{13} & L_{23} & L_{33} & L_{34} \\ L_{14} & L_{24} & L_{34} & L_{44} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

- General
- Reduces the circuit to matrix equations
- Numerically ill-conditioned in tightly-coupled case
- Complete model of four-winding transformer contains ten parameters

Equivalent-Circuit Approaches to Multiple-Winding Transformer Modeling

Reduced-order equivalent circuit

- Physically based
- Not general—Does not predict observed waveforms of flyback converter
- Difficult to apply to some geometries (for ex., toroidal)

Full-order equivalent circuits

- Allow circuit-oriented analysis of converter
- General

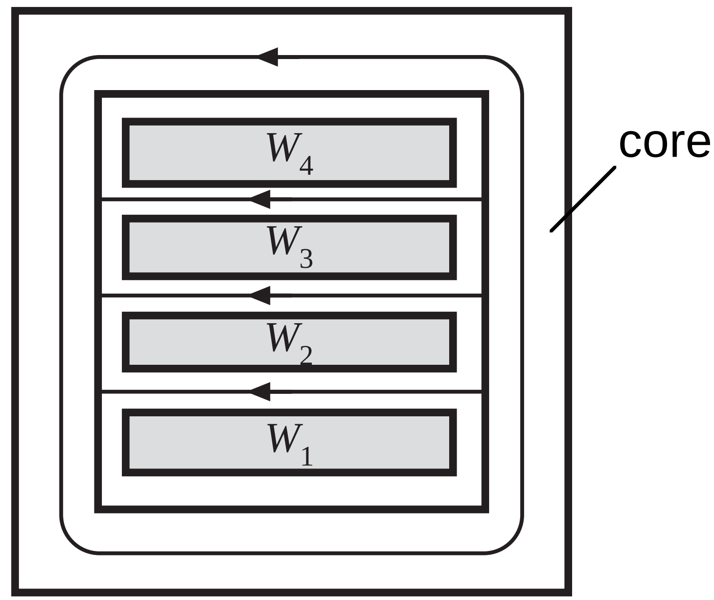
Flyback example

Physical-Based Reduced-Order Model
4 winding transformer example

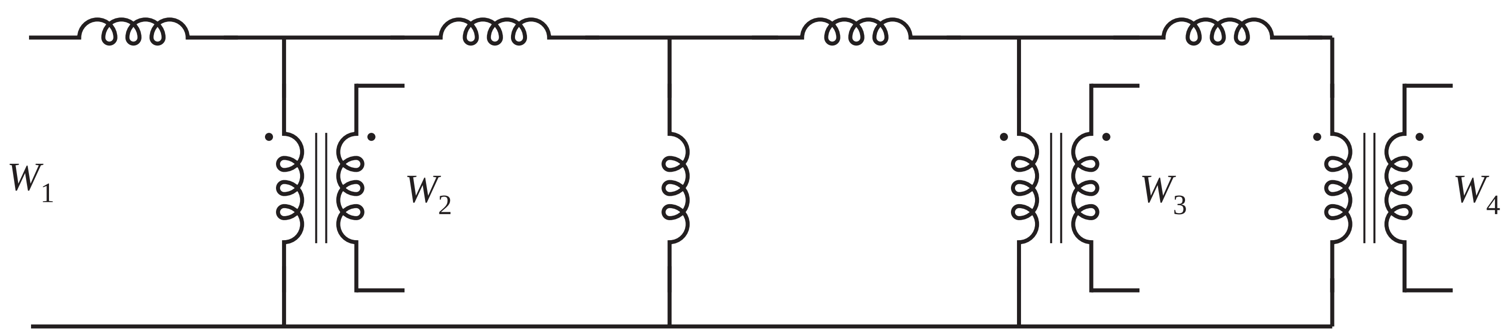
Four-winding transformer example

Physical modeling approach: equivalent circuit contains series-connected leakage inductances

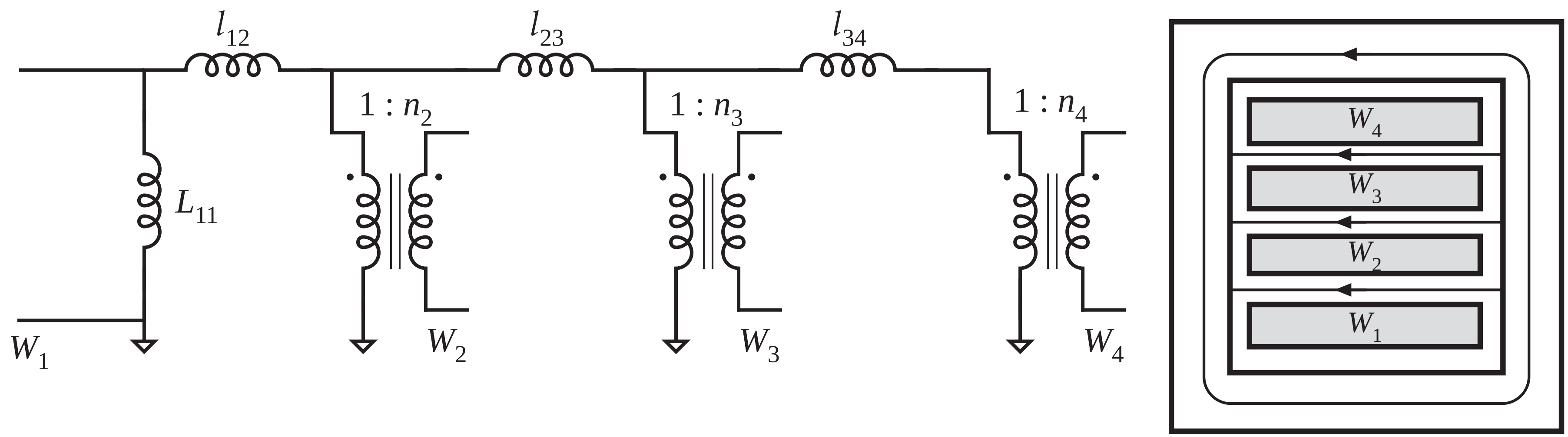
Reduced-order approximation based on winding geometry



Equivalent circuit proposed in [12]:

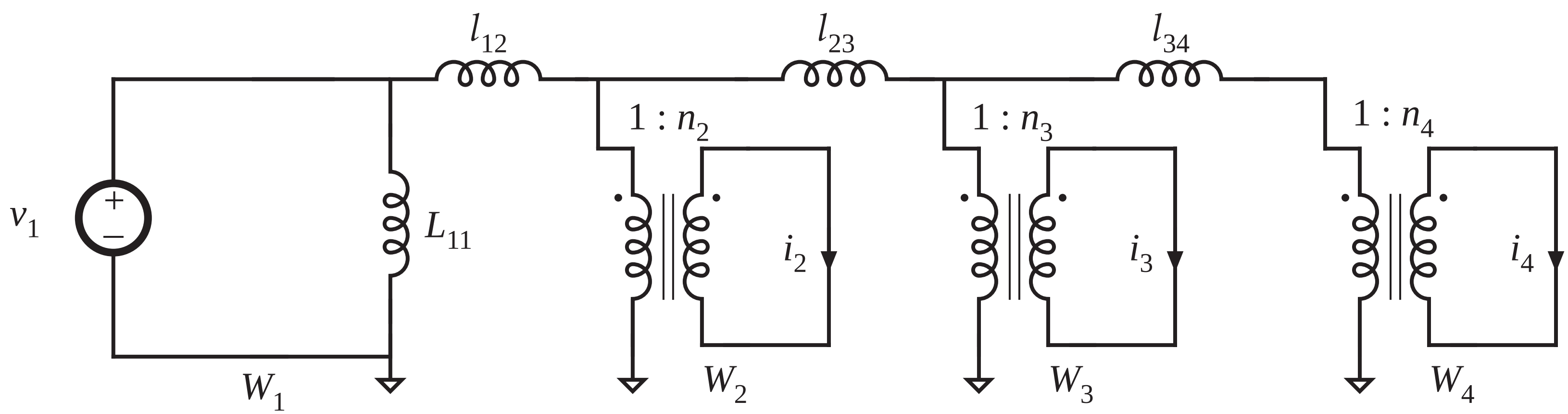


An Electrically-Equivalent Form
of the reduced-order model



- Contains seven independent parameters
- Inductance matrix of four-winding transformer contains ten independent parameters
- Is this model sufficient?

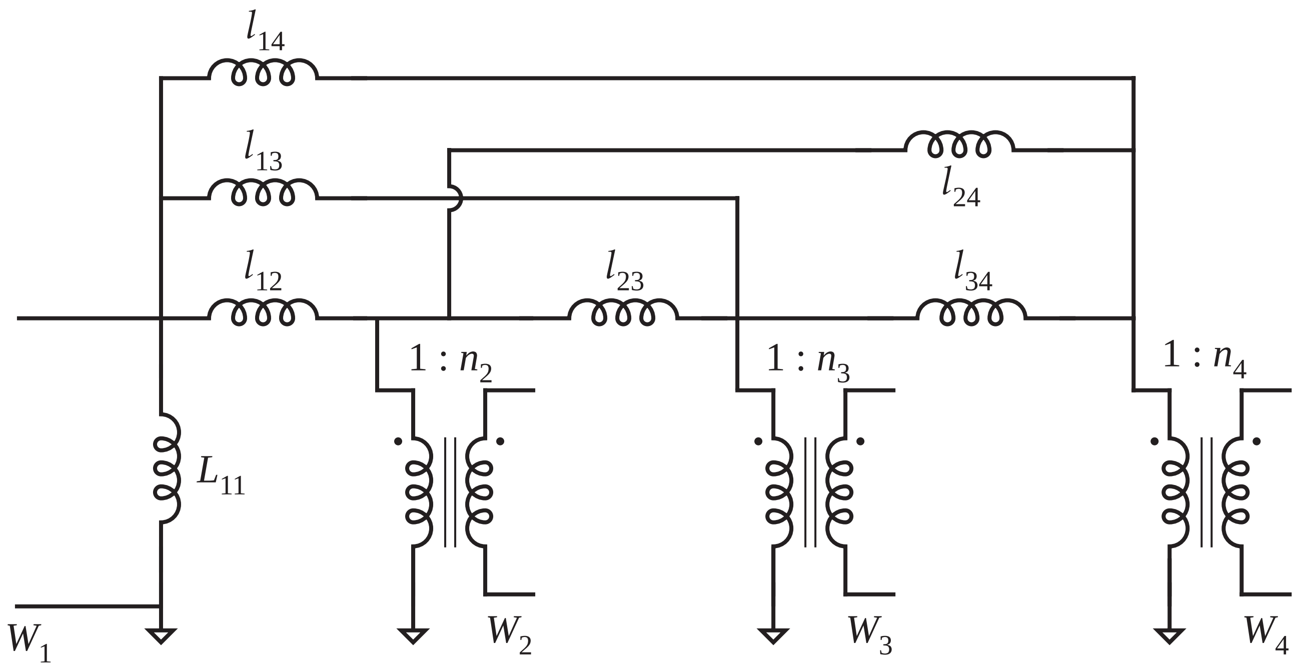
A Thought Experiment
using the 4-winding reduced-order model



Apply a voltage to winding 1, short windings 2, 3, and 4. Measure short-circuit currents in each winding.

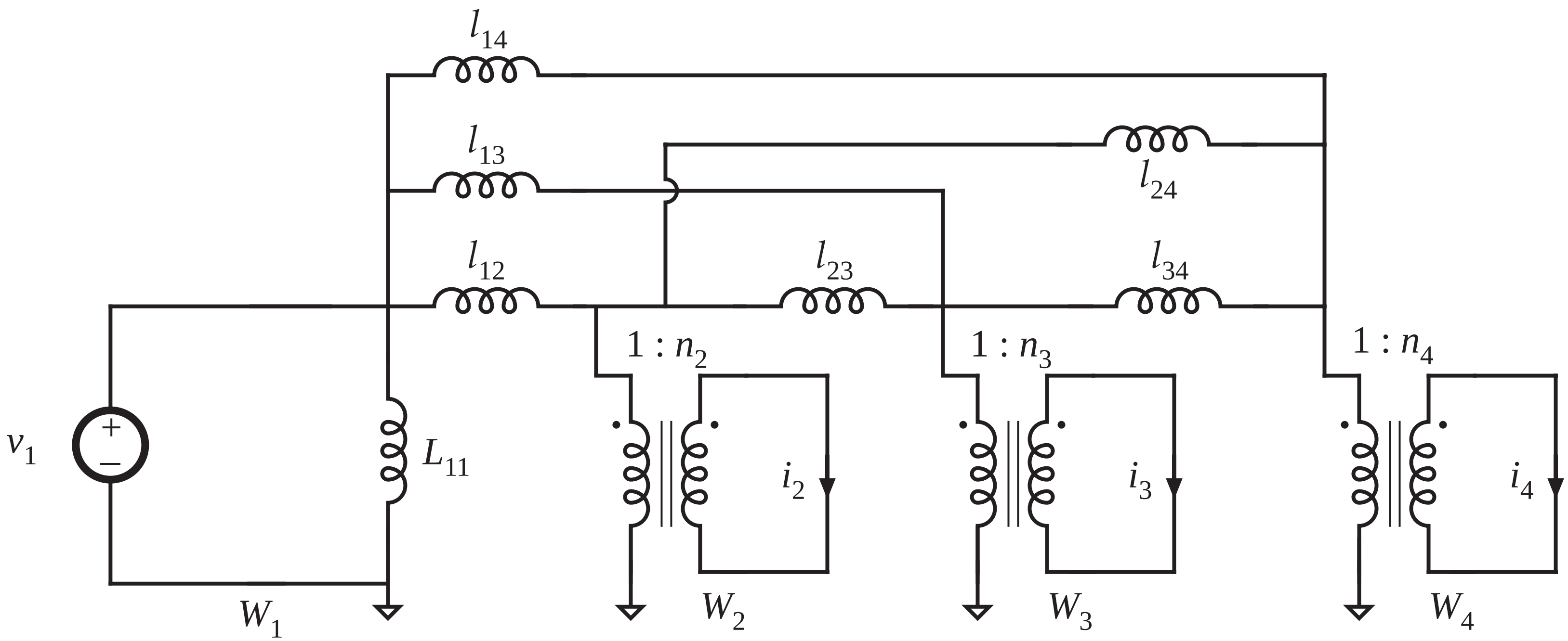
Model predicts that i_3 and i_4 are zero.

A Full-Order Model
Extended Cantilever Model



Include “leakage inductances” between each winding

Thought Experiment—Revisited



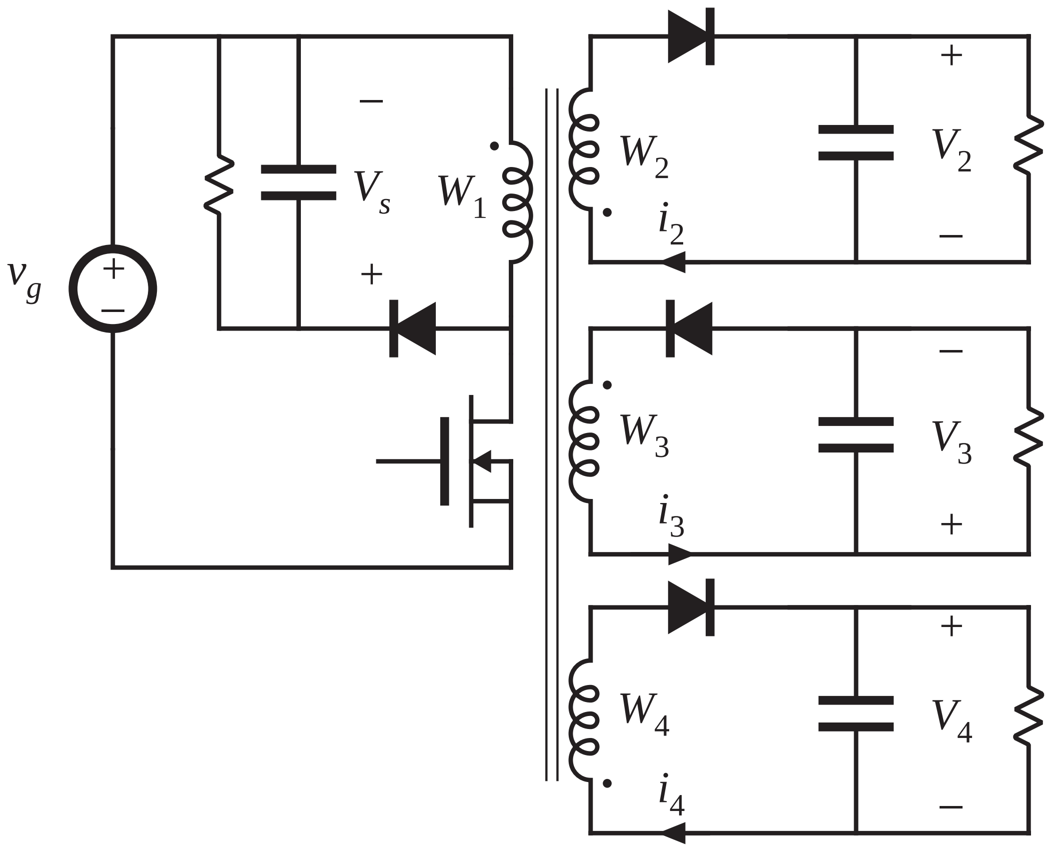
Again apply a voltage to winding 1, short windings 2, 3, and 4.
Measure short-circuit currents in each winding.

Model predicts nonzero i_3 and i_4.

Discussion

- It is always possible to connect a transformer such that a reduced-order model does not predict the actual waveforms
- Are such connections actually encountered in multiple-output converters?

Flyback converter circuit



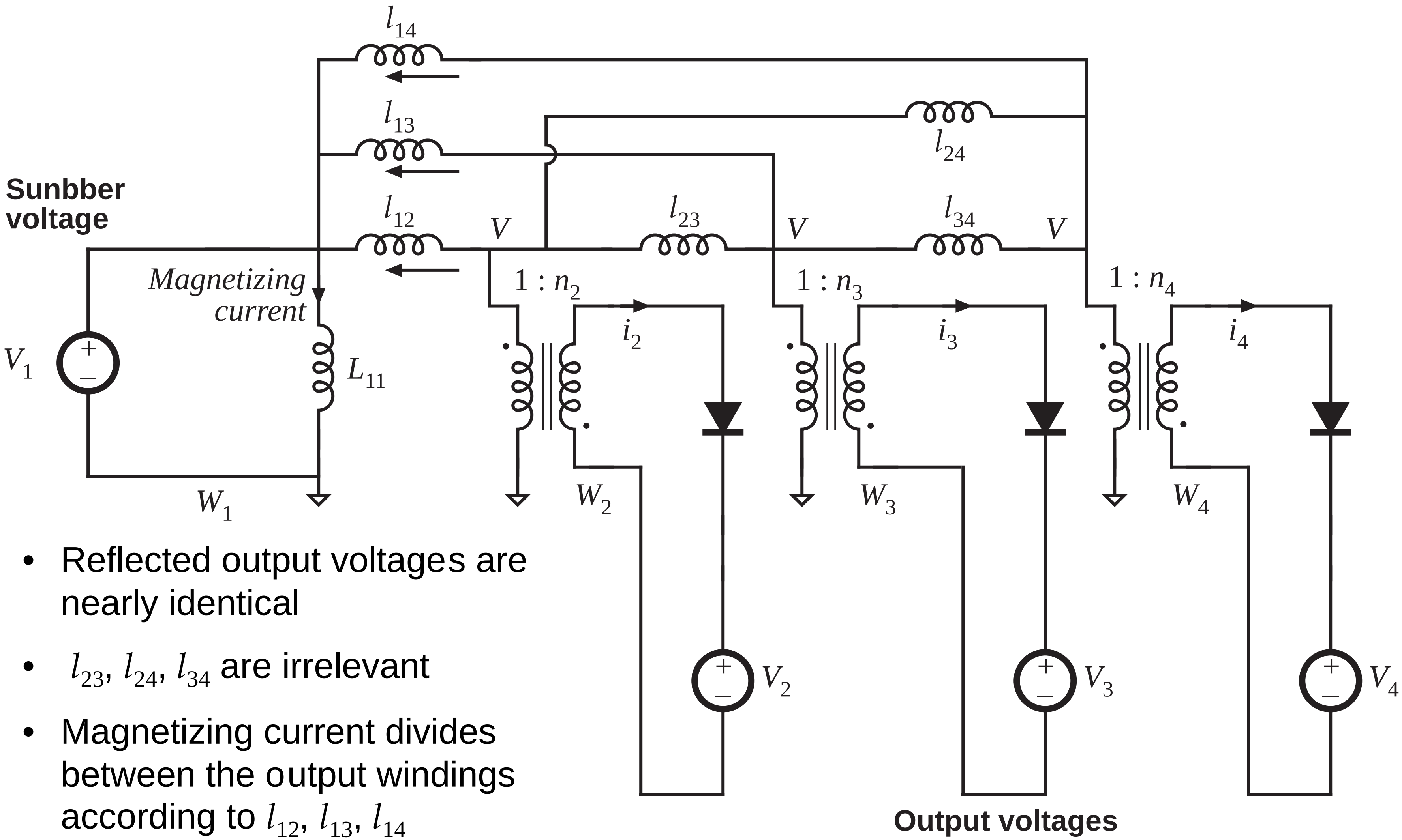
Three outputs

Primary-side voltage-clamp snubber

Cross-regulation is strongly influenced by *commutation interval*, when transistor turns off and magnetizing current shifts to secondary windings

Commutation Interval

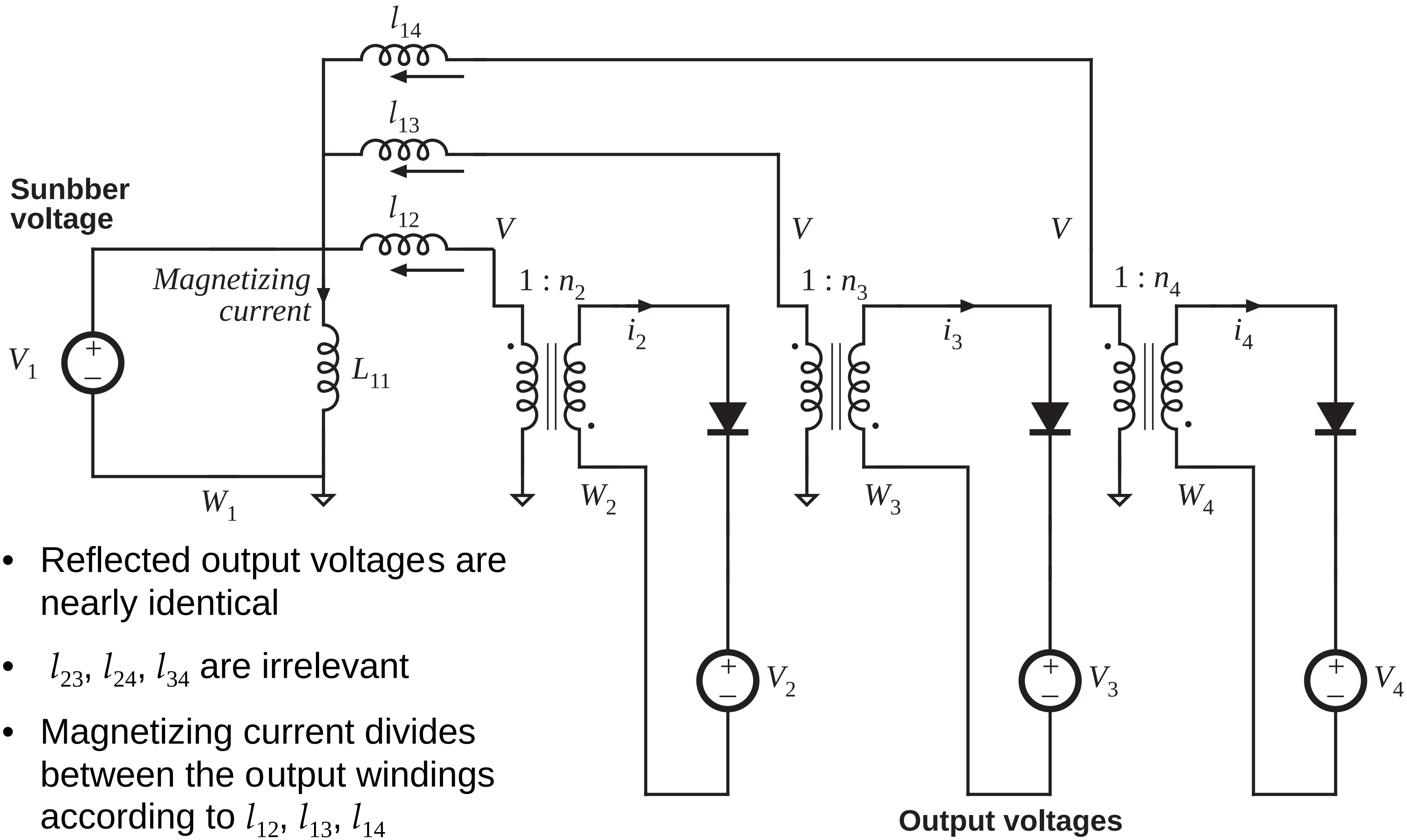
Flyback converter example—similar to thought experiment



- Reflected output voltages are nearly identical
- l_{23}, l_{24}, l_{34} are irrelevant
- Magnetizing current divides between the output windings according to l_{12}, l_{13}, l_{14}

Commutation Interval

Flyback converter example—similar to thought experiment



- Reflected output voltages are nearly identical
- l_{23}, l_{24}, l_{34} are irrelevant
- Magnetizing current divides between the output windings according to l_{12}, l_{13}, l_{14}

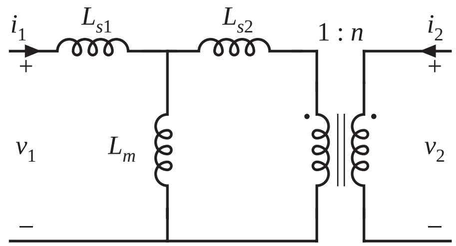
Conclusion: Reduced-Order Modeling

- Approximate reduced-order model derived via physical approach does not correctly predict behavior of multiple-output flyback converter
- Approximations must not be based solely on winding geometry
- Application and circuit behavior must be considered before attempting to reduce the order of the model
- Need a suitable full-order model

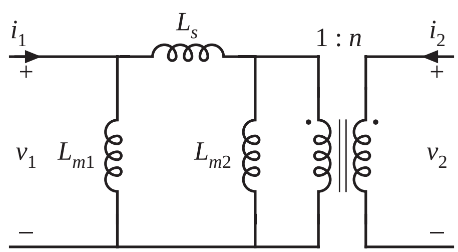
Transformer Equivalent Circuit Models

Two-winding transformer models

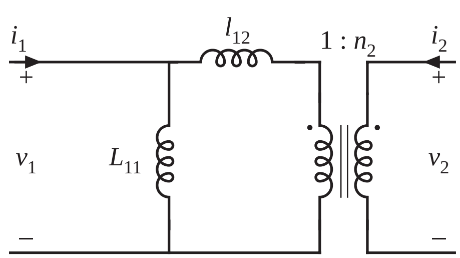
T



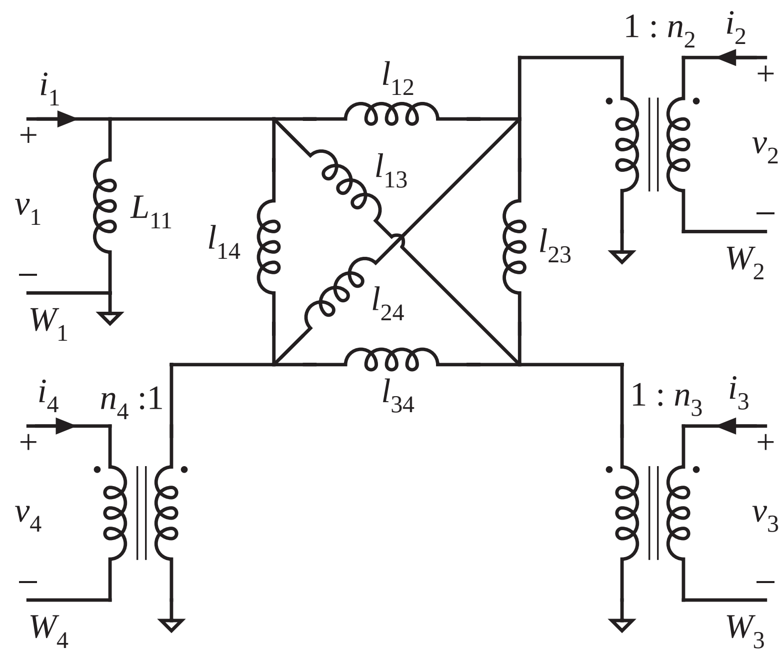
Pi



Cantilever

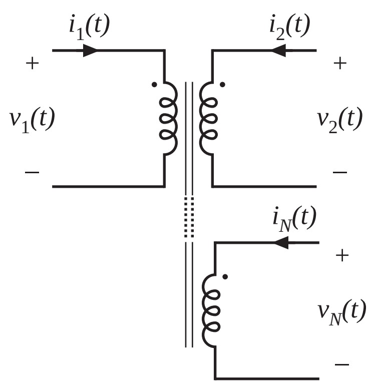


Extension of cantilever model to four-winding case

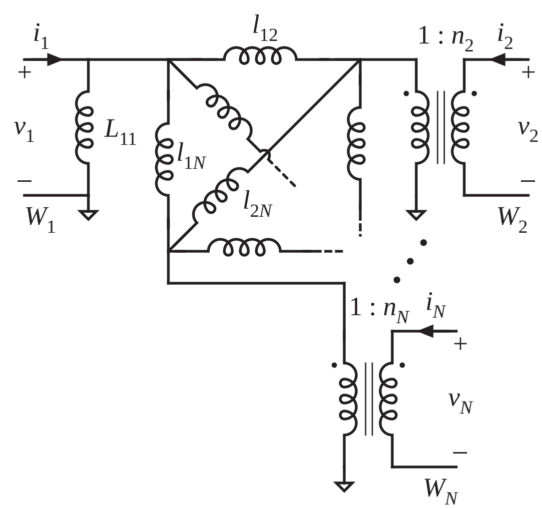


N-winding transformer models used here

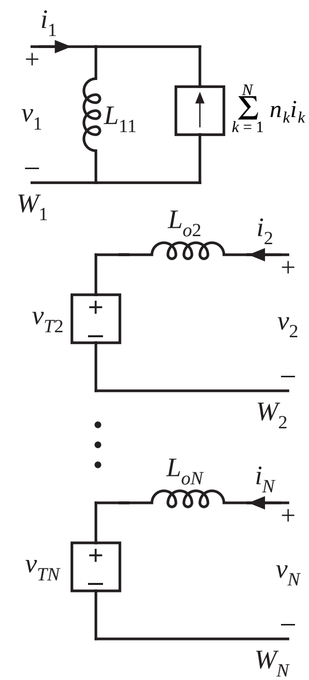
n-winding transformer



Extended cantilever model



n-port model



Relationship between inductance matrix and extended cantilever model

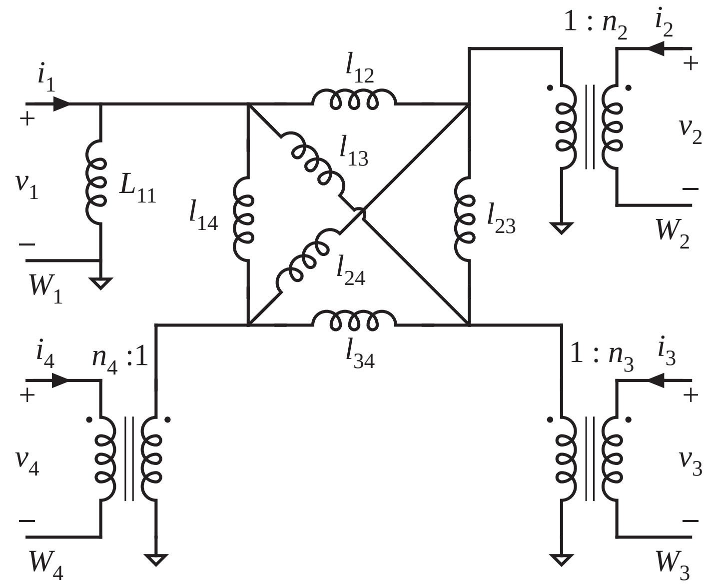
The inductance matrix:

$$\mathbf{v} = s\mathbf{L}\mathbf{i}$$
$$\mathbf{L} = \left\{ L_{jk} \right\} \quad \textit{inductance matrix}$$
$$\mathbf{B} = \mathbf{L}^{-1} = \left\{ b_{jk} \right\} \quad \textit{inverse inductance matrix}$$

For an N-winding transformer, contains $N(N + 1)/2$ independent parameters

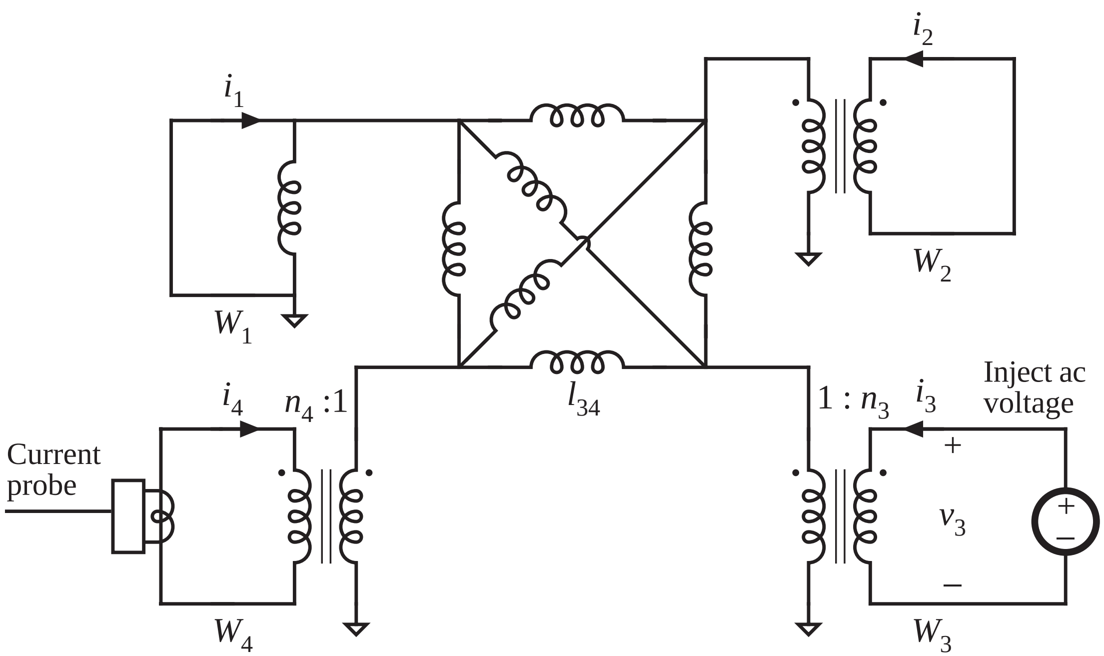
Extended cantilever model also contains $N(N + 1)/2$ independent parameters, related to the inductance matrix as follows:

$$L_{11} = L_{11}$$
$$n_j = \frac{L_{1j}}{L_{11}}$$
$$l_{jk} = -\frac{1}{n_j n_k b_{jk}}$$



Measurement of Leakage Inductance Parameters

To measure leakage inductance parameter l_{34}



- Measurement frequency must be sufficiently high, so that leakage reactance >> winding resistance

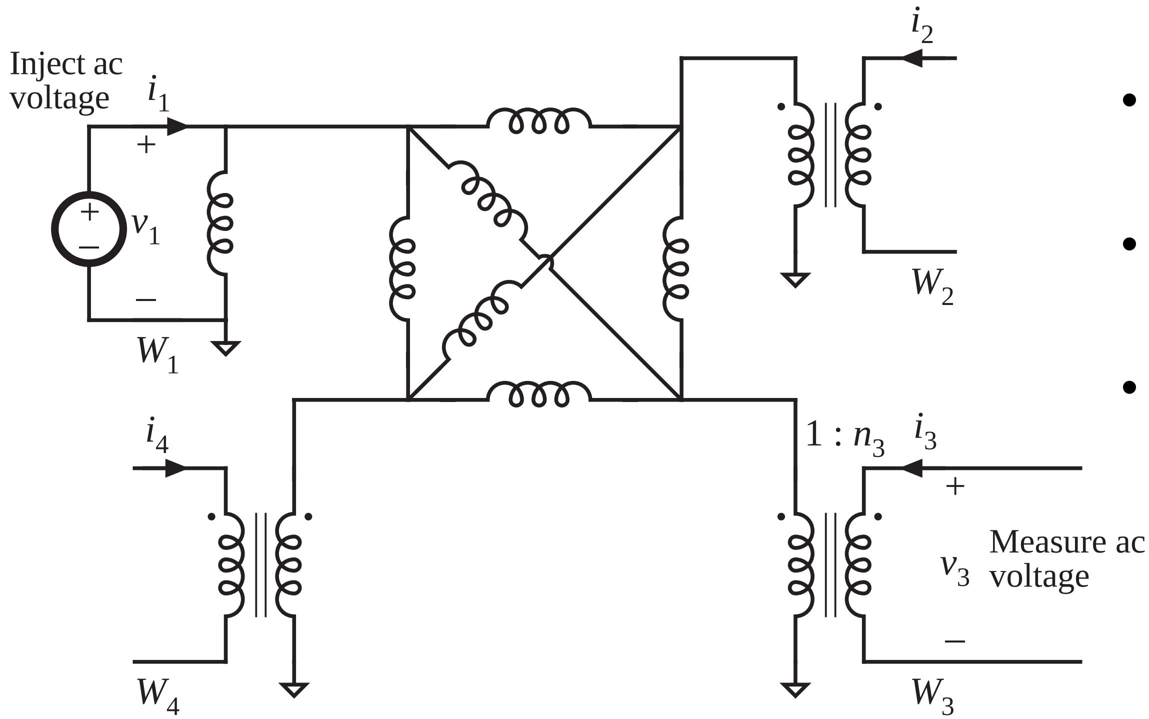
- Inject ac voltage at winding 3
- Short all other windings
- Measure current in winding 4
- l_{34} is given by

$$l_{jk} = \frac{v_j(s)}{s n_j n_k i_k(s)}$$

- Must carefully observe polarities, since l_{jk} can be negative

Measurement of Effective Turns Ratios

To measure effective turns ratio n_3



- Inject ac voltage at winding 1
- Open-circuit all other windings
- Measure voltage in winding 3
- n_3 is given by

$$n_k = \frac{v_k}{v_1}$$

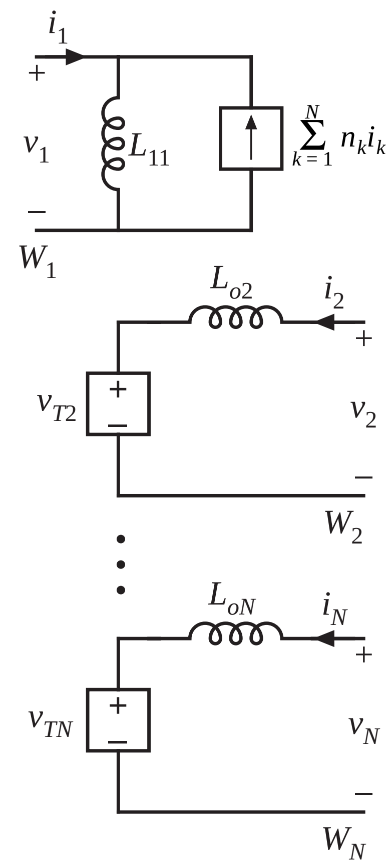
The *N*-Port Transformer Model

- Useful in deriving expressions for current ripples and zero-ripple condition, and for computer simulation
- Primary winding is represented by its current-controlled Norton equivalent
- Each secondary is modeled by a voltage-controlled Thevenin equivalent
- Secondary winding output inductance:

$$L_{ok} = n_k^2 \left(l_{1k} || l_{2k} || \cdots || l_{(k-1)k} || l_{(k+1)k} || \cdots || l_{Nk} \right)$$

- Secondary winding controlled voltage source:

$$v_{Tk} = \frac{L_{ok}}{n_k l_{1k}} v_1 + \frac{L_{ok}}{n_k n_2 l_{2k}} v_2 + \cdots + \frac{L_{ok}}{n_k n_{k-1} l_{(k-1)k}} v_{k-1} + \frac{L_{ok}}{n_k n_{k+1} l_{(k+1)k}} v_{k+1} + \cdots + \frac{L_{ok}}{n_k n_N l_{Nk}} v_N$$



Flyback Transformer Example

Experimental example:

Three-output flyback converter

Application specifications:

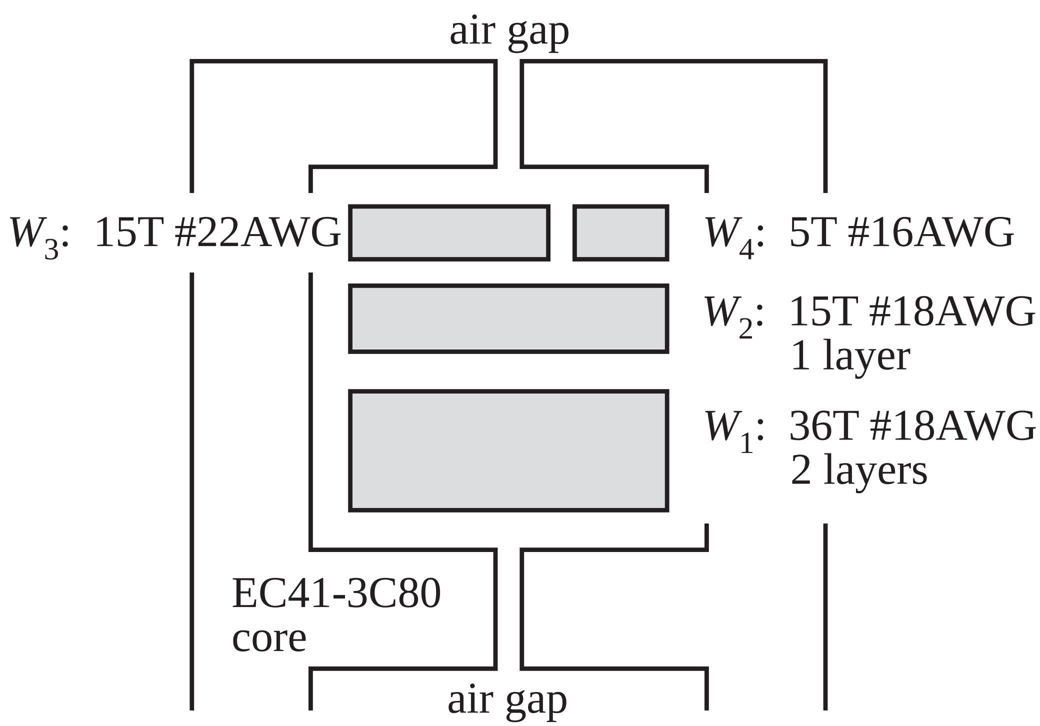
Input: 30 V (winding W_1)

Output: +12 V (winding W_2)

Output: –12 V (winding W_3)

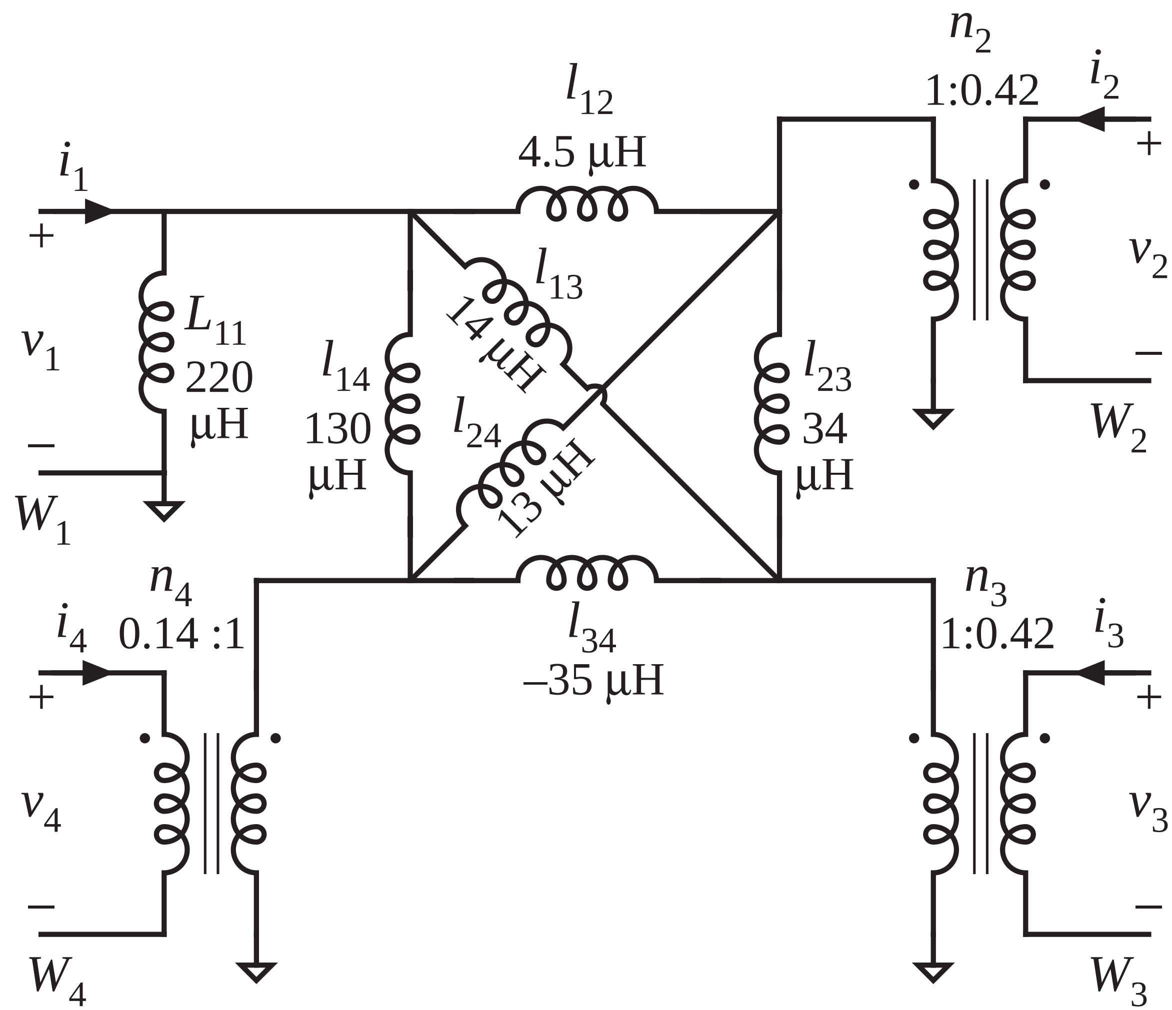
Output: +3.3 V (winding W_4)

Winding and core geometry



Measured Model

Flyback transformer extended cantilever model

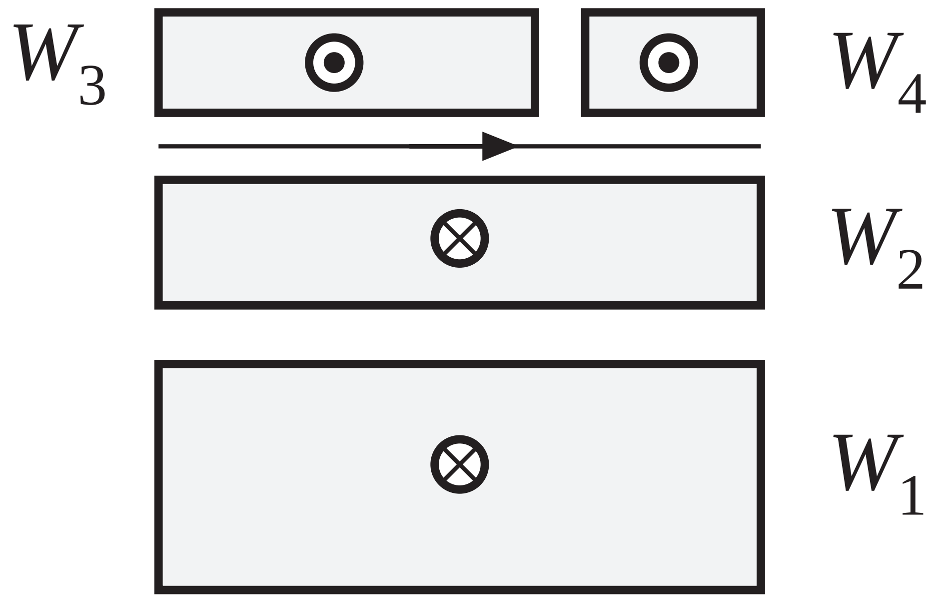


Negative l_{34}

Directions of induced winding currents, when winding W_3 is driven and windings W_1, W_2, and W_4 are shorted

Negative l_{34} indicates reversal of polarity of induced current i_4

Side-by-side winding geometry leads to negative leakage parameter



Measured n-port parameter model

Flyback transformer example

N-port parameters are computed from extended cantilever model parameters as follows:

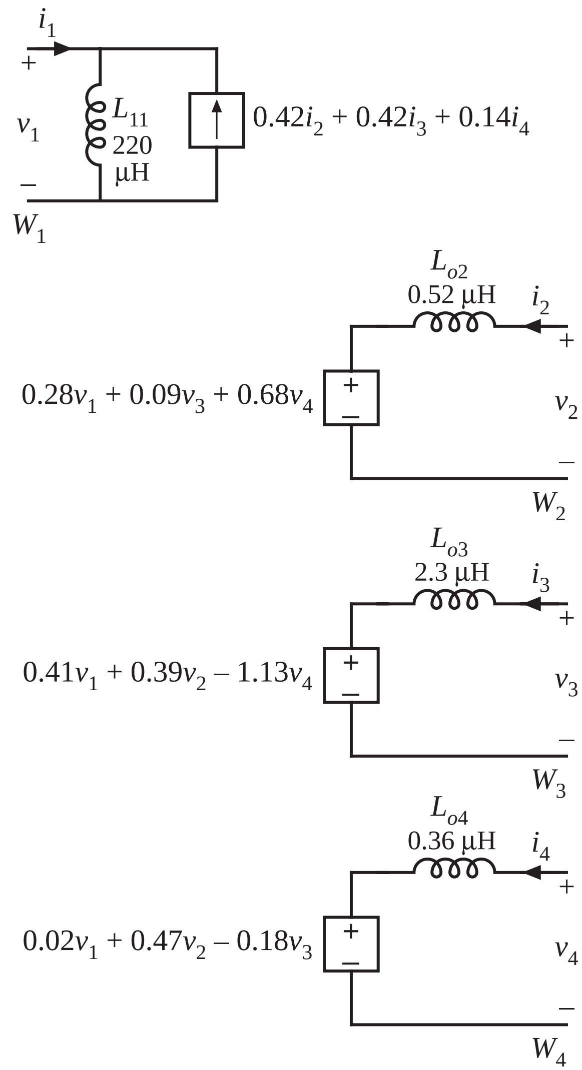
Winding output impedance

$$L_{ok} = n_k^2 \left(l_{1k} \| l_{2k} \| \cdots \| l_{(k-1)k} \| l_{(k+1)k} \| \cdots \| l_{Nk} \right)$$

Voltage-controlled voltage source

$$v_{Tk} = \frac{L_{ok}}{n_k l_{1k}} v_1 + \frac{L_{ok}}{n_k n_2 l_{2k}} v_2 + \cdots + \frac{L_{ok}}{n_k n_{k-1} l_{(k-1)k}} v_{k-1} + \frac{L_{ok}}{n_k n_{k+1} l_{(k+1)k}} v_{k+1} + \cdots + \frac{L_{ok}}{n_k n_N l_{Nk}} v_N$$

Alternatively, these parameters could be directly measured



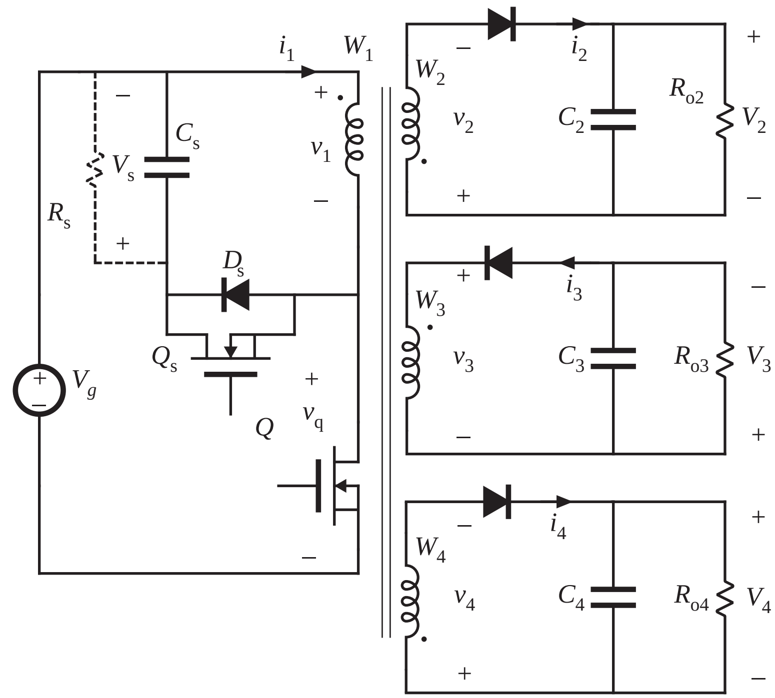
Summary—Part 1

- Extended cantilever model, and *N*-port model, correctly predict observed waveforms of multiple-output converters
- These models are full-order: the number of independent parameters is the same as in the inductance matrix, and the parameters are directly related to the entries of the inverse inductance matrix
- Each model parameter can be directly measured, and the model can be checked using several other measurements
- Reduced-order models generally do not predict the observed phenomena of multiple-output converters

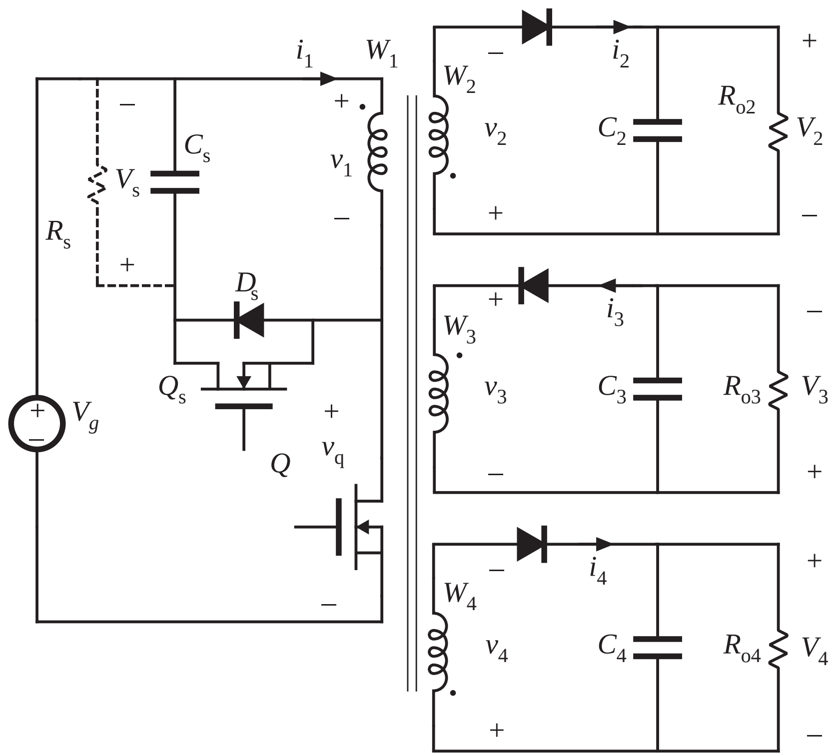
Next: the mechanisms of cross-regulation in flyback converters can be explained using the extended cantilever model

2. Cross-Regulation in Flyback Converters

- Widespread applications, usually at low to medium power levels
- Multiple-output flyback transformer design is usually based on practical experience, trial and error
- Operation, steady-state and dynamic properties are strongly affected by transformer leakage inductances
- Modeling is considered intractable (especially if the number of outputs exceeds two)
- Very few analytical results or models are available to aid the designer
- Poor cross-regulation is often observed in practice



Flyback Converter Circuit



Example: three outputs

Two snubber configurations:

- Passive voltage-clamp snubber: D_s, C_s, R_s
- Active-clamp snubber: Q_s/D_s, C_s

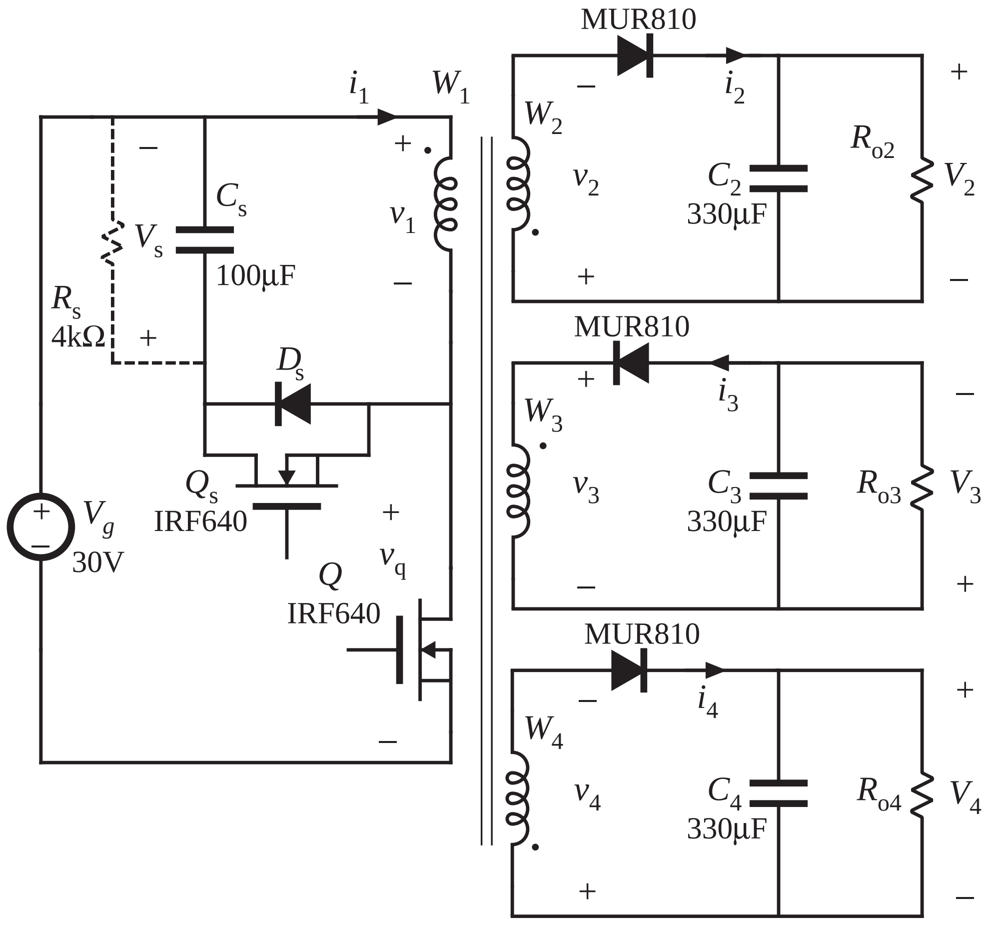
Cross-regulation affected by:

- Conduction losses
- Transformer leakage inductances

Objectives

- Explain qualitative behavior using the extended cantilever magnetics model for the transformer
- Derive general steady-state analytical model capable of predicting static cross-regulation for any number of outputs and arbitrarily complex magnetics configuration
- Compare model predictions with experimental results
- Discuss model implications and strategies for improvement of static cross regulation
- Point to dynamic response considerations

Experimental 3-Output Flyback Converter



Application specifications:

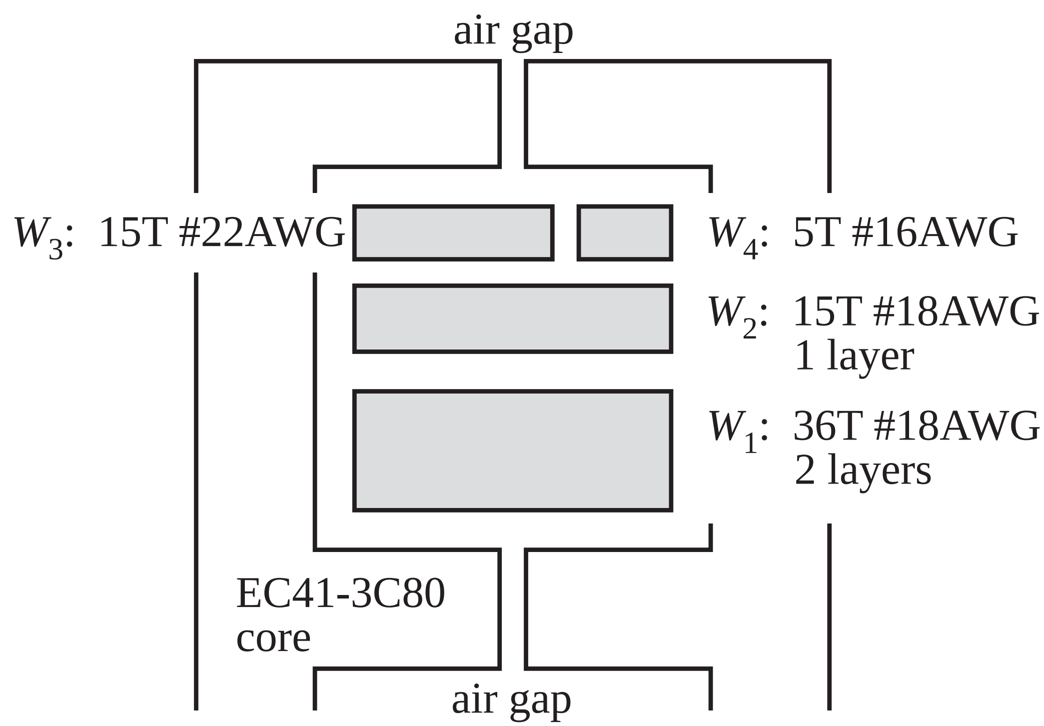
- Input: 30 V (winding W_1)
- Output: +12 V (winding W_2)
- Output: –12 V (winding W_3)
- Output: +3.3 V (winding W_4)

Flyback Transformer Example

Experimental example:

Three-output flyback converter

Winding and core geometry

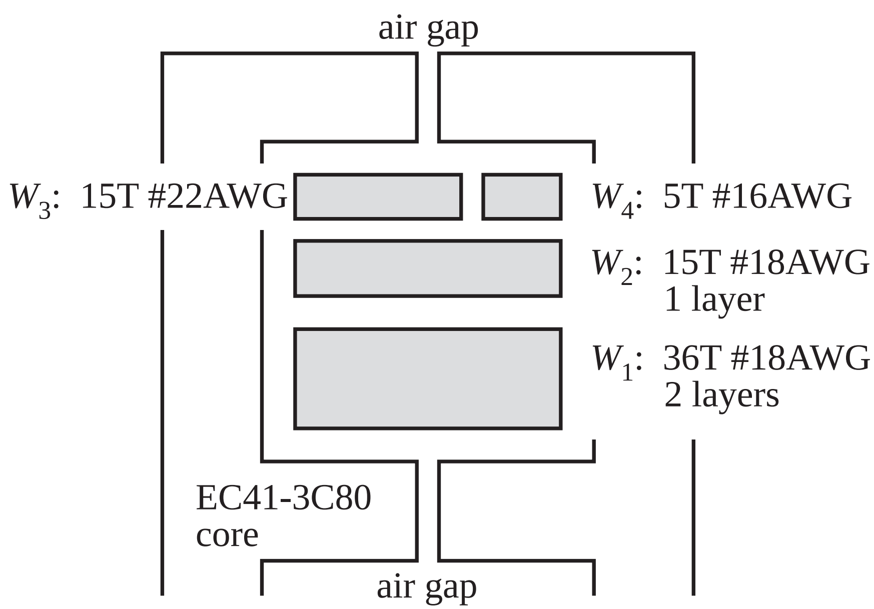


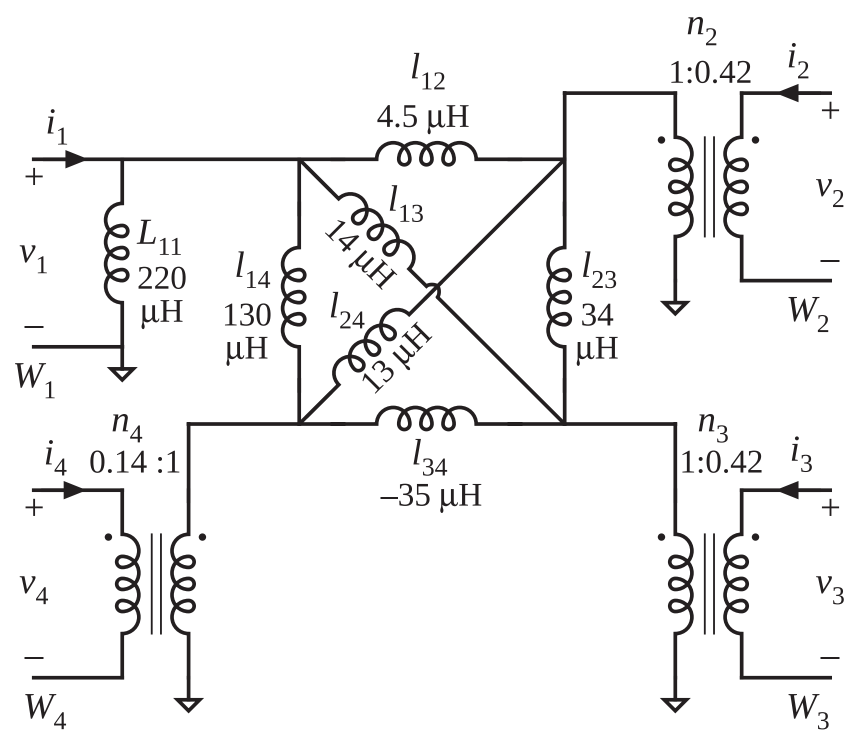
Application specifications:

Input: 30 V (winding W_1)
Output: +12 V (winding W_2)
Output: –12 V (winding W_3)
Output: +3.3 V (winding W_4)

Measured Model

Flyback transformer extended cantilever model



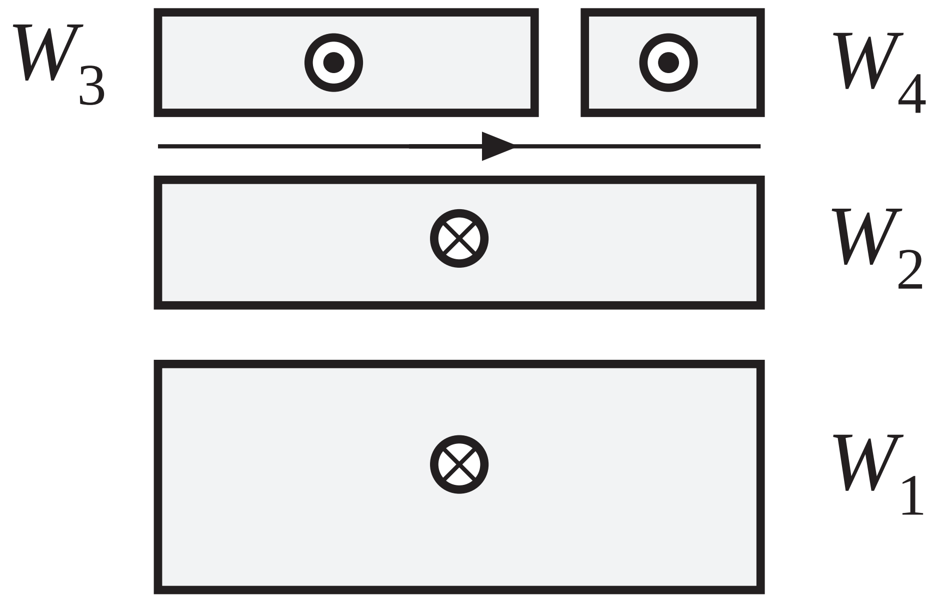


Negative l_{34}

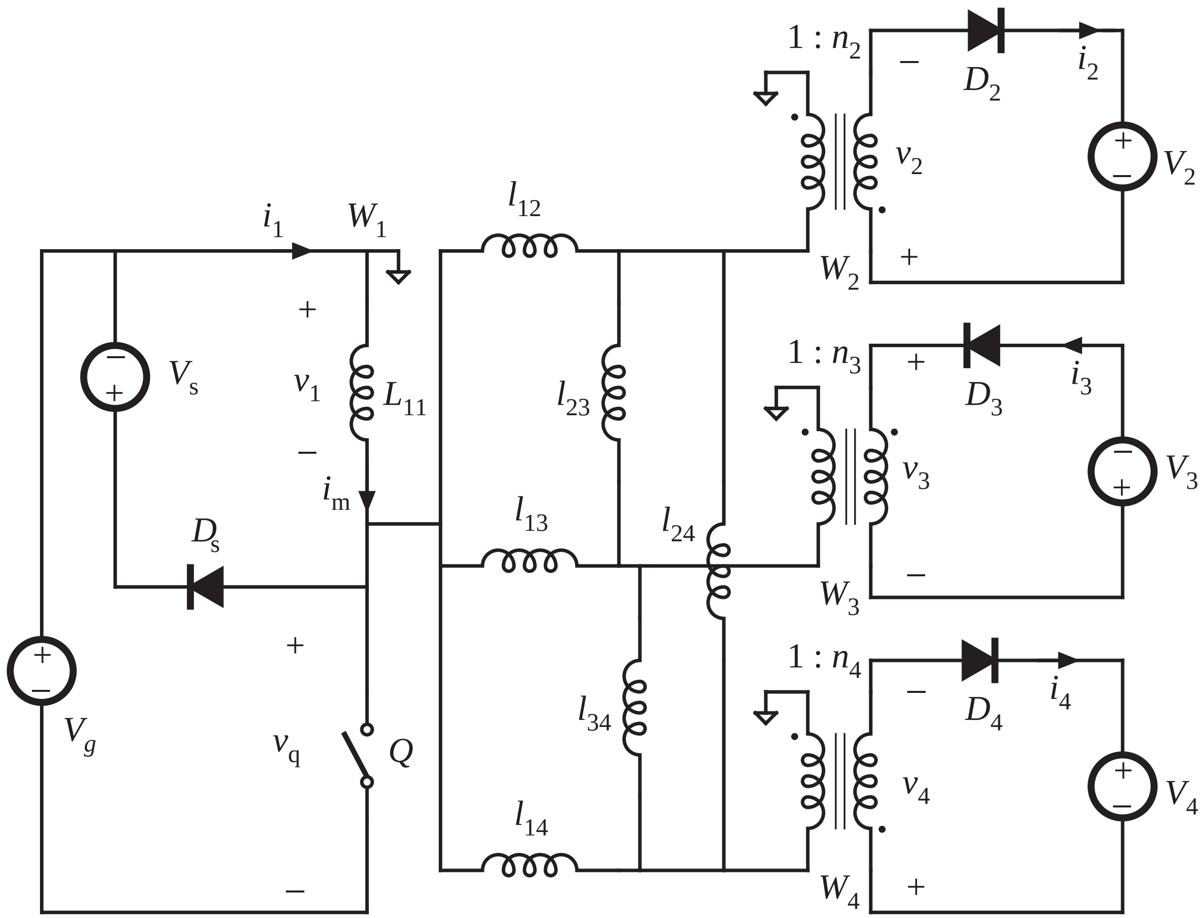
Directions of induced winding currents, when winding W_3 is driven and windings W_1, W_2, and W_4 are shorted

Negative l_{34} indicates reversal of polarity of induced current i_4

Side-by-side winding geometry leads to negative leakage parameter



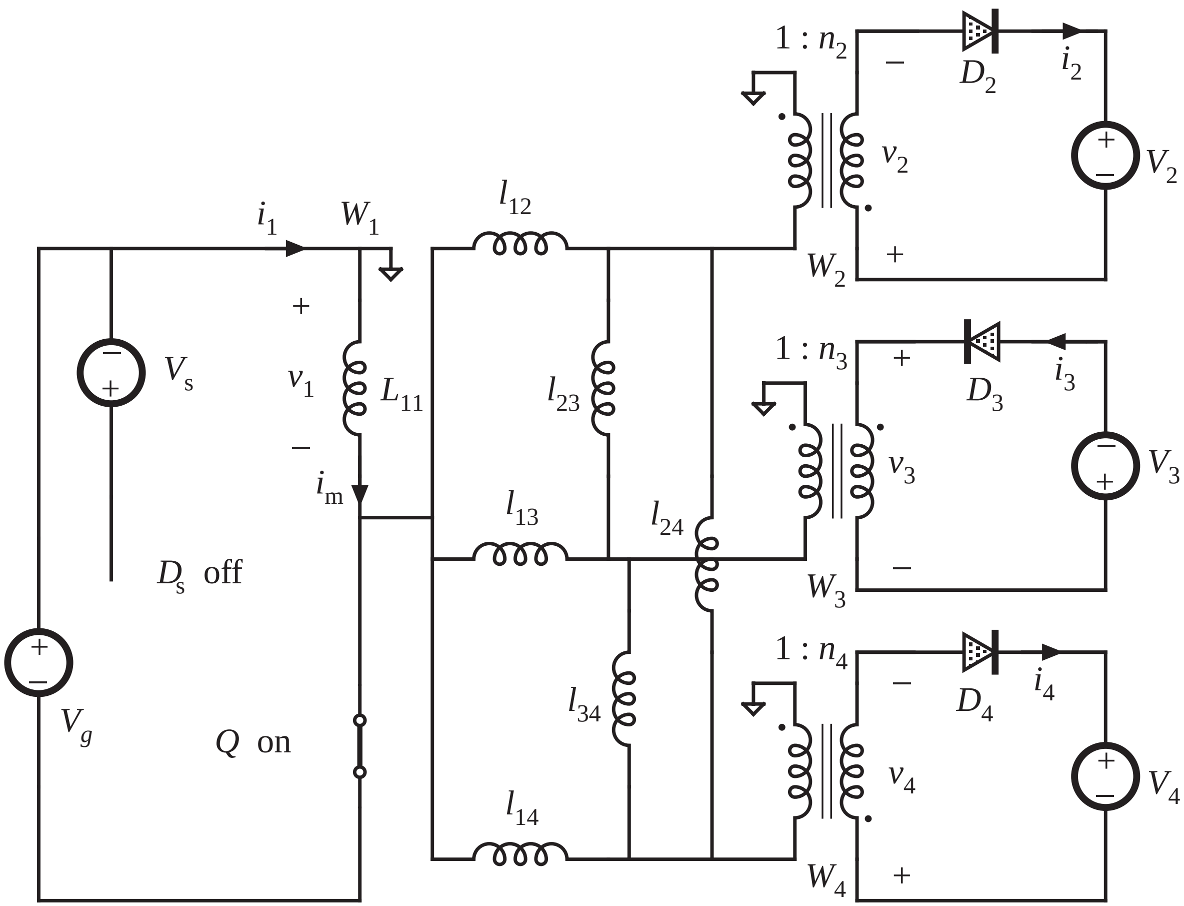
Qualitative Behavior
Case 1: Passive Voltage-Clamp Snubber



Analysis:

- Use the extended cantilever model for the transformer
- Neglect losses (other than snubber losses due to leakage inductances)
- Neglect capacitor voltage ripples

Start at the Time When the Main Switch is Turned ON: (1) Diode Turn-Off Interval t_d

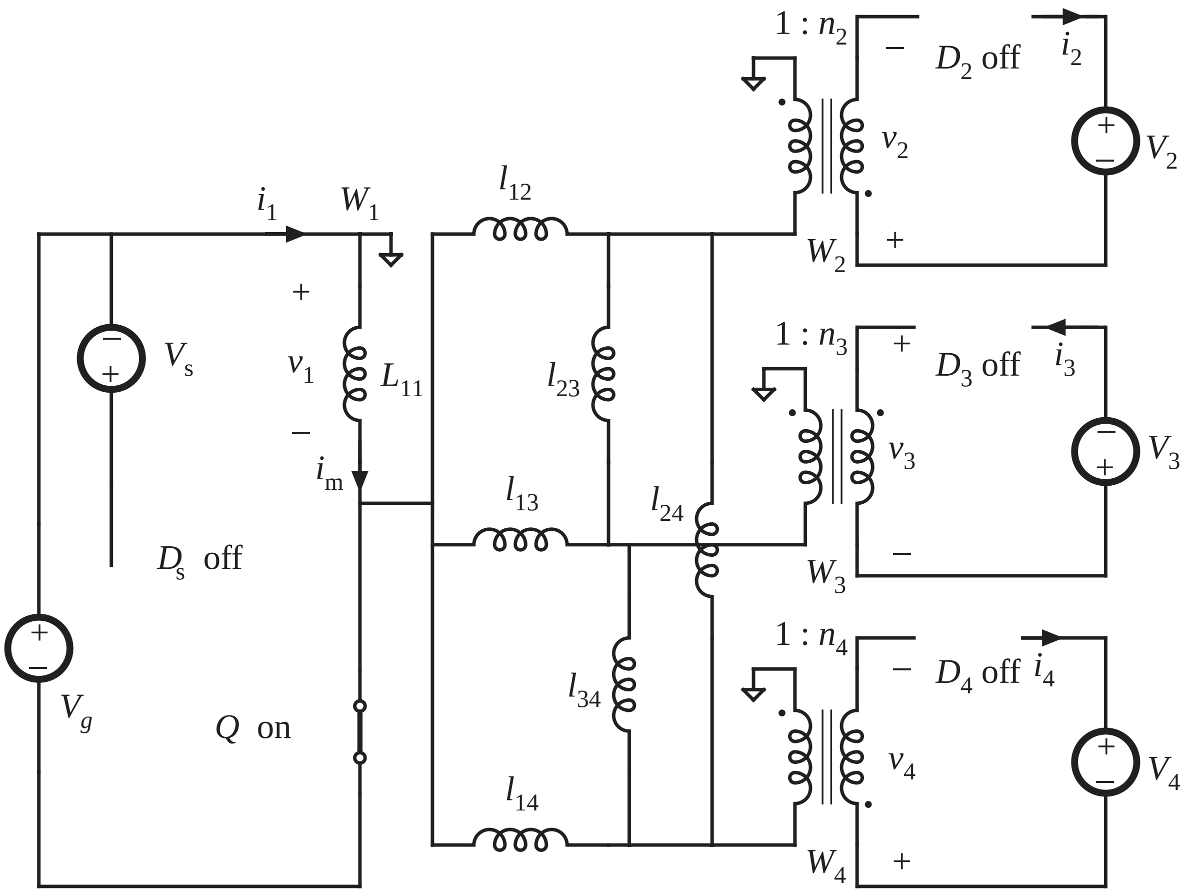


Currents through the leakage inductances decrease to zero after the main switch *Q* turns on

Secondary diodes turn off

Diode turn-off times are not equal

(2) Main switch conduction interval

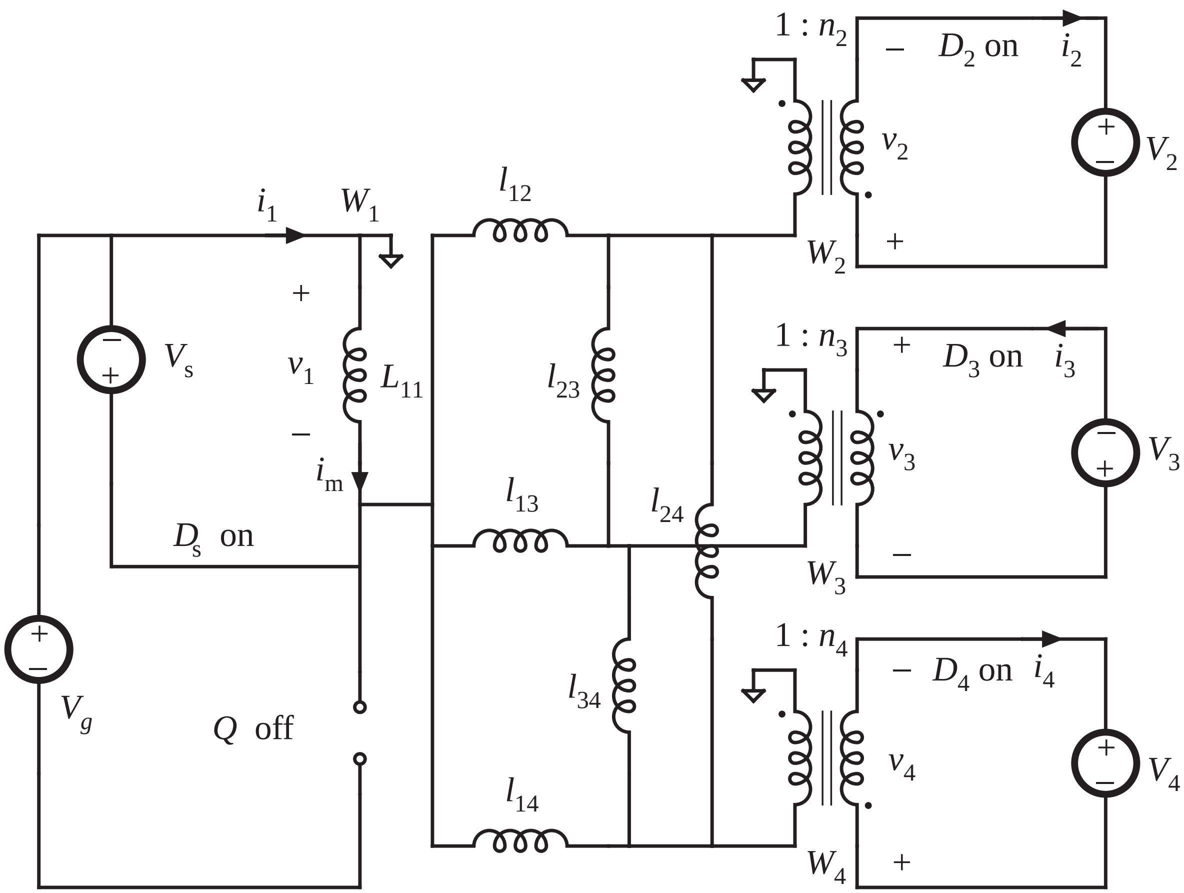


The main switch Q is on, all diodes are off

The magnetizing current increases

All leakage inductor currents are zero

(3) Commutation interval t_c



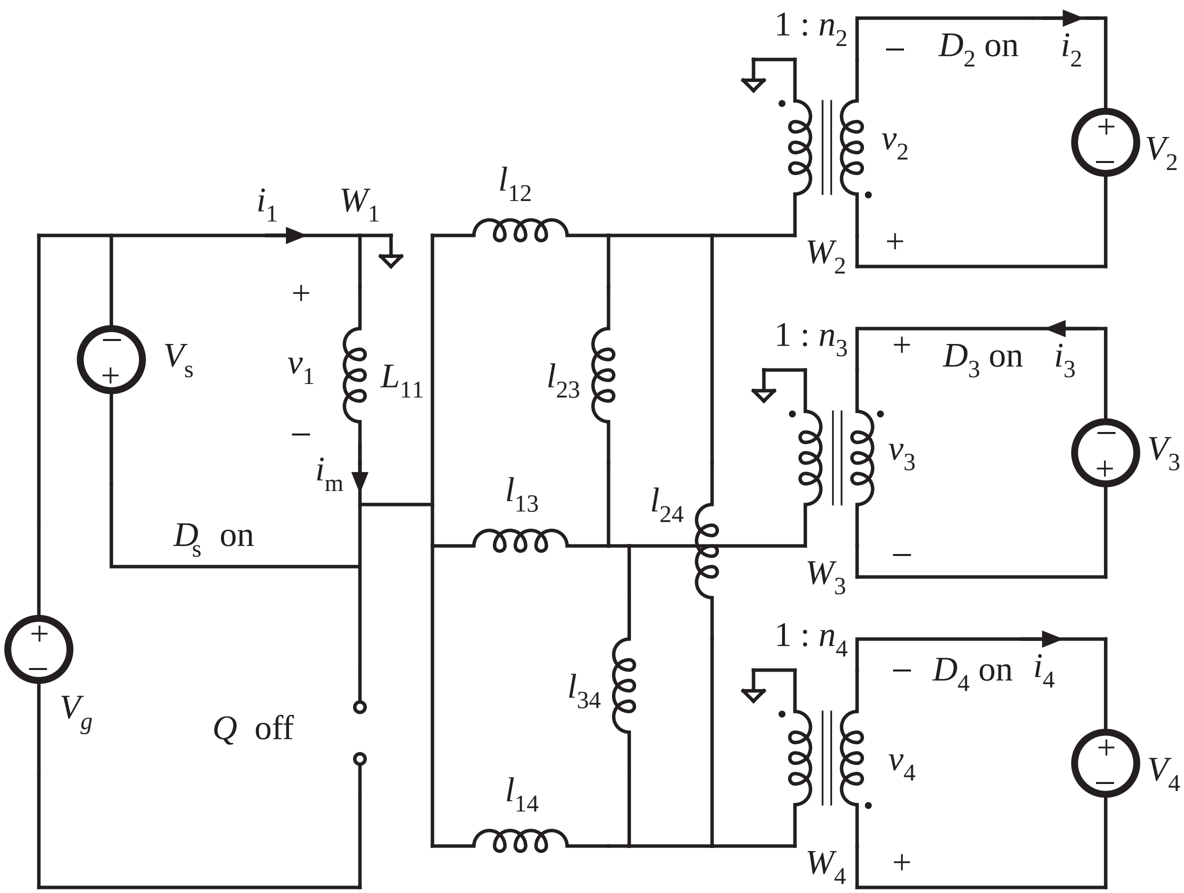
The main switch Q turns off, the snubber diode D_s turns on

The secondary diodes turn on, and the secondary winding current start increasing

The interval ends when the reflected secondary currents add to i_m

The magnetizing current divides between the widnings according to l_{12}, l_{13}, l_{14}

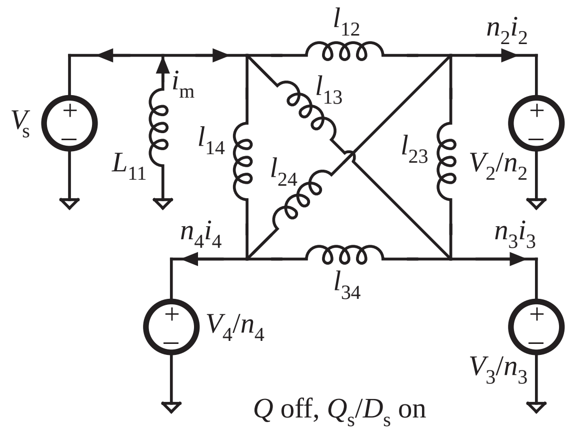
(4) Diode conduction interval



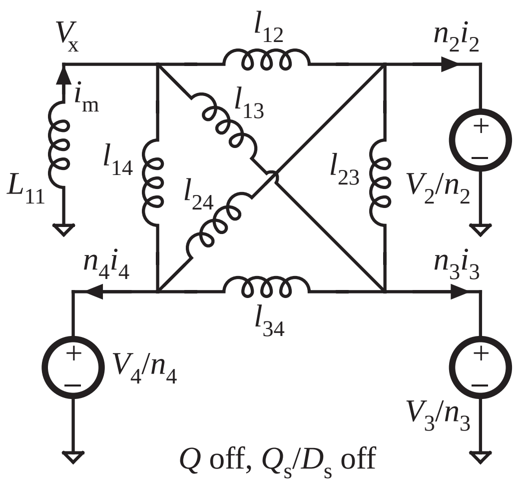
The main switch Q is off, the snubber diode D_s is off

The secondary diodes are on, and the secondary winding currents *increase or decrease* at the rates that depend on the initial values and the loads

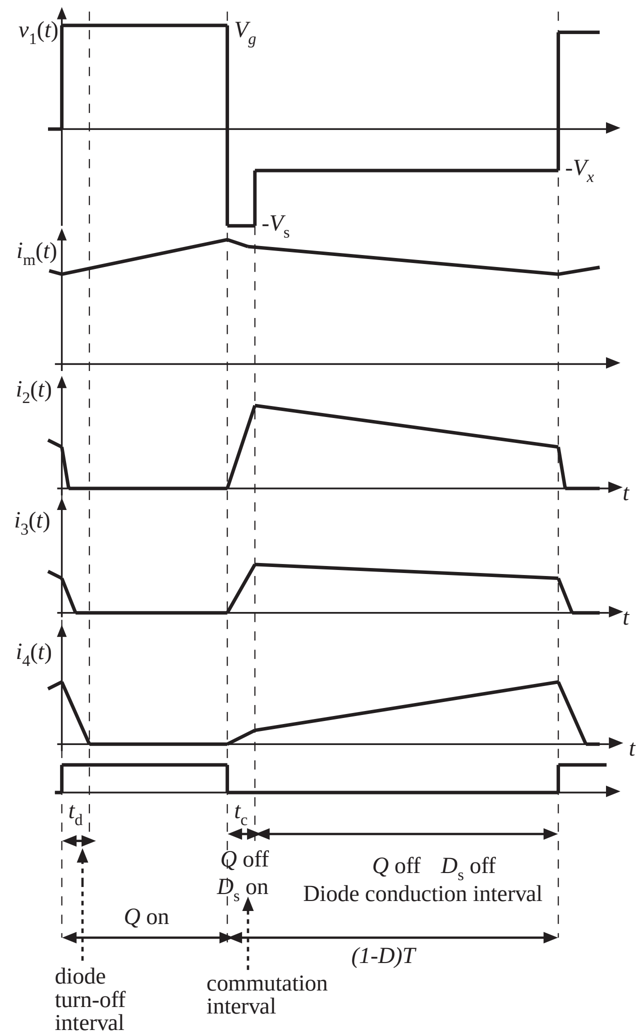
Qualitative Behavior
Case 1: Passive Voltage-Clamp Snubber



Commutation interval



Diode conduction interval

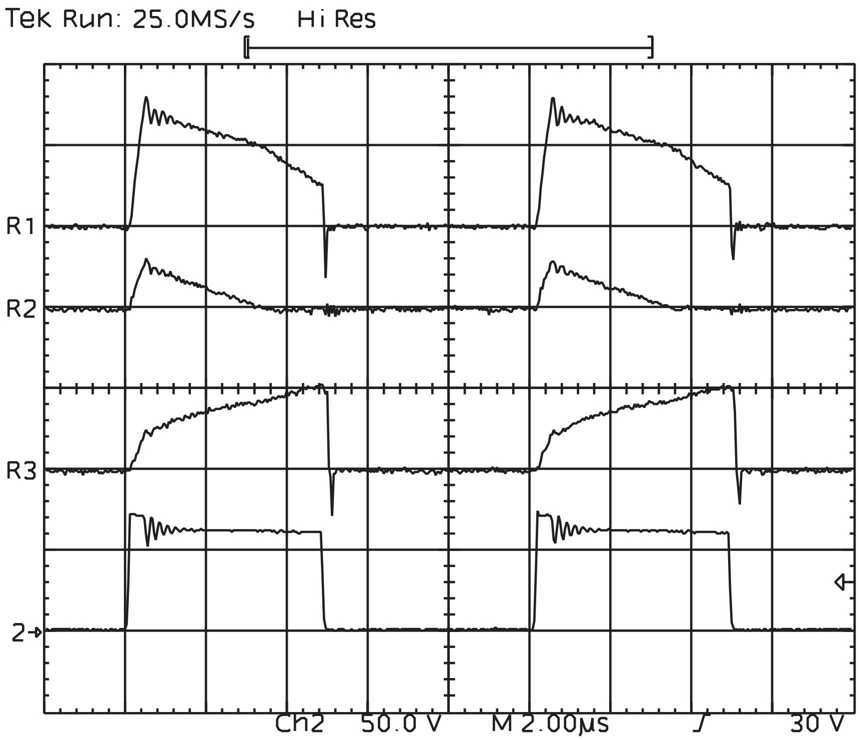


All outputs in CCM

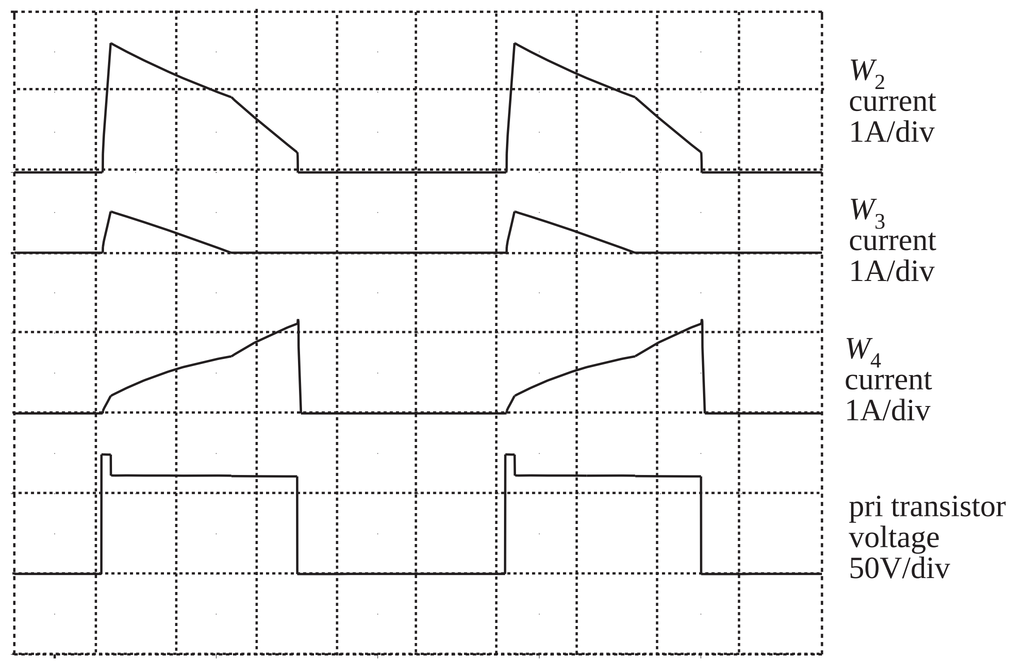
Measured and predicted waveforms

Flyback converter example with passive snubber

Measured waveforms



Simulated waveforms
using extended cantilever model

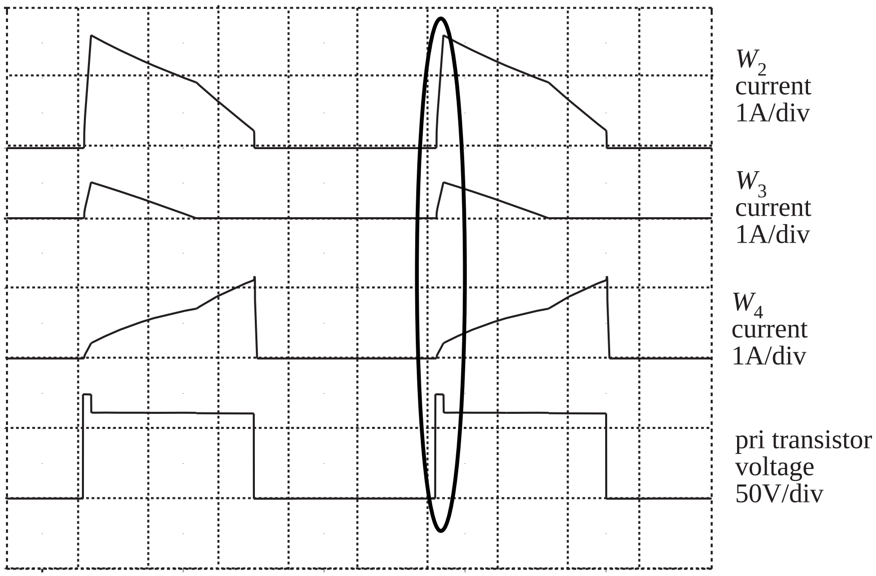


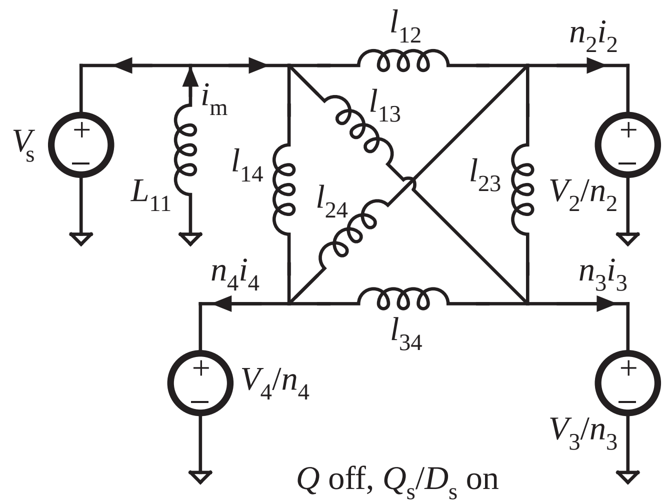
Winding 2: CCM with negative ripple

Winding 3: DCM

Winding 4: CCM with positive ripple

Secondary current waveforms
Commutation interval

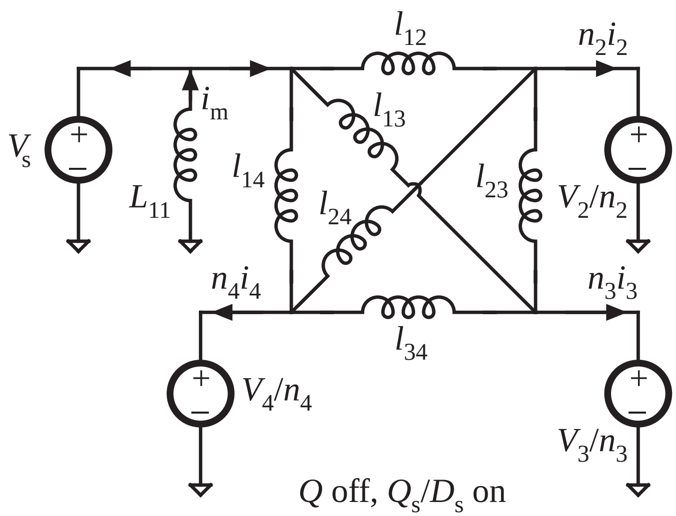




Circuit model referred to the primary side

- Magnetizing current commutes from primary winding to secondary windings
- Reflected output winding voltages are nearly equal
- Essentially zero voltage is applied across l_{23}, l_{23}, and l_{23}
- Large voltage is applied across l_{12}, l_{13}, and l_{14}
- Magnetizing current divides between secondary windings according to relative values of l_{12}, l_{13}, and l_{14}

Commutation interval analysis



Secondary winding current (W2 example):

$$n_2 \frac{di_2}{dt} = \frac{V_s - V_2/n_2}{l_{12}} + \frac{V_3/n_3 - V_2/n_2}{l_{23}} + \frac{V_4/n_4 - V_2/n_2}{l_{24}};$$

Reflected output voltages are nearly equal:

$$n_2 \frac{di_2}{dt} \approx \frac{V_s - V_2/n_2}{l_{12}}$$

At the end of the commutation interval the reflected secondary currents add up to the magnetizing current:

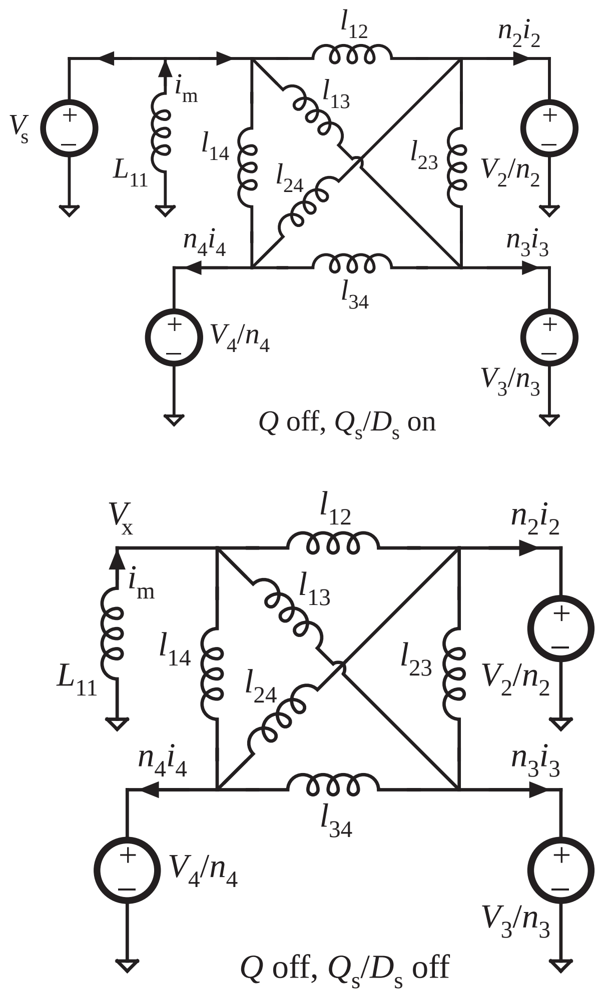
$$n_2 i_2 + n_3 i_3 + n_4 i_4 = i_m$$

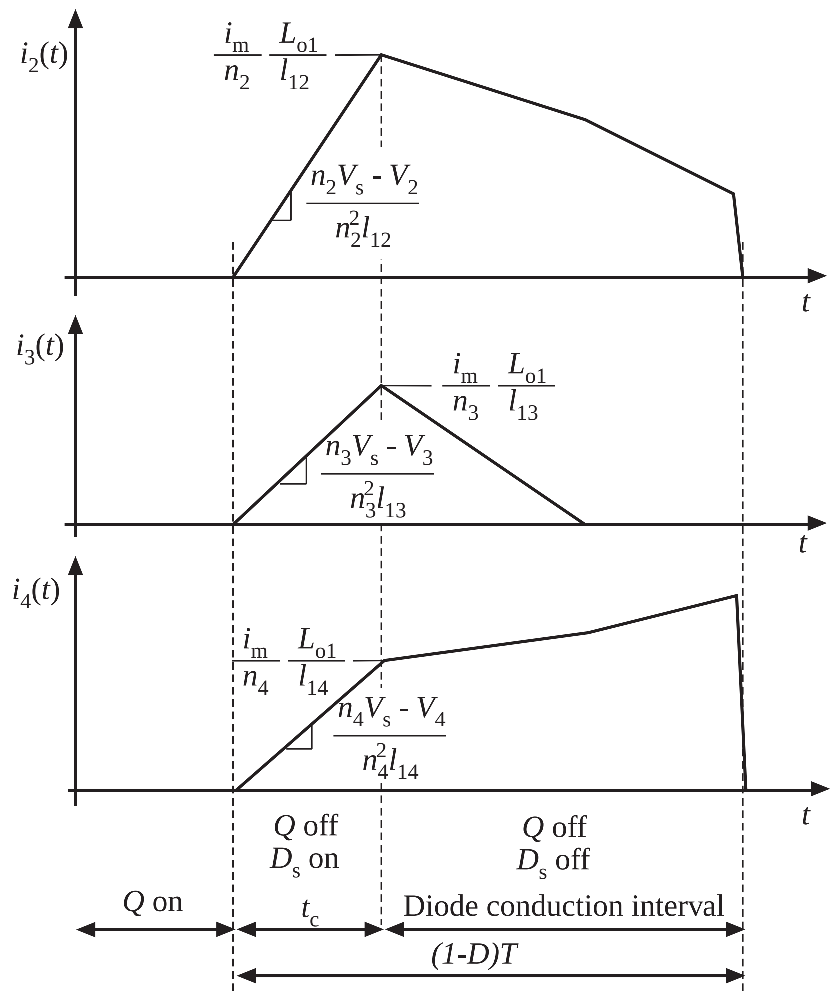
The magnetizing current divides according to relative values of l_{12}, l_{13}, and l_{14}, not directly related to the loads:

$$\boxed{n_k i_k = \frac{L_{o1}}{l_{1k}} i_m} \qquad L_{o1} = l_{12}||l_{13}||l_{14}$$

Secondary Current Waveforms

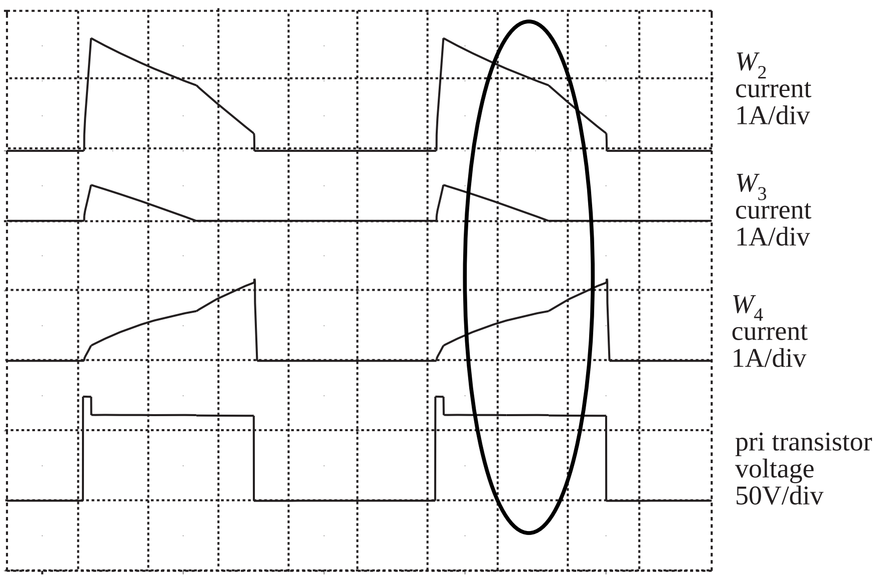
Case 1: Passive Voltage-Clamp Snubber

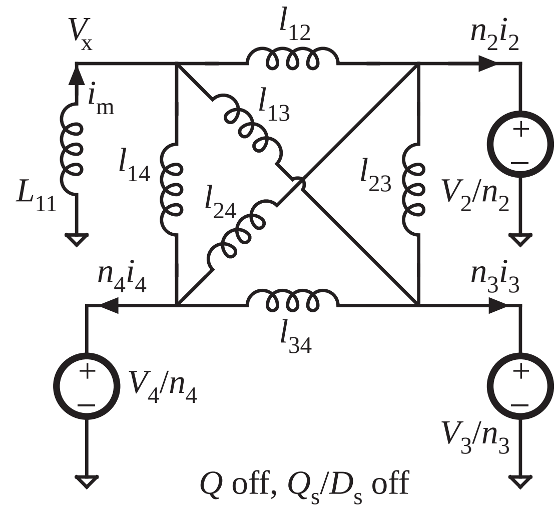




Secondary current waveforms

Diode conduction interval





Circuit model referred to the primary side

- The sum of the reflected secondary currents equals the magnetizing current
- Current slopes can be positive or negative, depending on the initial values and the loads
- Increased output voltage reduces slope of winding current waveform, leading to reduced average output current
- Decreased output voltage increases slope of winding current waveform, leading to increased average output current

Diode conduction interval analysis

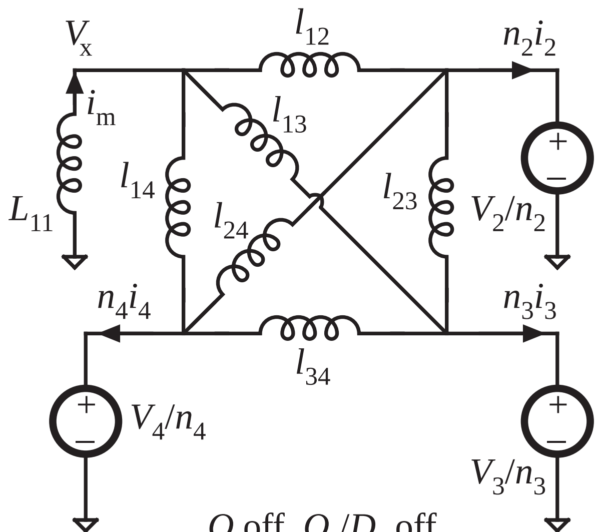
Secondary winding current (W2 example):

$$n_2 \frac{di_2}{dt} = \frac{V_x - V_2/n_2}{l_{12}} + \frac{V_3/n_3 - V_2/n_2}{l_{23}} + \frac{V_4/n_4 - V_2/n_2}{l_{24}}$$

$$n_2 \frac{di_2}{dt} = -\frac{V_2/n_2 - V_x}{L_{o2}} + \frac{V_3/n_3 - V_x}{l_{23}} - \frac{V_4/n_4 - V_x}{l_{24}}$$

Winding Thevenin output inductances (ref.to primary):

$$\begin{aligned} L_{o2} &= l_{12}||l_{23}||l_{24} \\ L_{o3} &= l_{13}||l_{23}||l_{34} \\ L_{o4} &= l_{14}||l_{24}||l_{34} \end{aligned}$$



Q off, Q_s/D_s off

All secondary winding currents:

$$\frac{d\mathbf{i}'}{dt} = \mathbf{B}_1(\mathbf{V}' - \mathbf{u}V_x)$$

$$\mathbf{B}_1 = \begin{bmatrix} -\frac{1}{L_{o2}} & \frac{1}{l_{23}} & \frac{1}{l_{24}} \\ \frac{1}{l_{23}} & -\frac{1}{L_{o3}} & \frac{1}{l_{34}} \\ \frac{1}{l_{24}} & \frac{1}{l_{34}} & -\frac{1}{L_{o4}} \end{bmatrix}, \; \mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

General Steady-State Solution

Case 1: Passive voltage-clamp snubber, all outputs in CCM

Averaging of the secondary winding currents gives:

$$\mathbf{B}_1(\mathbf{V}' - \mathbf{u}V_x) = \frac{2f_s}{(1-D)^2}\mathbf{B}_2\mathbf{I}' - \mathbf{b}_2\frac{L_{o1}}{L_{11}}V_x,$$

$$\mathbf{B}_2 = \begin{bmatrix} 1 - \frac{L_{o1}}{l_{12}} & -\frac{L_{o1}}{l_{12}} & -\frac{L_{o1}}{l_{12}} \\ -\frac{L_{o1}}{l_{13}} & 1 - \frac{L_{o1}}{l_{13}} & -\frac{L_{o1}}{l_{13}} \\ -\frac{L_{o1}}{l_{14}} & -\frac{L_{o1}}{l_{14}} & 1 - \frac{L_{o1}}{l_{14}} \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} \frac{1}{l_{12}} \\ \frac{1}{l_{13}} \\ \frac{1}{l_{14}} \end{bmatrix}$$

where V_x is found from the volt-second balance on L_{11}:

$$V_x \approx V_g\frac{D}{1-D} - \frac{f_s L_{o1}}{(1-D)^2}(\mathbf{u}^T\mathbf{I}')$$

Thevenin Output Resistance Matrix

Passive voltage-clamp snubber, all outputs in CCM

Referred to the primary side:

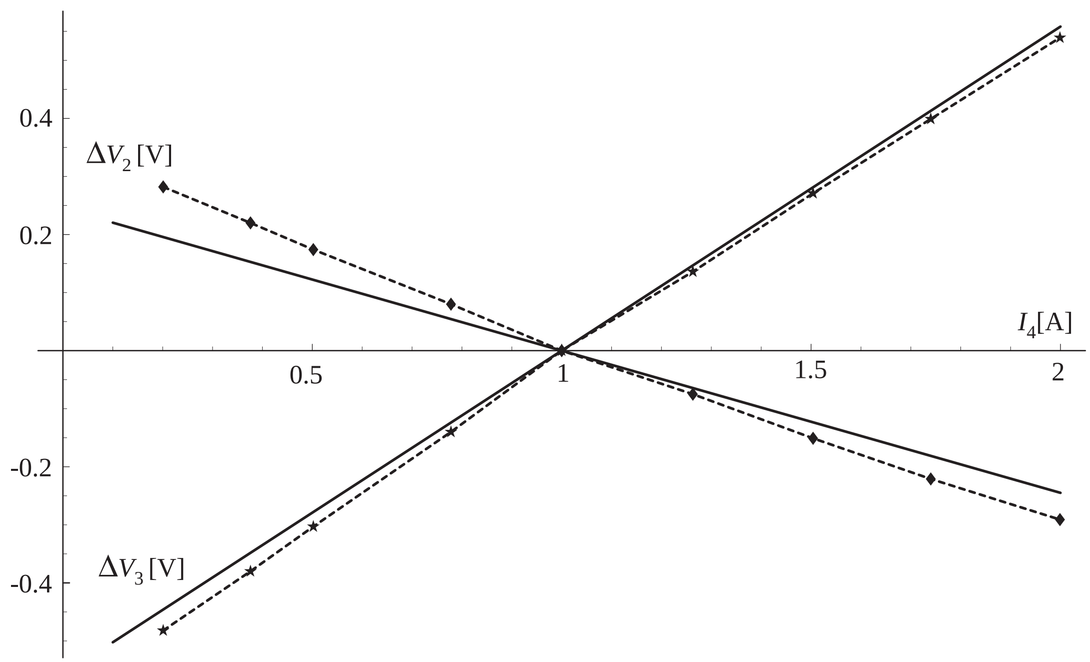
$$\mathbf{R}' = -\frac{2f_s}{(1-D)^2}\left(\mathbf{B}_1^{-1}\mathbf{B}_2 - \frac{L_{o1}}{2}\mathbf{u}\mathbf{u}^T\right)$$

Referred to the secondaries: $\Delta\mathbf{V} = -\mathbf{R}\Delta\mathbf{I}$

$$\mathbf{R} = \mathbf{N}\mathbf{R}'\mathbf{N}$$

$$\mathbf{N} = \begin{bmatrix} n_2 & 0 & 0 \\ 0 & n_3 & 0 \\ 0 & 0 & n_4 \end{bmatrix}$$

Predicted and measured cross-regulation (with passive voltage-clamp snubber)



Operating conditions: D=0.52, I_2=I_3=0.4A, I_4=0.1A to 2.0A

Changing load on the main output has opposite effects on the two auxiliary outputs !

Predicted and measured output resistance matrix

Operating point: D=0.52, I_2=I_3=0.4A, I_4=1A

Predicted:

$$\begin{bmatrix} \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = - \begin{bmatrix} 0.42 & -0.32 & 0.24 \\ -0.32 & 2.15 & -0.56 \\ 0.24 & -0.56 & 0.54 \end{bmatrix} \begin{bmatrix} \Delta I_2 \\ \Delta I_3 \\ \Delta I_4 \end{bmatrix}$$

Measured variations in the output voltages corresponding to load changes:

$$\begin{bmatrix} \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = - \begin{bmatrix} 1.22 & -0.2 & 0.24 \\ -0.35 & 3.2 & -0.52 \\ 0.26 & -0.56 & 0.90 \end{bmatrix} \begin{bmatrix} \Delta I_2 \\ \Delta I_3 \\ \Delta I_4 \end{bmatrix}$$

Experimental verification of the model

- Waveforms obtained by simulation match experimentally observed waveforms: the extended cantilever magnetics model can be used to predict complex operation of multiple-output flyback converters
- The extended cantilever model allows easy qualitative explanation of the converter operation, as well as derivation of a general steady-state cross-regulation model
- Good agreement between predictions of the steady-state cross-regulation model and measured results
- Off-diagonal terms (“mutual resistances”) in the Thevenin output resistance matrix are determined mainly by the transformer leakage inductances. Prediction of these terms is very good
- Diagonal terms (“self resistances”) in the Thevenin output resistance matrix are affected by conduction losses. Hence the measured values are greater than predicted by the model.

Discontinuous conduction modes

$$n_2 i_2 + n_3 i_3 + n_4 i_4 = i_m$$

$$n_k i_k = \frac{L_{o1}}{l_{1k}} i_m$$

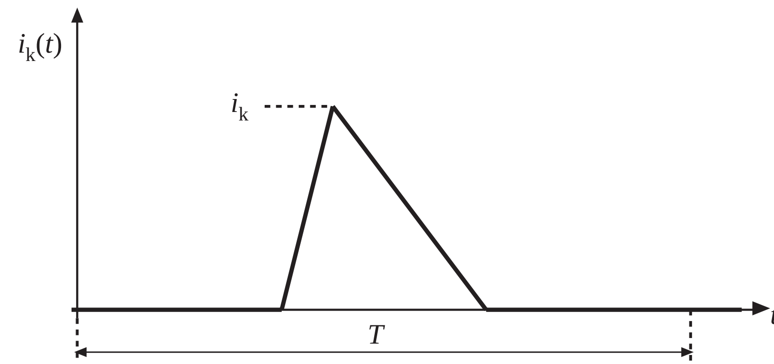
$$L_{o1} = l_{12} || l_{13} || l_{14}$$

- Initial secondary winding currents at the end of the commutation interval are proportional to the magnetizing current, i.e. to the *total* load, not to the load on the individual output

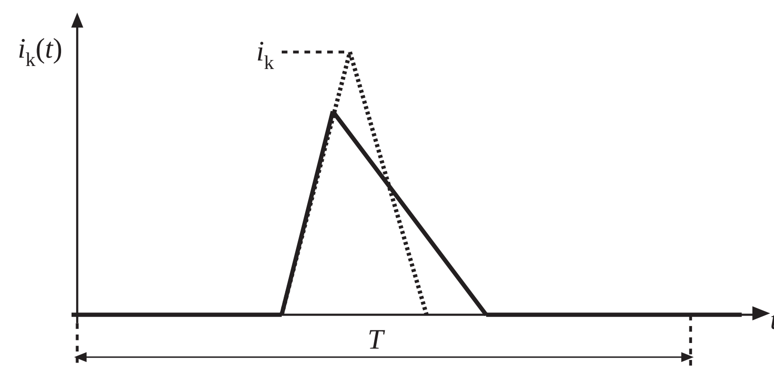
- The magnetizing current is divided according to the relative values of l_{12}, l_{13}, and l_{14}, *not* according to individual loads

Discontinuous conduction modes

- A lightly loaded output well coupled to the primary (l_{12} small)
- Starts with large initial value i_k
- Large negative slope in the diode conduction interval => DCM with much increased output voltage



- Increasing load on another output increases i_k further
- Deeper DCM, even larger output voltage
- Very poor cross-regulation (often referred to as “peak detection”)



CCM boundaries

- General result for operation of all outputs in CCM:

$$\begin{bmatrix} 2\frac{l_{12}}{L_{o1}}-1 & -1 & -1 \\ -1 & 2\frac{l_{13}}{L_{o1}}-1 & -1 \\ -1 & -1 & 2\frac{l_{14}}{L_{o1}}-1 \end{bmatrix} \begin{bmatrix} n_2 I_2 \\ n_3 I_3 \\ n_4 I_4 \end{bmatrix} > \frac{(1-D)^2 V_x}{2 f_s L_{11}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- The winding best coupled to the primary is most likely to operate in DCM
- Increasing load on one winding eventually drives all other outputs into DCM
- Improper relative primary-to-secondary couplings and the resulting DCM modes are the major cause of poor cross-regulation
- General rule: the winding with the widest load range should be best coupled to the primary

CCM boundaries: experimental example

- Experimental example:

$$\begin{bmatrix} 0.73 & -0.42 & -0.14 \\ -0.42 & 3.0 & -0.14 \\ -0.42 & -0.42 & 10.9 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} > \begin{bmatrix} 0.17\text{A} \\ 0.17\text{A} \\ 0.17\text{A} \end{bmatrix}$$

- Predicted CCM conditions:

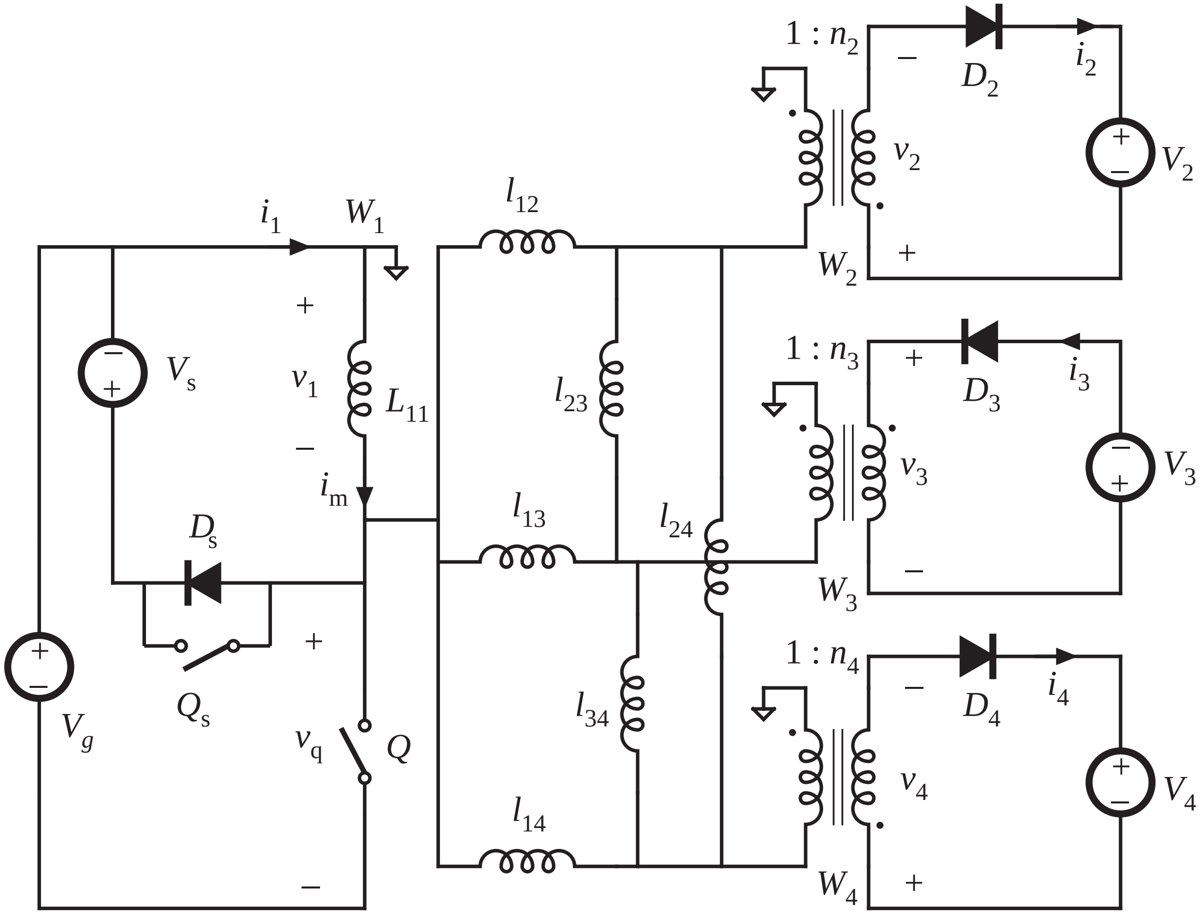
$$I_2 > 0.23\text{A} + 0.58 I_3 + 0.19 I_4$$

$$I_3 > 0.06\text{A} + 0.14 I_2 + 0.05 I_4$$

$$I_4 > 0.016\text{A} + 0.045 I_2 + 0.045 I_3$$

- Good agreement with experiment

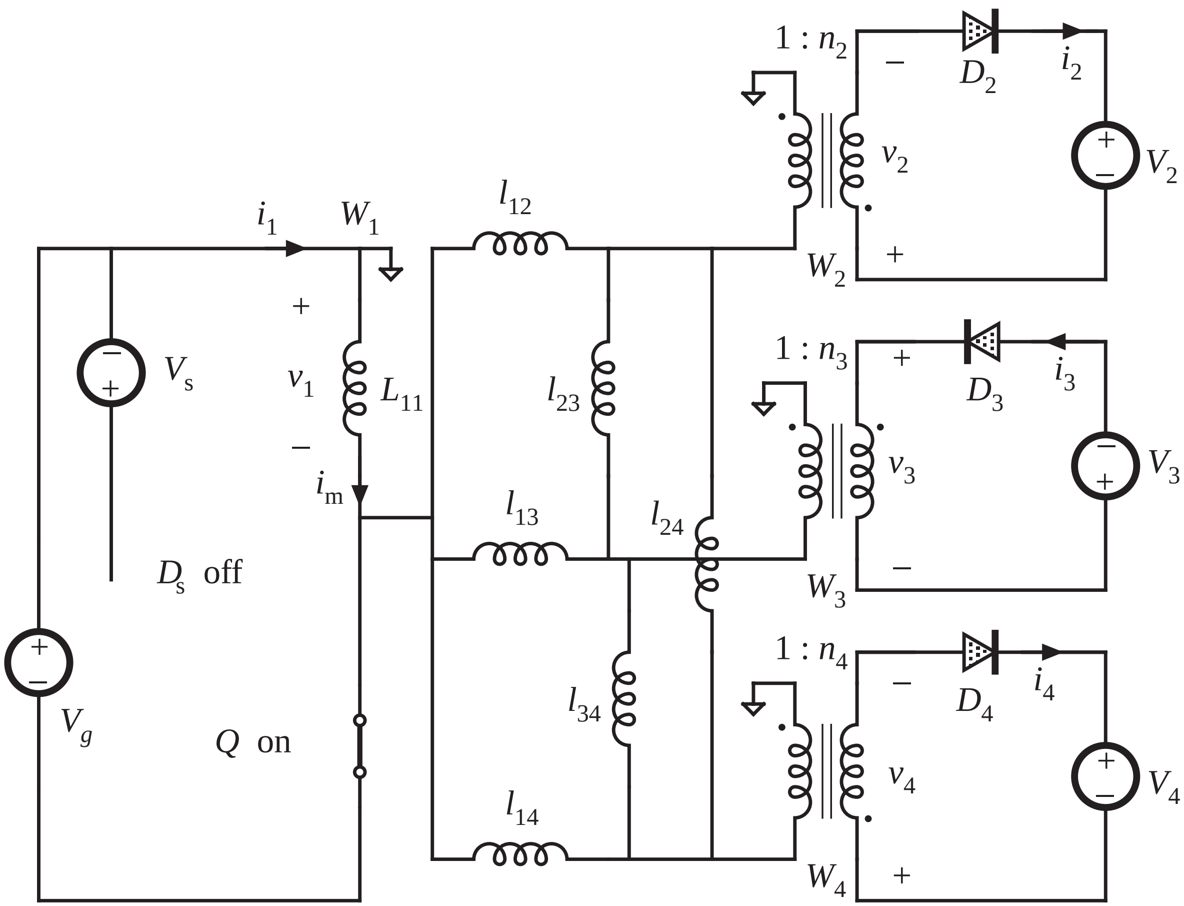
Qualitative Behavior
Case 2: Active-Clamp Snubber



Analysis:

- Use the extended cantilever model for the transformer
- Neglect losses (other than losses due to leakage inductances)
- Neglect capacitor voltage ripples

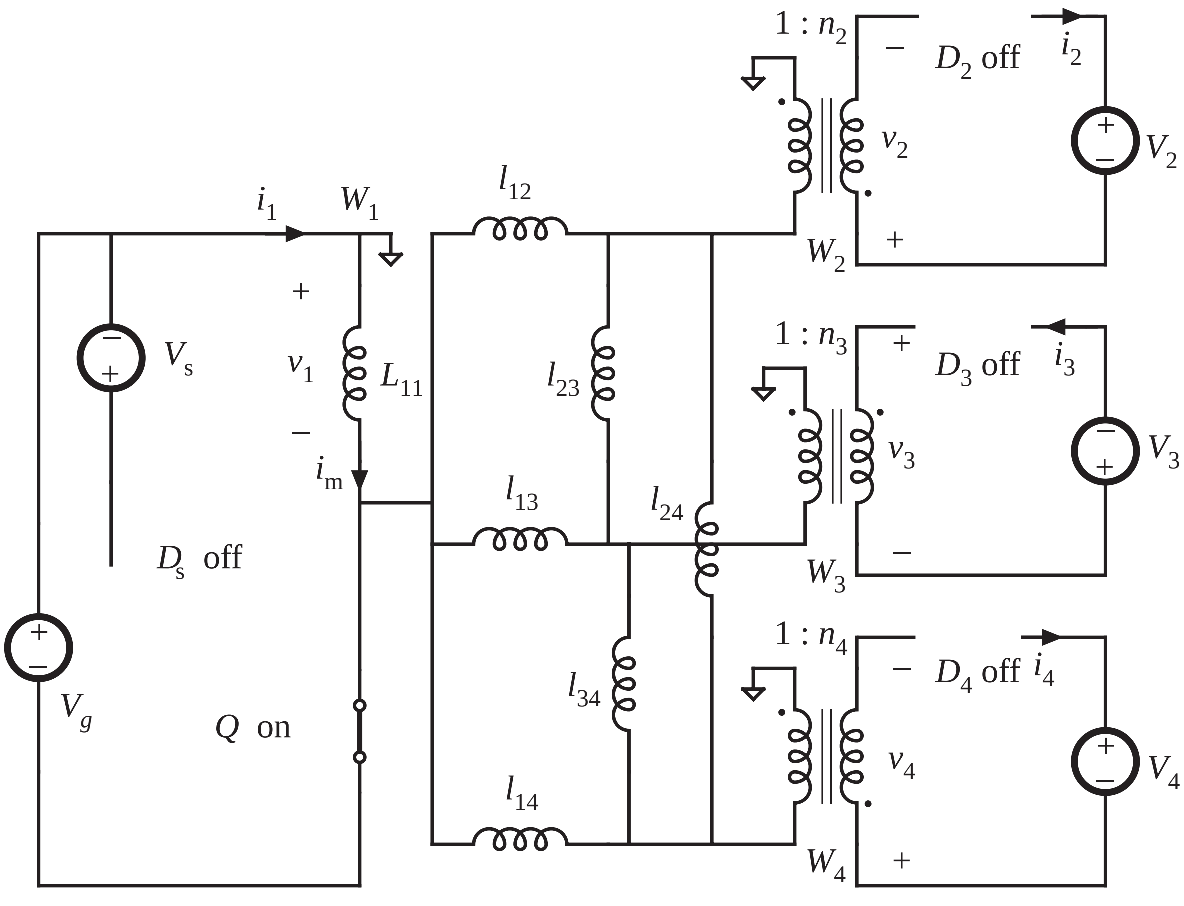
Diode turn-off interval t_d (same as in Case 1)



Currents through leakage inductances decrease to zero after the main switch Q turns on; secondary diodes turn off

Diode turn-off times are not equal

Main switch conduction interval (same as in Case 1)

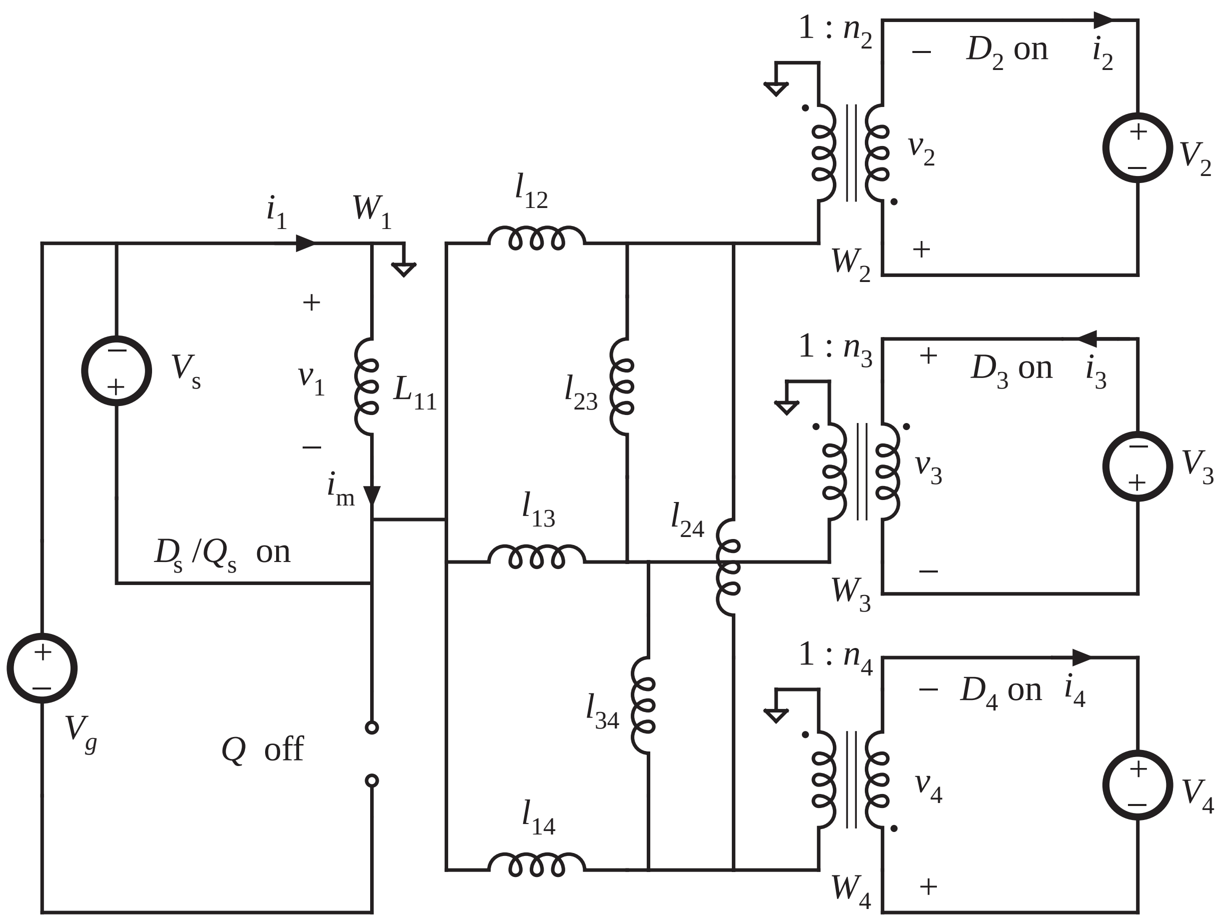


The main switch Q is on, all diodes are off

The magnetizing current increases

All leakage inductor currents are zero

Commutation & diode conduction interval merge into one

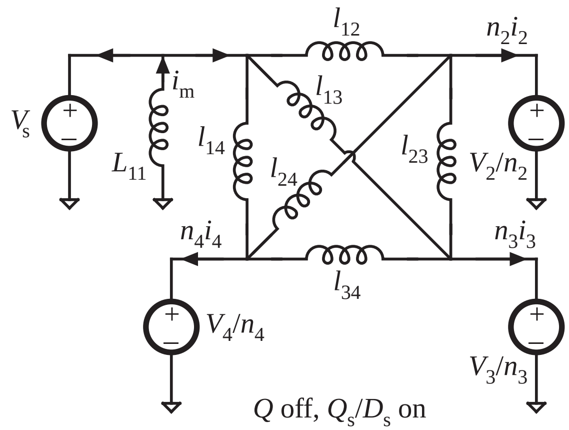


The main switch *Q* turns off, the snubber diode D_s turns on; Q_s is turned on

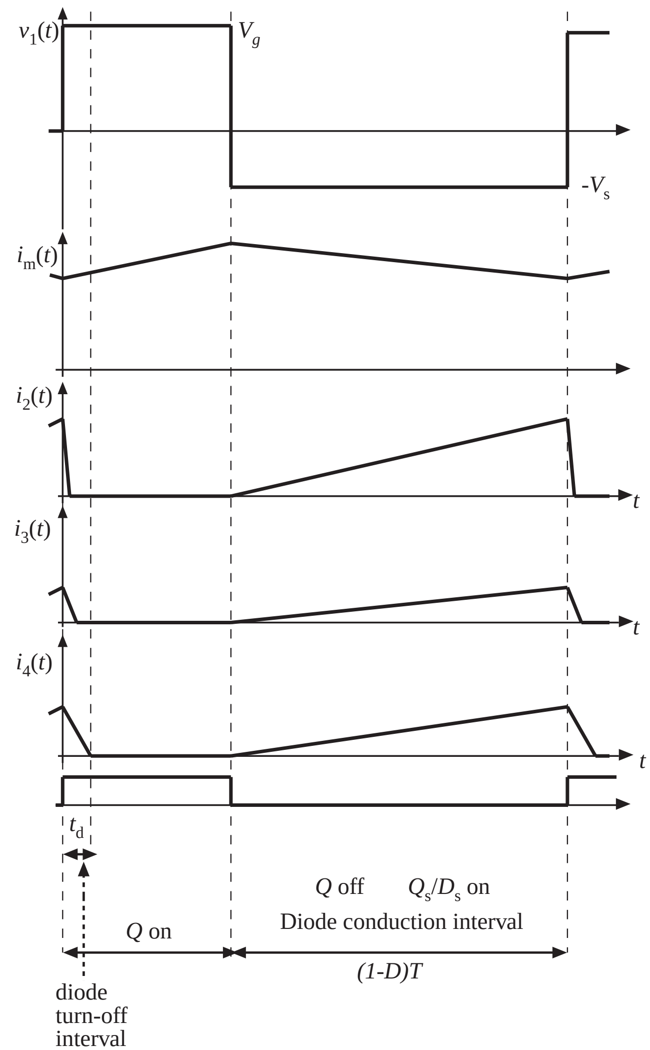
The secondary diodes turn on, and the secondary winding currents start *increasing* at the rates that depend on the leakage inductances and loads

<u>All outputs operate in CCM always</u>

Qualitative Behavior
Case 2: Active-Clamp Snubber



Diode conduction interval

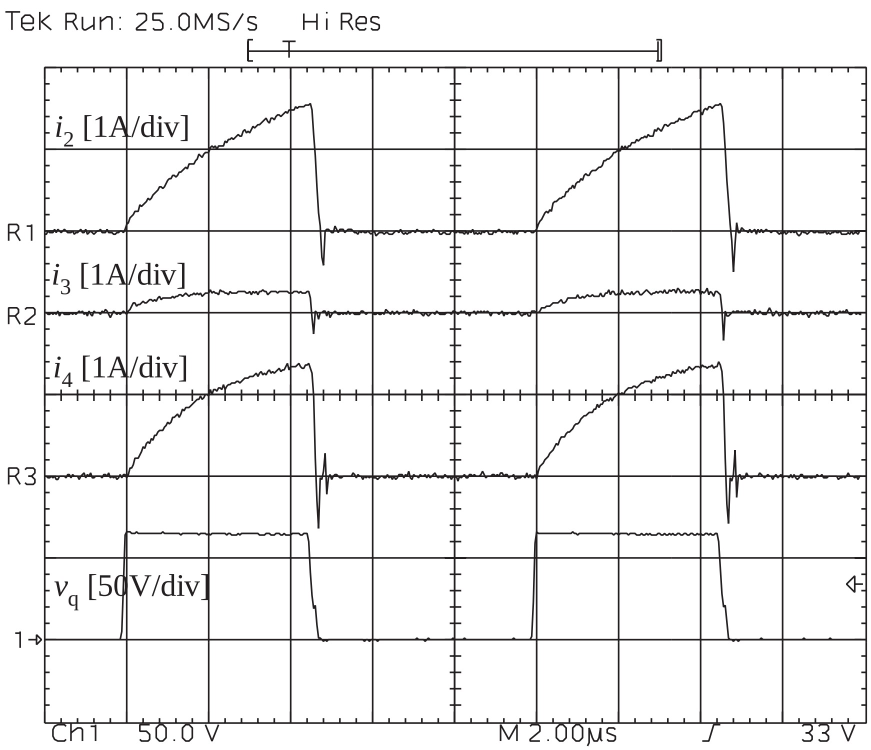


- All secondary winding currents have positive slope
- There is only one operating mode: CCM
- Current slopes can be calculated as in the passive snubber case

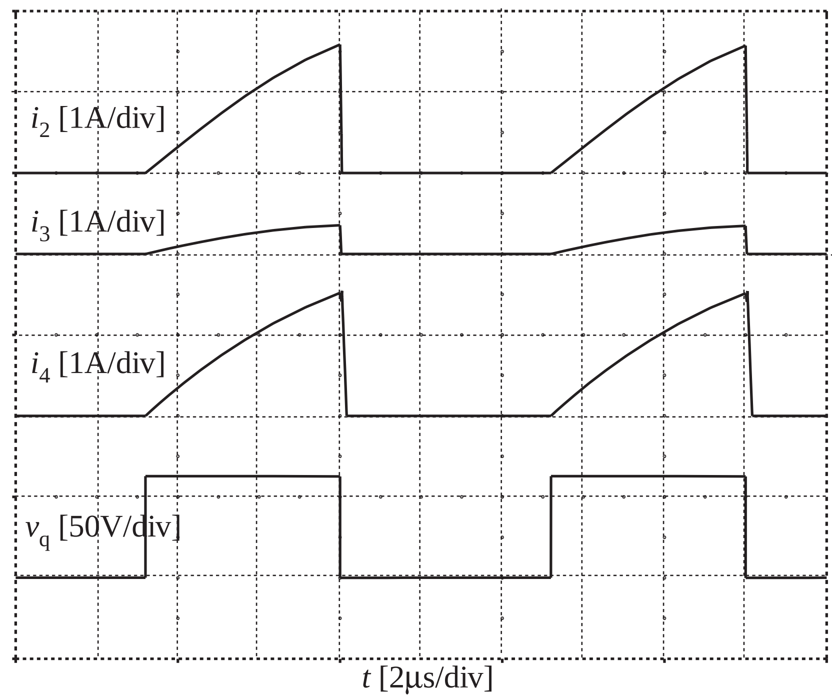
Measured and predicted waveforms

Flyback converter example: Active-Clamp Snubber

Measured waveforms

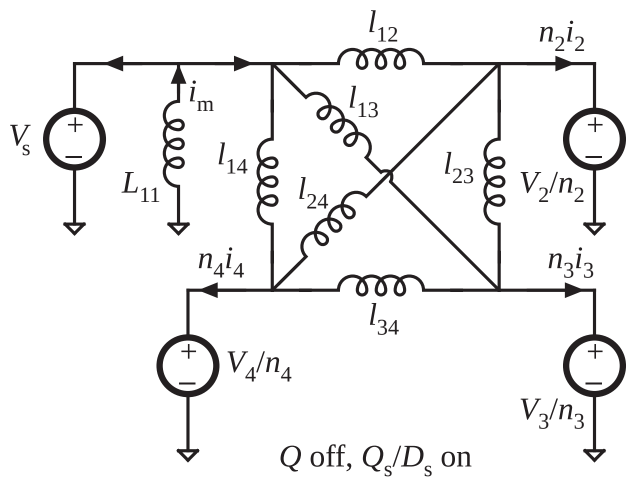


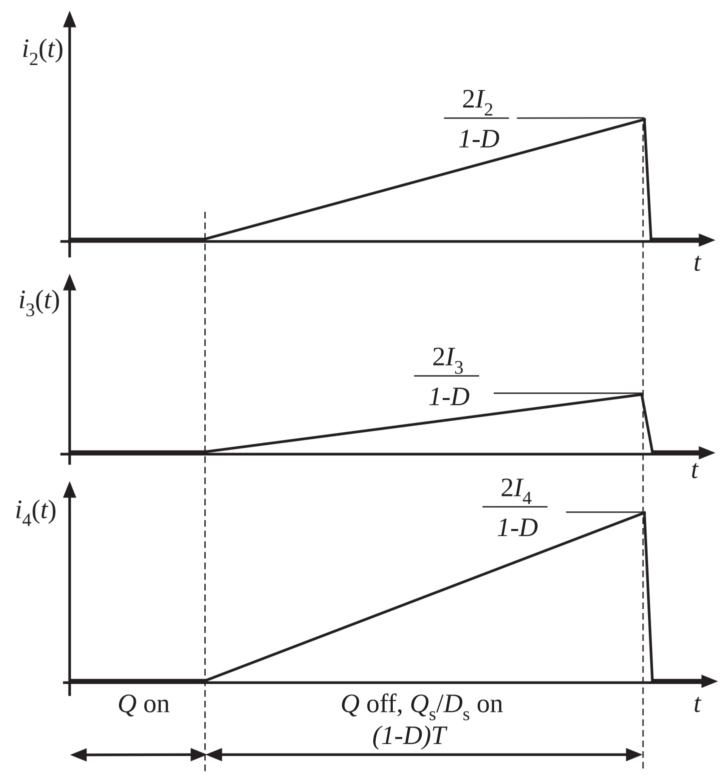
Simulated waveforms
using extended cantilever model



Simulation model included resistive conduction losses

Secondary Current Waveforms
Case 2: Active-Clamp Snubber





Secondary currents increase at the rates determined by the leakage inductances and the loads

Current slopes can be calculated as in the passive snubber case

General Steady-State Solution (Active-clamp snubber)

Averaging of the secondary winding currents gives:

$$\mathbf{B}_1(\mathbf{V}' - \mathbf{u}V_s) = \frac{2f_s}{(1-D)^2}\mathbf{I}'$$

where V_s is found from the volt-second balance on L_{11}:

$$V_s = \frac{D}{1-D}V_g$$

Thevenin Output Resistance Matrix (Active-clamp snubber)

Referred to the primary side:

$$\mathbf{R}' = -\frac{2f_s}{(1-D)^2}\mathbf{B}_1^{-1}$$

Referred to the secondaries: $\Delta\mathbf{V} = -\mathbf{R}\Delta\mathbf{I}$

$$\mathbf{R} = \mathbf{N}\mathbf{R}'\mathbf{N}$$

$$\mathbf{N} = \begin{bmatrix} n_2 & 0 & 0 \\ 0 & n_3 & 0 \\ 0 & 0 & n_4 \end{bmatrix}$$

Predicted and measured output resistance matrix

Operating point: D=0.52, I_2=I_3=0.4A, I_4=1A

Predicted:

$$\begin{bmatrix} \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = - \begin{bmatrix} 0.67 & -0.07 & 0.33 \\ -0.07 & 2.40 & -0.48 \\ 0.33 & -0.48 & 0.56 \end{bmatrix} \begin{bmatrix} \Delta I_2 \\ \Delta I_3 \\ \Delta I_4 \end{bmatrix}$$

Measured variations in the output voltages corresponding to load changes:

$$\begin{bmatrix} \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = - \begin{bmatrix} 1.5 & -0.3 & 0.25 \\ -0.16 & 3.3 & -0.64 \\ 0.25 & -0.7 & 0.90 \end{bmatrix} \begin{bmatrix} \Delta I_2 \\ \Delta I_3 \\ \Delta I_4 \end{bmatrix}$$

Compare to the passive-snubber case: resistance values can be significantly different

Discussion

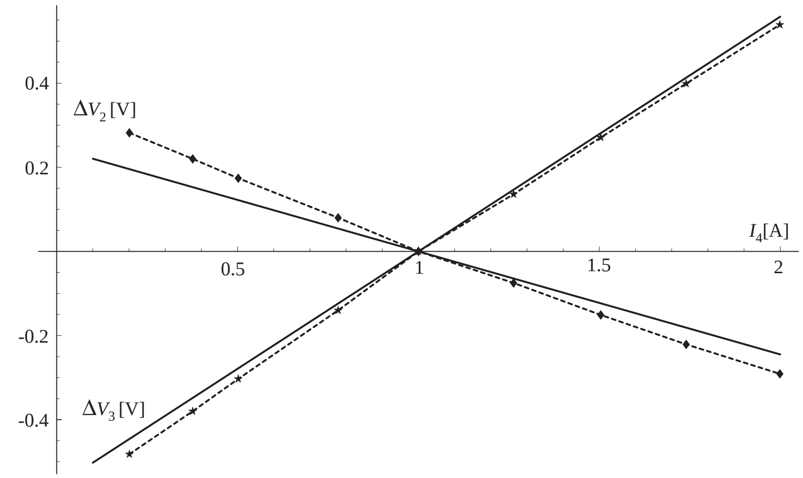
- Cross-regulation (open loop) is in general better if the terms in the Thevenin equivalent output resistance matrix are smaller
- Output resistances are directly proportional to the leakage inductances between the secondaries. Therefore, tighter coupling between the secondaries improves cross-regulation
- On lightly loaded outputs, tight primary-to-secondary coupling may lead to DCM operation and significantly worse cross regulation
- Relative values of primary-to-secondary leakage inductances are important for good cross regulation: the output with the widest load range should have the best coupling to the primary
- Perfect closed-loop cross-regulation can be obtained even if leakage inductances are not small: attempt to match rows of the resistance matrix **R'** referred to the primary side

Closed-loop cross-regulation

- Open-loop cross-regulation model: $\Delta \mathbf{V} = -\mathbf{R}\Delta \mathbf{I}$
- Closed-loop operation:
  - Increasing load on the regulated output increases duty ratio *D* to compensate for the load-induced drop
  - All other output voltages increase with *D*
  - If an auxiliary output has the same open-loop dependence on load currents, i.e., the same rows of the resistance matrix **R'** referred to the primary side, then this auxiliary output will have perfect cross-regulation

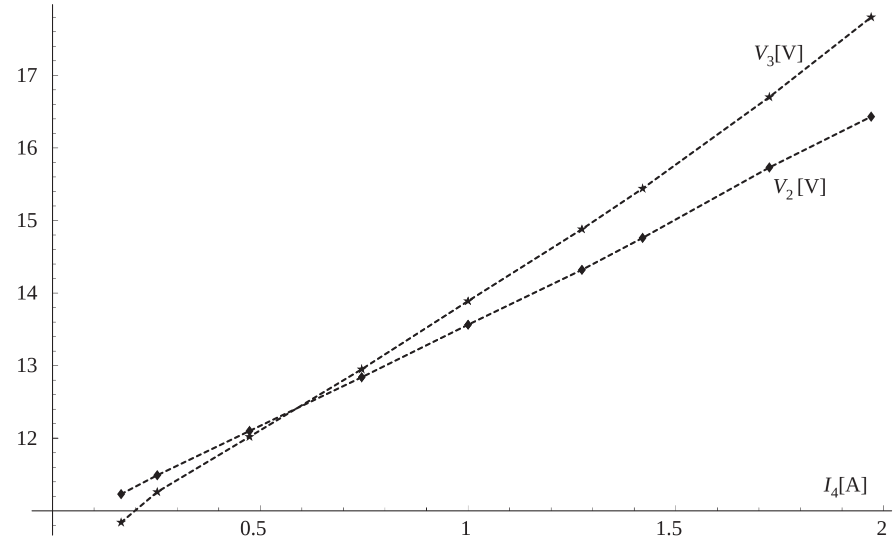
Experimental example

$$\mathbf{R}' = \left[\begin{array}{ccc} 3.8 & -0.4 & 5.5 \\ -0.4 & 13.6 & -8.1 \\ 5.5 & -8.1 & 28.7 \end{array} \right] \Omega$$



- Winding W4 (the last row) is closed-loop regulated
- Winding W2 has better match of the resistance terms with W4 than winding W3
- Winding W3 has the resistance terms of opposite sign !
- Expect better closed-loop cross-regulation on the W2 output (as long as the outputs operate in CCM)

Experimental Closed-loop Cross-regulation



Possible improvements:

- Avoid opposite-sign terms (which in this example came from the negative leakage of the side-by-side winding W3 next to W4)
- Improve matching of resistance terms by better primary-to-regulated output coupling
- Avoid DCM operation by relating primary-to-secondary couplings to loads

Prediction of small-signal dynamics

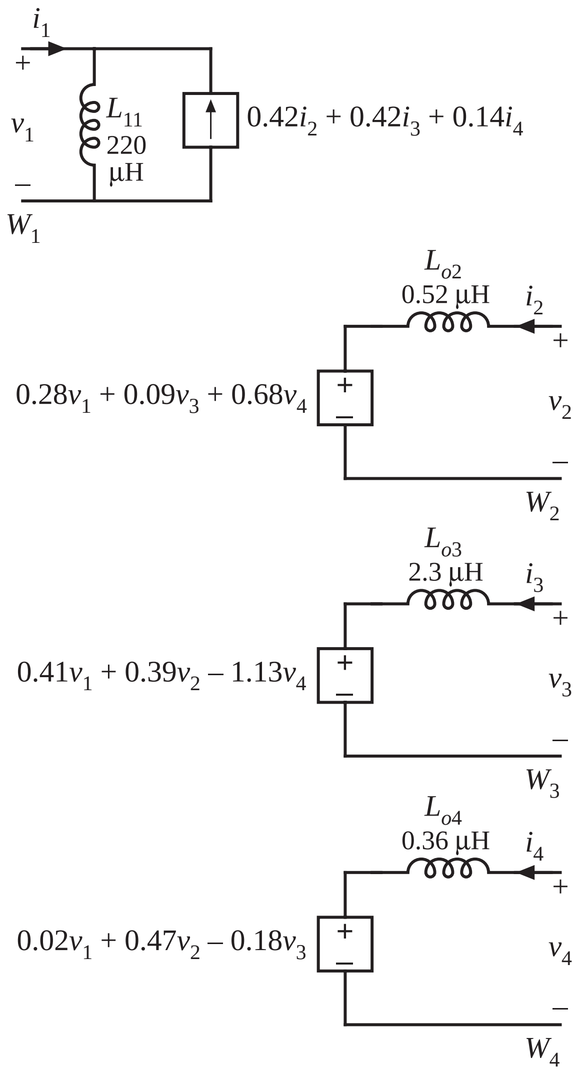
The extended cantilever model can also be used to predict the converter control-to-output transfer function

- Depends on operating modes of auxiliary windings
- Significant changes observed when auxiliary winding changes from DCM to CCM

Computer modeling method is described in reference [18]

- Small-signal frequency response is generated by *Mathematica*, based on converter impulse responses generated by PSPICE or PETS
- Approach automatically accounts for changes in operating mode
- Transformer was simulated using N-port model
- Simulations converged quickly and easily, even though system contained eight states

Measured *N*-port parameter model
Flyback transformer example



The n-port model is well-suited for simulation in PSpice or PETS

The number of inductances is equal to the number of states in a multiple-winding transformer

Measured and predicted transfer functions

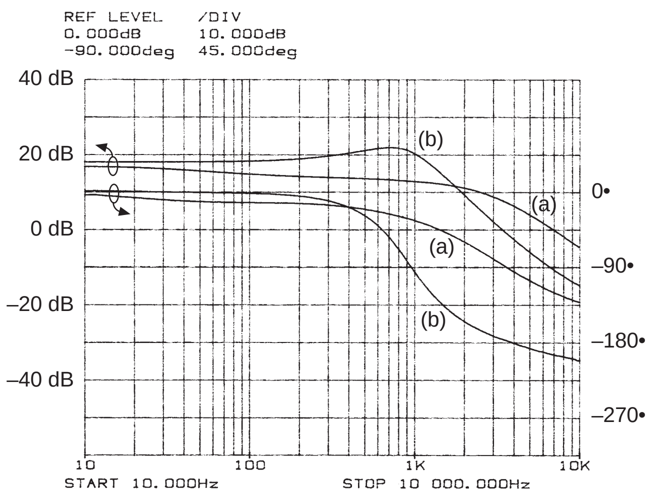
Flyback converter example

Small-signal CCM duty-cycle to W_4 output transfer functions

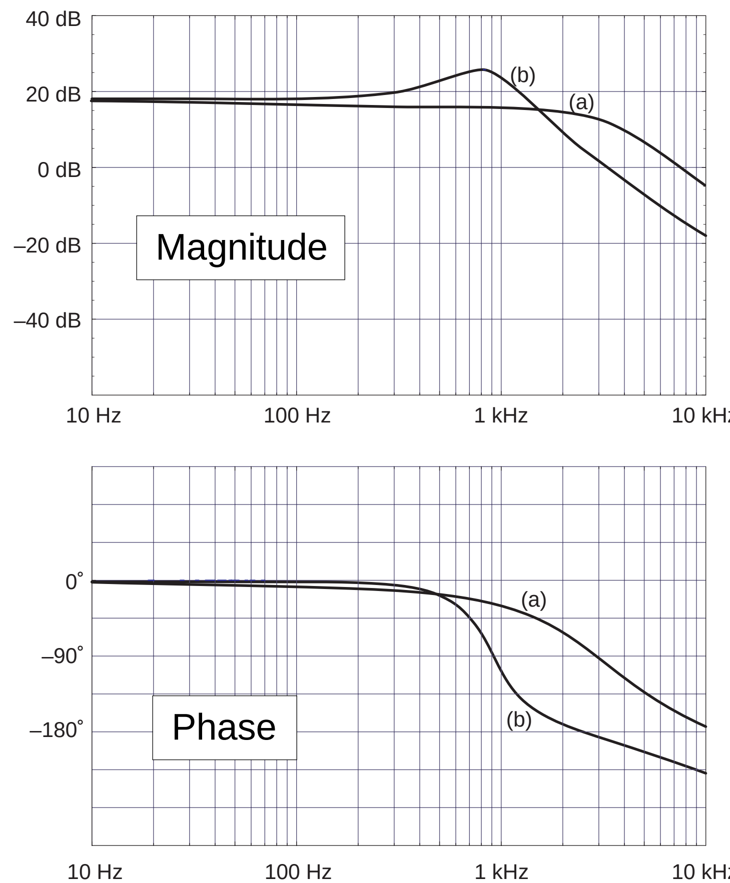
(a) with W_2 and W_3 outputs operating in DCM

(b) with W_2 and W_3 outputs operating in CCM

Measurements



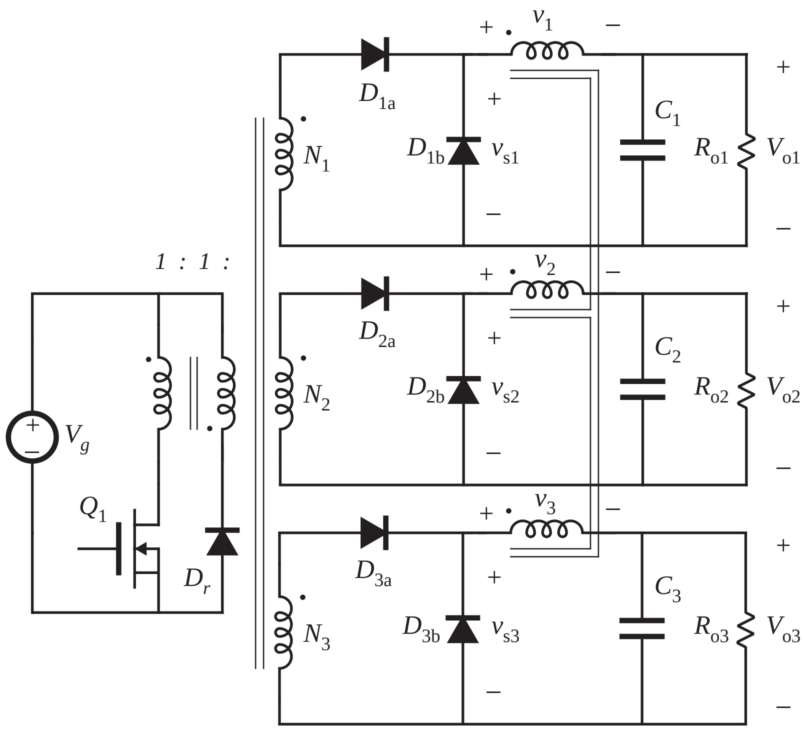
Predictions of model



Summary - Part 2

- Two flyback converter cases considered: (1) passive voltage-clamp snubber and (2) active-clamp snubber
- Multiple-output flyback converter has complex operation strongly affected by the transfomer leakage inductances
- Extended cantilever model offers easy qualitative explanation of all details of operation
- General analytical models derived to predict static cross-regulation in converters with arbitrary number of outputs
- Cross-regulation mechanisms: DCM operation and output resistances due to leakage inductances are now well understood
- The models can be used to evaluate and compare magnetics designs, and to test various approaches to improve cross-regulation
- The extended cantilever model can also be used to correctly predict the converter control-to-output transfer function, and to investigate frequency-responses at various load conditions

3. Cross Regulation in Forward Converters with Coupled Inductors

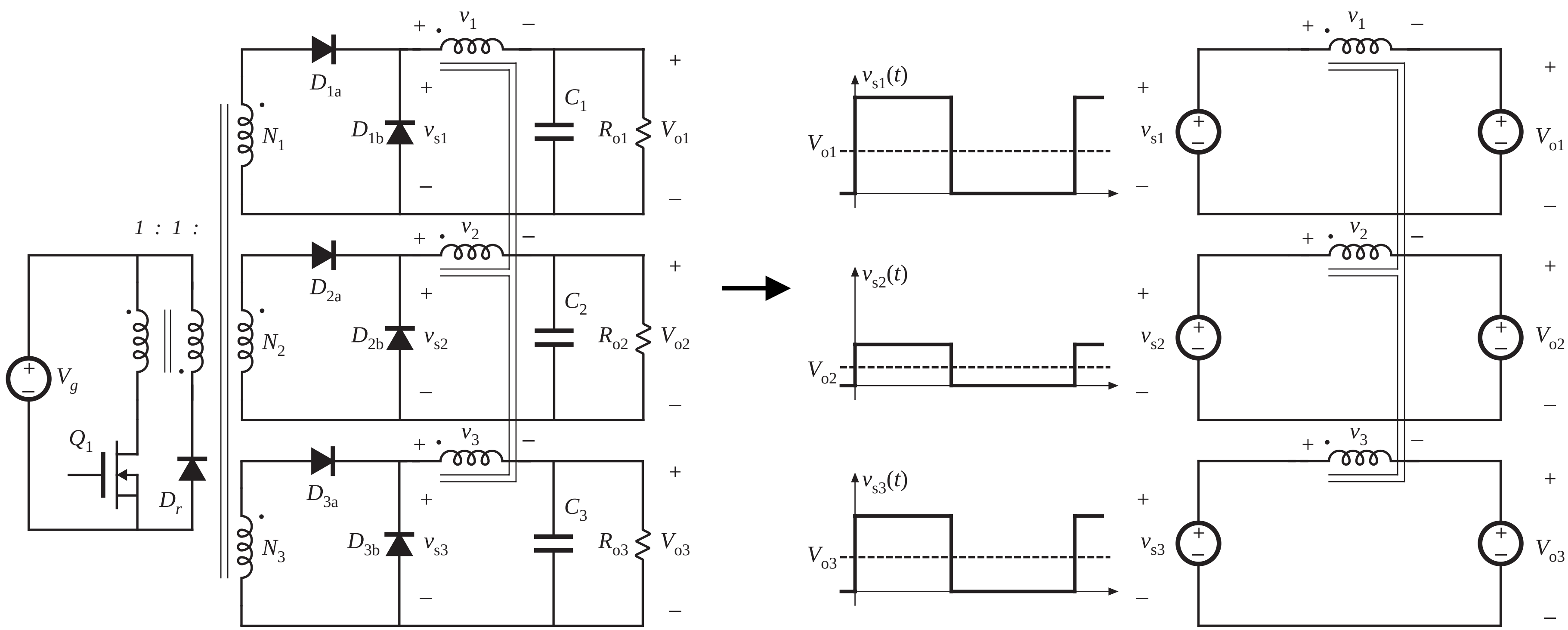


- Conduction losses
  - diodes
  - windings
  - esr
- Unequal diode conduction times due to the transformer leakage inductances
- <u>Discontinuous conduction modes</u>

Objectives

- Explain effects of inductor coupling on cross-regulation using the extended cantilever magnetics model
- Find general solution for the discontinuous-mode boundaries and steady-state conversion ratio
- Discuss approaches to coupled-inductor design:
  - Near-ideal coupling
  - Moderate coupling
  - “Zero-ripple” approach
- Compare predictions with experimental results
- Point to dynamic response considerations

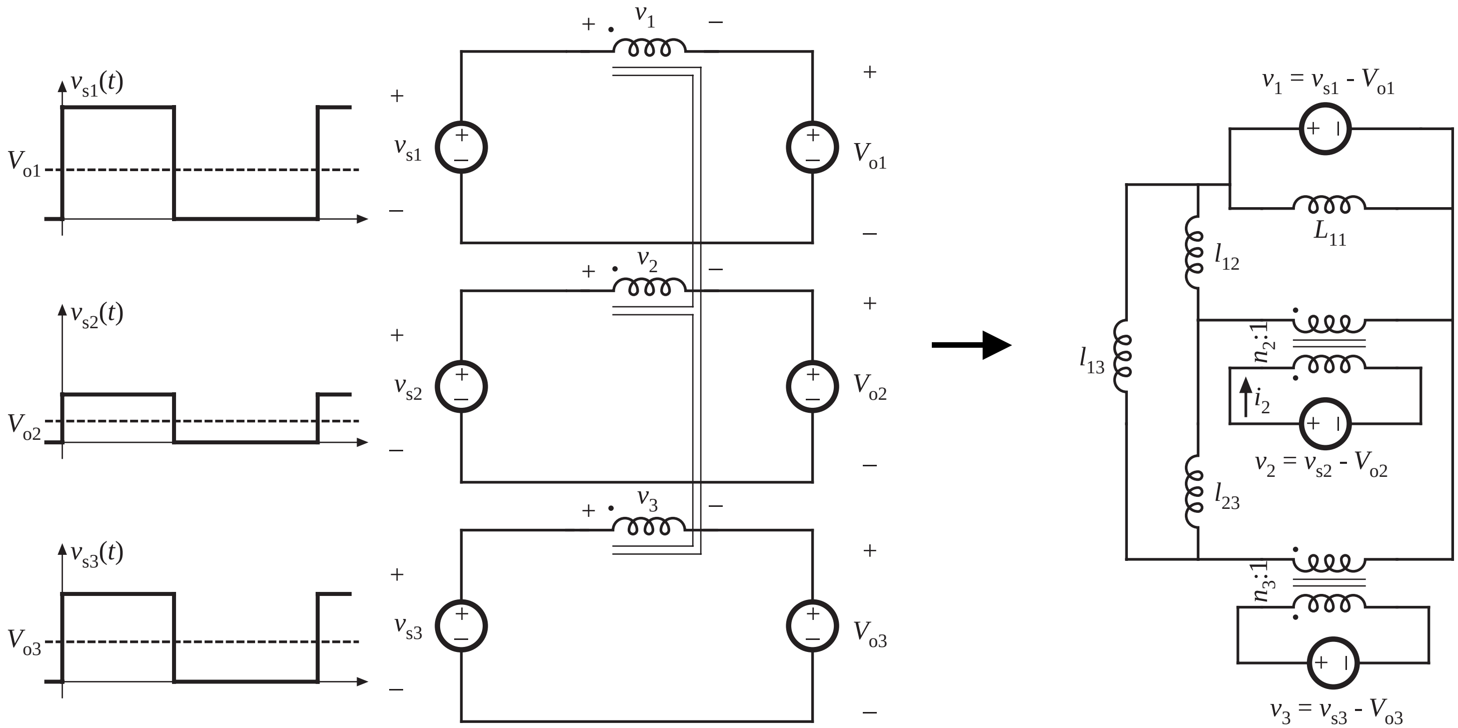
DCM Analysis Using the Extended Cantilever Model



Step 1: Voltage waveforms across coupled-inductor windings

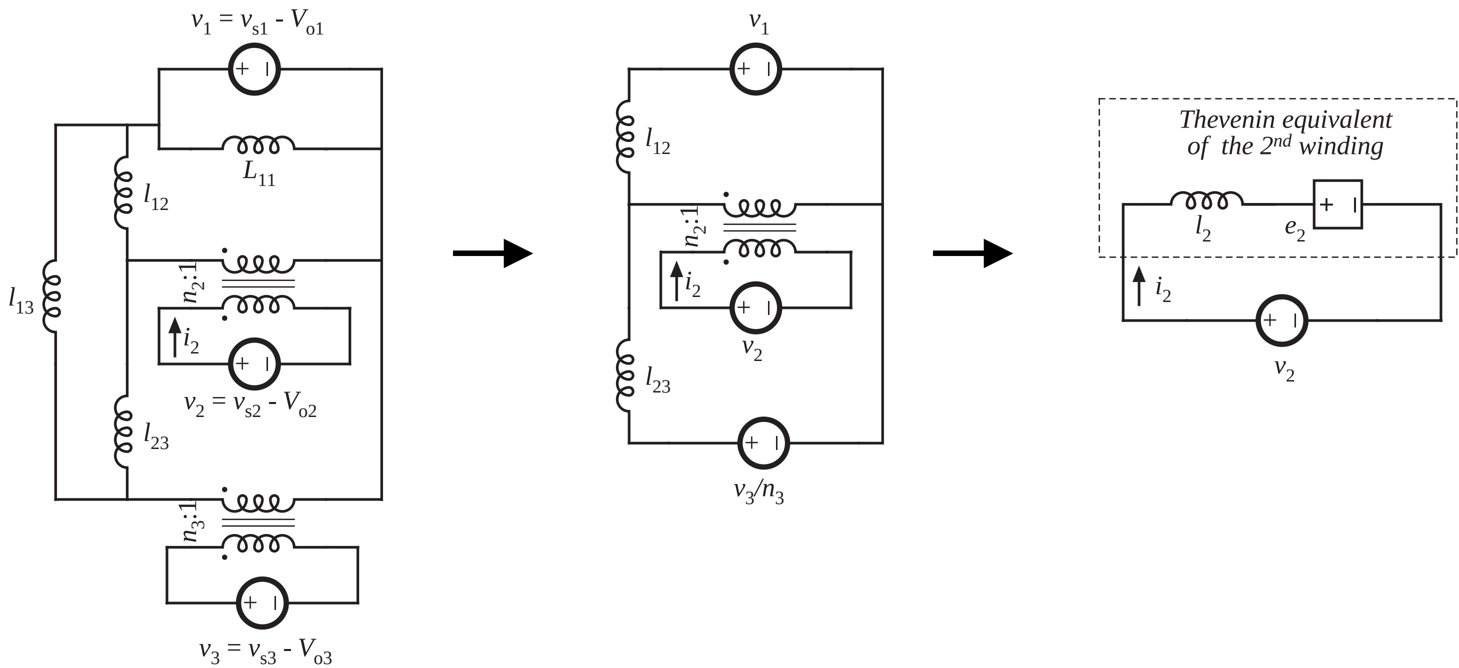
(assume all outputs operate in continuous conduction mode)

DCM Analysis Using the Extended Cantilever Model



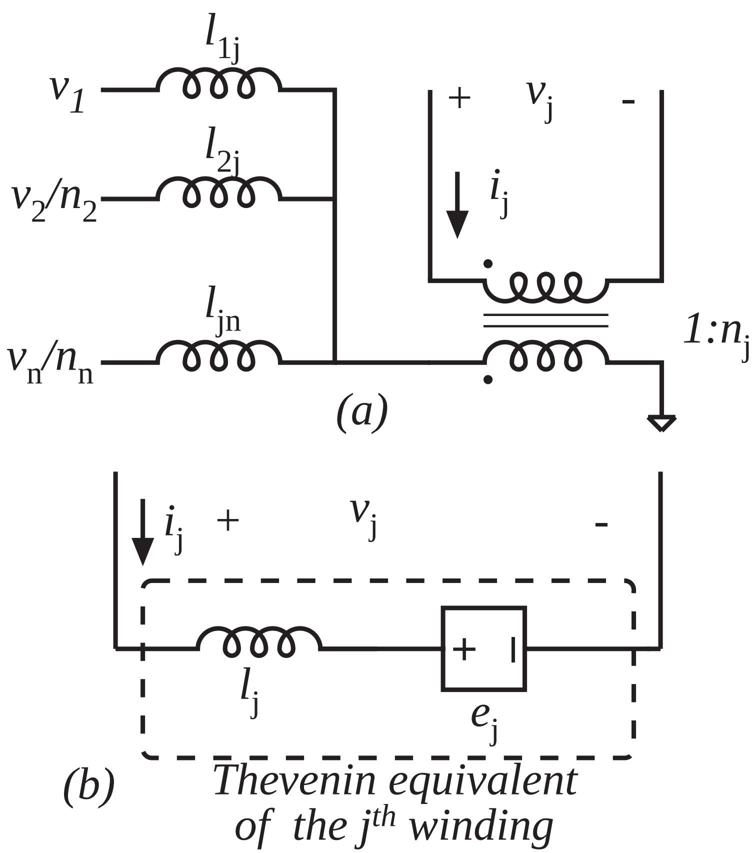
Step 2: Use the extended cantilever model for the coupled inductor

DCM Analysis Using the Extended Cantilever Model



Step 3: Find Thevenin equivalent for one of the windings

Thevenin Equivalent for the j^{th} Winding

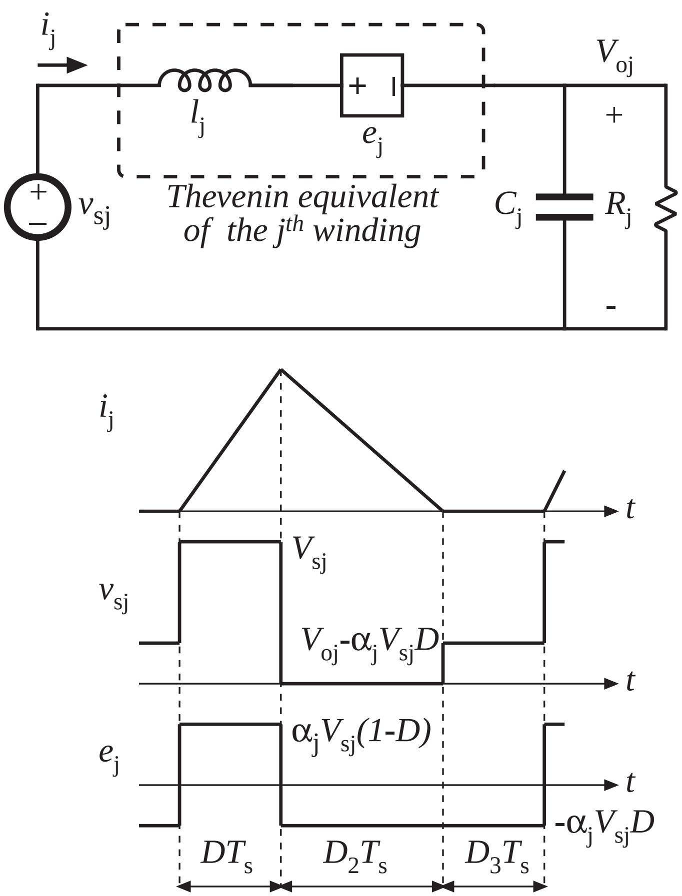


$$e_j = \sum_{\substack{k=1,\ldots,n \\ k\neq j}} a_{jk} v_k$$

$$a_{jk} = \frac{1}{n_j n_k} \frac{l_j}{l_{jk}}$$

$$\frac{1}{l_j} = \frac{1}{n_j^2} \sum_{\substack{k=1,\ldots,n \\ k\neq j}} \frac{1}{l_{jk}}$$

CCM Condition for the j^{th} Winding



$$\frac{v_1}{N_1} = \frac{v_2}{N_2} = \cdots = \frac{v_n}{N_n}$$

$$e_j = \alpha_j v_j\,, \quad \alpha_j = \sum_{\substack{k=1,\ldots,n\\ k\neq j}} a_{jk}\frac{N_j}{N_k}$$

$$l_j\frac{di_j}{dt} = \begin{cases} (V_{sj} - V_{oj})(1-\alpha_j), & 0 \le t \le DT_s \\ -V_{oj}(1-\alpha_j), & DT_s < t \le T_s \end{cases}$$

$$\Delta i_j = \frac{V_{oj}|1-\alpha_j|}{2l_j f_s}(1-D)$$

$$k_j \ge (1-D)\,, \quad k_j = \frac{2l_j f_s}{R_j}\frac{1}{|1-\alpha_j|} = \frac{K_j}{|1-\alpha_j|}$$

DC Conversion Ratio

- In CCM:

$$\frac{V_{oj}}{V_{sj}} = D$$

- Output *j* in DCM, other outputs in CCM:

$$\frac{V_{oj}}{V_{sj}} = \frac{2\left(1-(1-D)\alpha_j\right)}{1-\frac{k_j\alpha_j}{D}+\left(1+\frac{k_j\alpha_j}{D}\right)\sqrt{1+\frac{4k_j(1-\alpha_j)}{(D+k_j\alpha_j)^2}}}$$

DC Conversion Ratio

- $\alpha_j = 0$:

No coupling

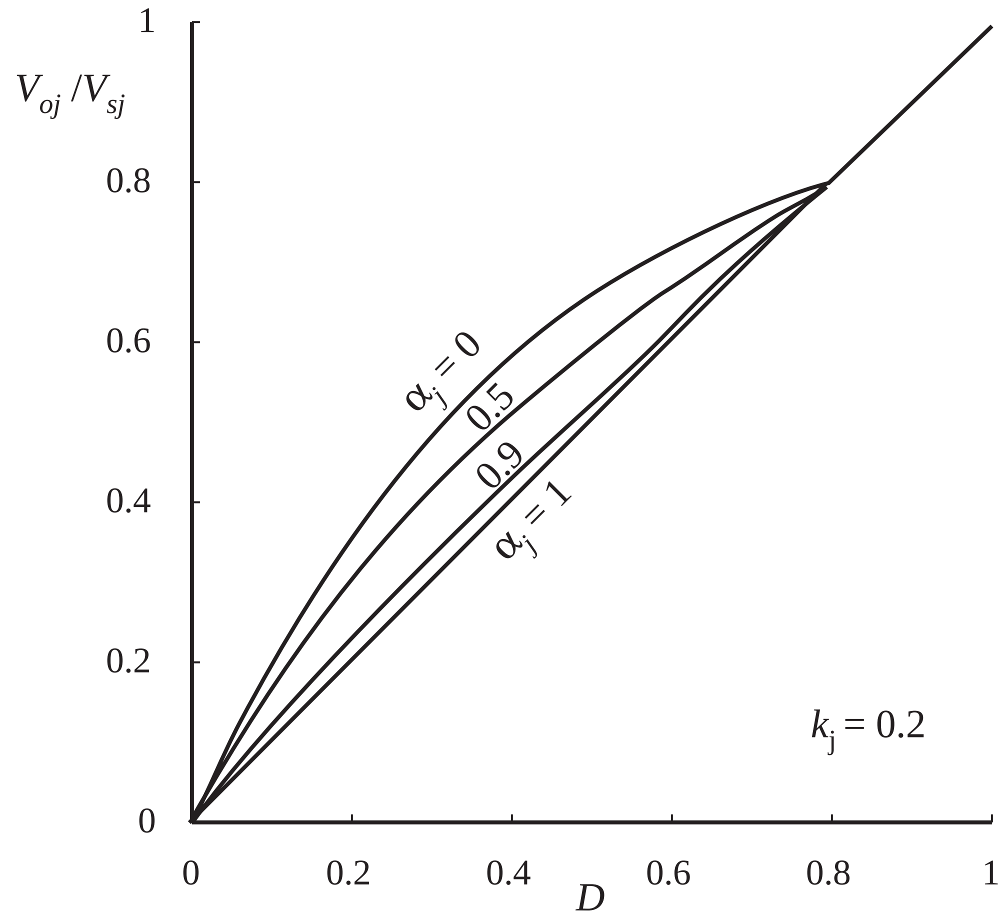
- $\alpha_j = 1$:

"Zero-ripple" approach, CCM at any load

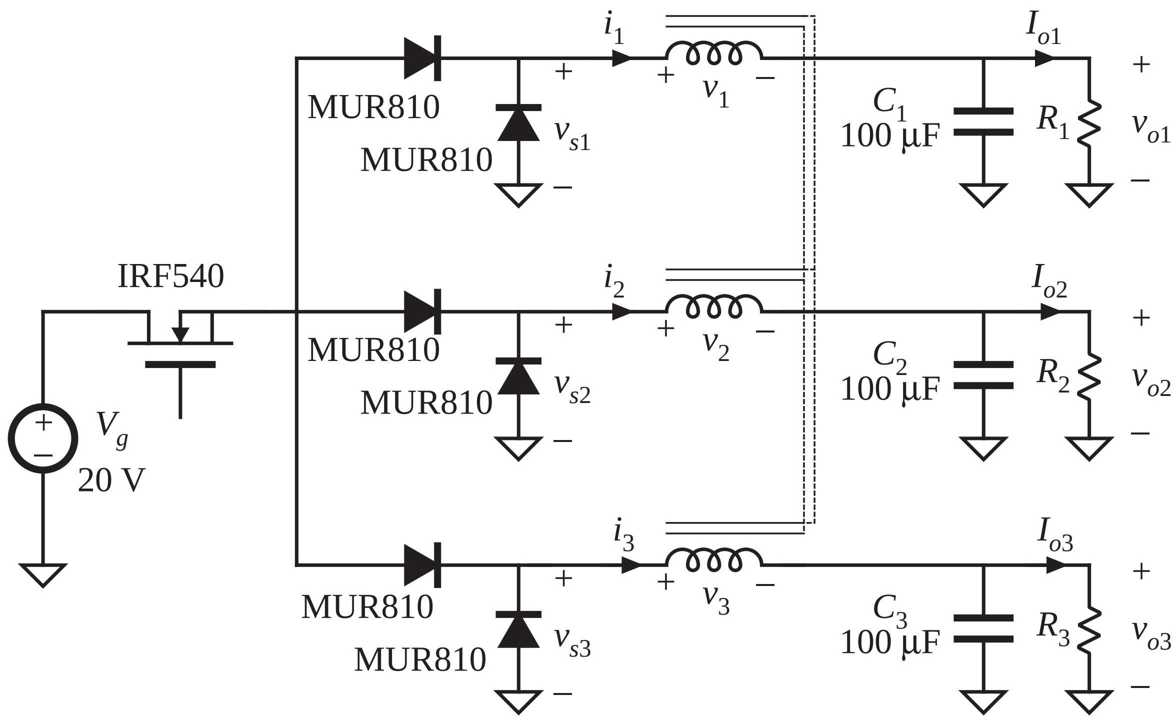
- Smaller l_j :

DCM operation more likely;

Worse cross-regulation in DCM

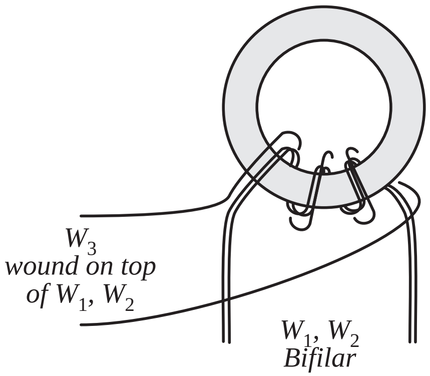


Experimental Example



3-output converter, V_{o1} is the main regulated output at 5.1V

Coupled-Inductor Design Examples

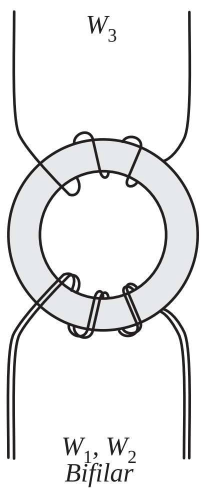


Design #1:

W1 turns: 24

W2 turns: 24, tightly coupled to W1

W3 turns: 24, moderately coupled to W1



Design #2:

W1 turns: 24

W2 turns: 24, tightly coupled to W1

W3 turns: 28, poorly coupled to W1, # of turns selected for “zero-ripple”

Magnetics Inc. 58254 powdered iron core

Model parameters

Design #1:

n_2=1.004, n_3=0.919, l_{12}=0.36uH, l_{13}=21.3uH, l_{23}=16.4uH

l_2=0.36uH, α_2=1.006 (W2 tightly coupled to W1)

l_3=7.81uH, α_3=0.919 (W3 moderately coupled to W1)

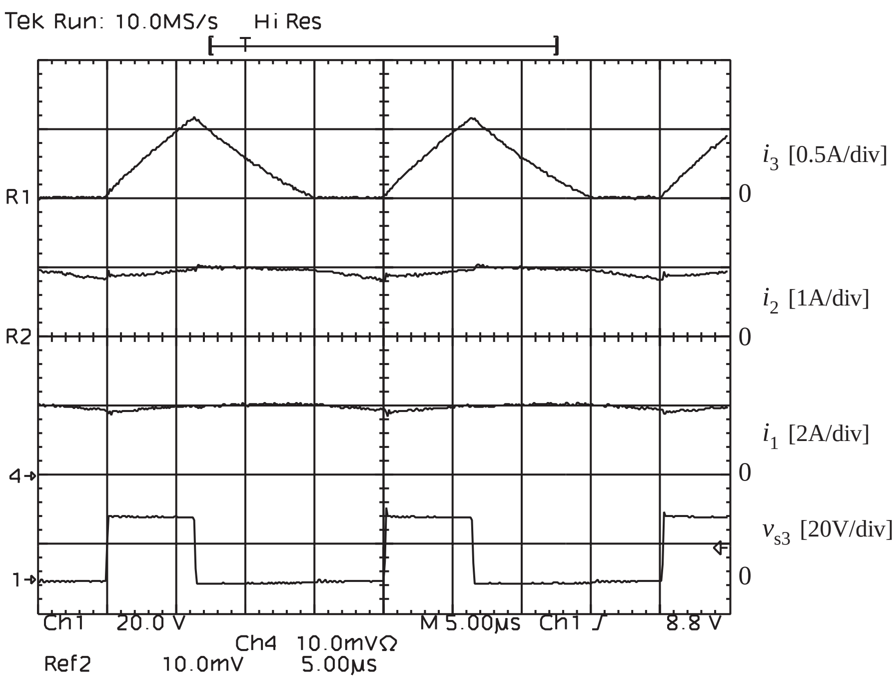
Design #2

n_2=0.997, n_3=0.994, l_{12}=0.36uH, l_{13}=21.3uH, l_{23}=16.4uH

l_2=0.38uH, α_2=0.99 (W2 tightly coupled to W1)

l_3=44.3uH, α_3=0.994 (“zero-ripple” approach in W3-to-W1 coupling)

Near-Ideal Coupling vs. Moderate Coupling
Coupled-Inductor Design #1

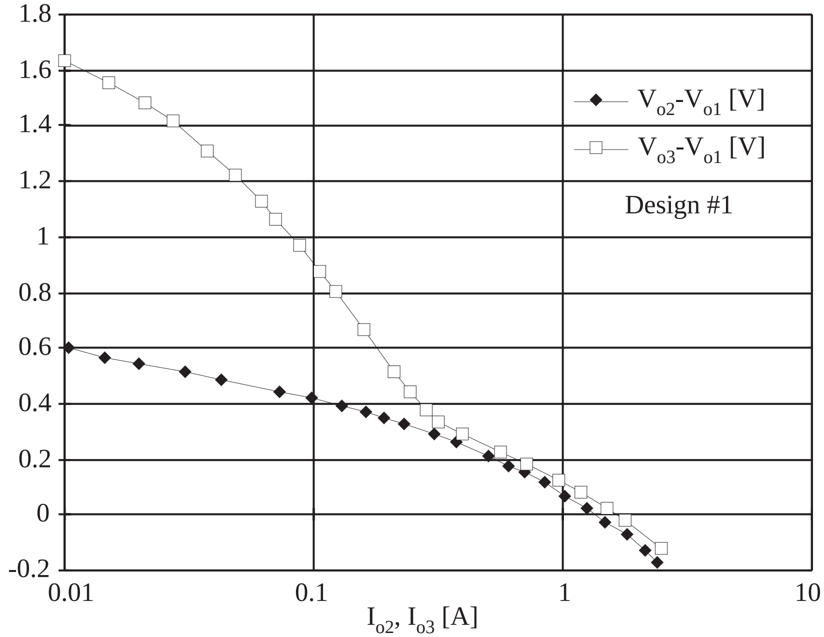


W3 lightly loaded, operates in DCM

V_{o1}=5.1V, I_{o1}=2A, V_{o2}=5.3V, I_{o2}=0.97A, V_{o3}=5.6V, I_{o3}=0.2A, V_g=20V, f_s=100kHz

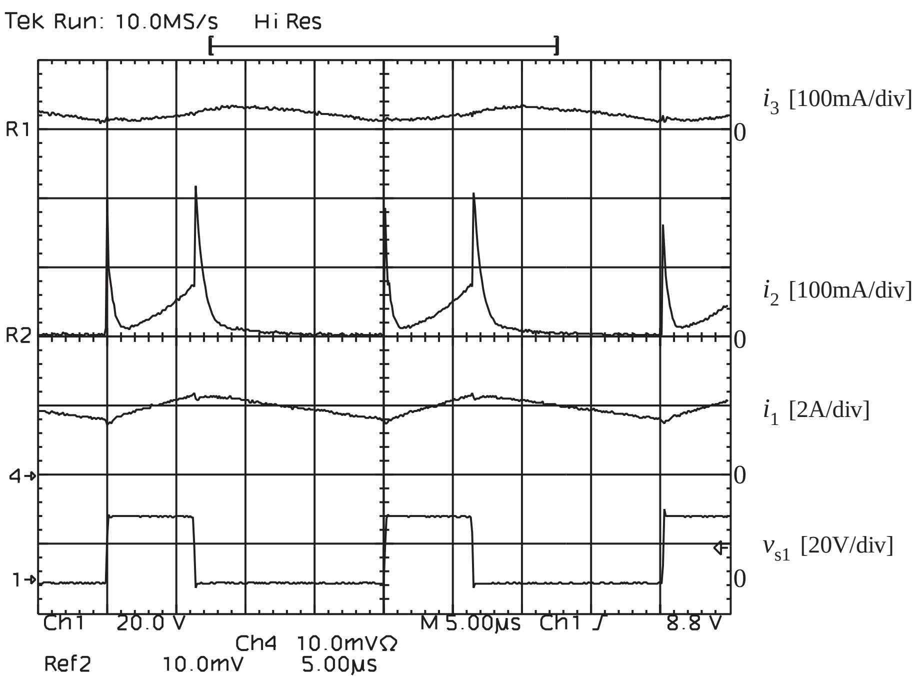
Near-Ideal Coupling vs. Moderate Coupling Coupled-Inductor Design #1

Measured closed-loop cross-regulation, V_{o1} regulated at 5.1V, I_{o1}=2A



Worse cross-regulation on the moderately-coupled W3 because of DCM operation

Near-Ideal Coupling vs. Zero-Ripple Approach Coupled-Inductor Design #2

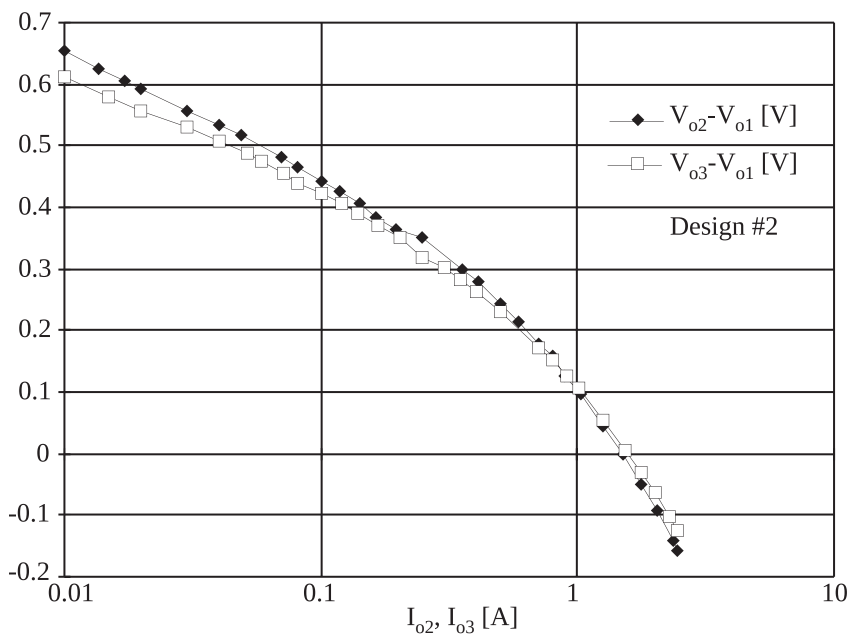


Current spikes and DCM operation on the tightly coupled W2 output

CCM operation and small current ripple on the W3 output

V_{o1}=5.1V, I_{o1}=2A, V_{o2}=5.7V, I_{o2}=0.2A, V_{o3}=5.7V, I_{o3}=0.2A

Near-Ideal Coupling vs. Zero-Ripple Approach Coupled-Inductor Design #2



Measured closed-loop cross-regulation, V_{o1} regulated at 5.1V, I_{o1}=2A

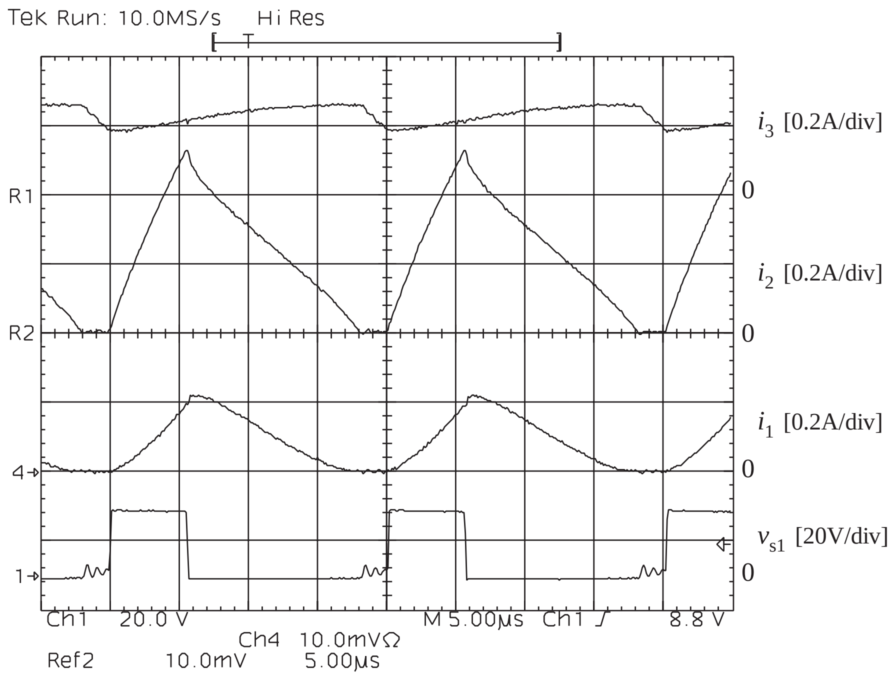
The tightly-coupled output operates in DCM and has large current spikes

Tight coupling may not be easy to achieve with larger number of outputs

The “zero-ripple” approach gives slightly better cross-regulation than tight coupling, and can in practice be achieved easily for any number of outputs

In the “zero-ripple” approach, one output operates with non-zero ripple determined by L_{11}

Zero-Ripple Approach: DCM on Other Outputs



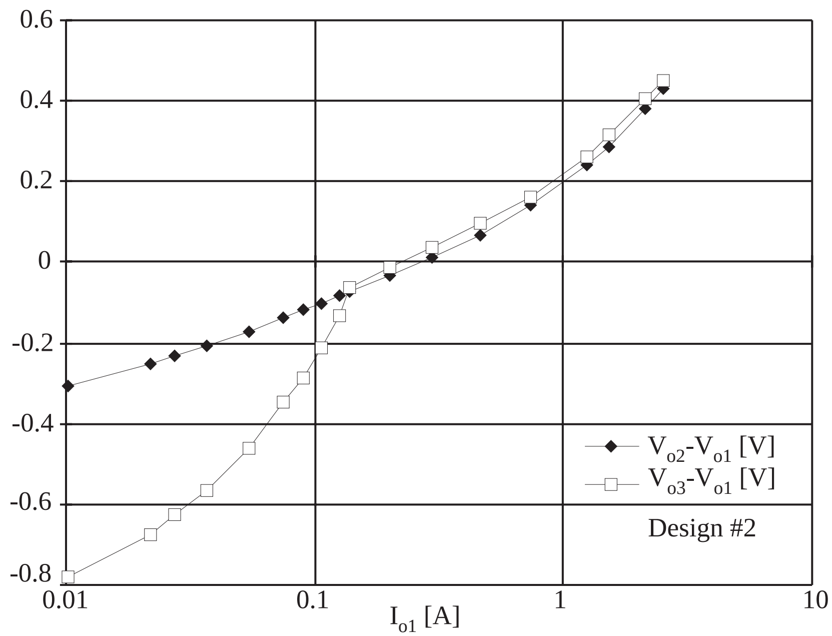
W1 and W2 in DCM, so:

Voltage waveforms across W1 and W3 are no longer propotional

Cross-regulation between the regulated W1 output and the W3 output is affected because of relatively poor coupling between W1 and W3

W3 current has increased ripple

Zero-Ripple Approach: DCM on Other Outputs



Since W3 is poorly coupled to W1, DCM on W1 causes poor cross-regulation on W3

"Zero-ripple" approach works best if the non-zero-ripple output always operates in CCM

Possible approach:

- Use a heavily loaded output as the non-zero-ripple output
- Adjust the number of turns on the other windings to scale effective turns ratios and achieve the "zero-ripple" condition of equal induced and applied voltages to the winding

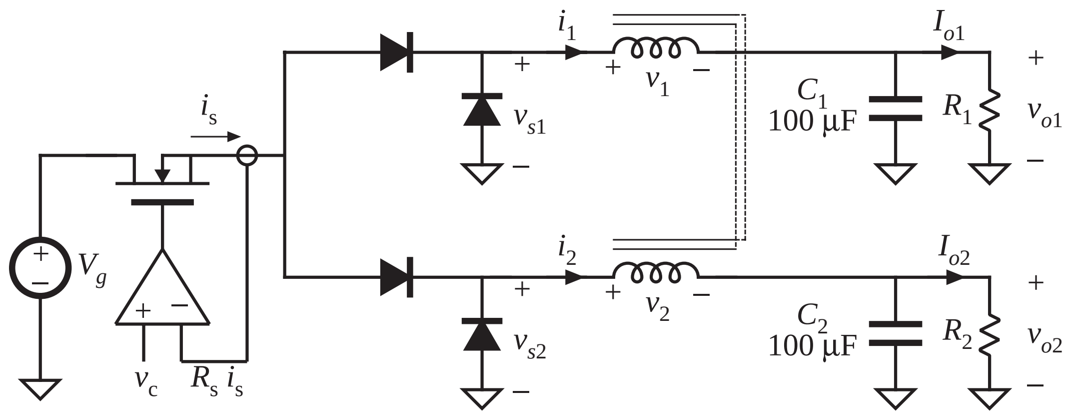
Coupled-Inductor Design Approaches

- Near-ideal coupling: (very small l_j, $\alpha_j \approx 1$)
  - Good cross-regulation even in DCM
  - Exact matching of turns ratios in necessary
  - Significant current spikes, larger ripples
  - May be difficult to achieve in practice
- Moderate coupling (moderate l_j, $\alpha_j \neq 1$)
  - Degraded cross-regulation in DCM
- "Zero-ripple" approach (moderate to large l_j, $\alpha_j \approx 1$)
  - Effective turns ratios match the ratios of imposed voltages
  - Effective turns ratios differ from the turns ratios of physical windings
  - CCM operation down to almost zero load
  - Very small ripples
  - Best static cross-regulation if non-zero-ripple output always operates in CCM

Frequency-Response Considerations

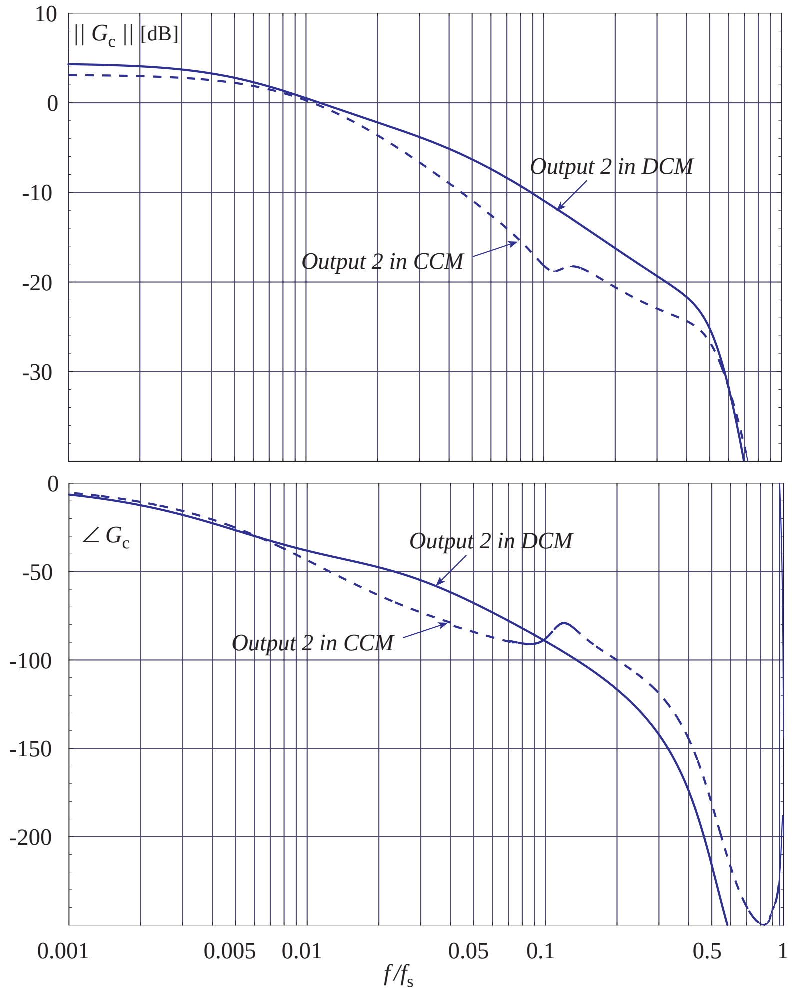
- Experimental 5-output forward converter with current-mode programming used in the feedback loop
- Coupled-inductor had moderately coupled auxiliary windings with physical turns ratios matching the physical turns ratios on the transformer secondaries
- Load variations on the auxiliary outputs produced very large changes in the experimental control-to-output response and cross-over frequency of the feedback loop
- Observed behavior can be explained once discontinuous conduction modes on the auxiliary outputs are taken into account

2-output Circuit Model



- V_{o1} is the main regulated output
- Coupled-inductor design is using windings W1, W3 of the design #1 with W3 moderately coupled to W1
- Control-to-output frequency response was found using the method of reference [18]
- Load on the auxiliary output changed to move from CCM to DCM

Control-to-output responses

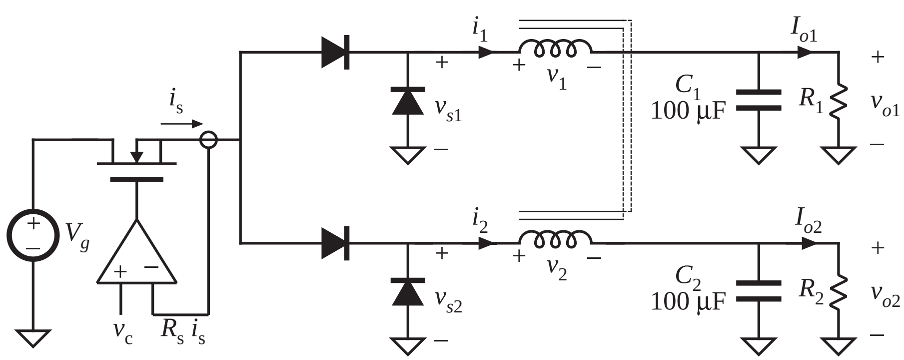


The model correctly predicts a significant change in the magnitude and phase responses

A factor of two change in the cross-over frequency f_c would result if f_c is between 5% and 20% of the switching frequency

Explanation

- Simple model for current-mode programming predicts a single-pole response with pole frequency inversely proportional to the output filter capacitance
- When the auxiliary output operates in CCM, the effective filter capacitance referred to the main output is C_1+C_2
- When the auxiliary output operates in DCM, the effective filter capacitance referred to the main output reduces to C_1; C_2 is “disconnected” from the main feedback loop because the auxiliary output operates in DCM



Summary - Part 3

- Extended cantilever model used to explain operation of the coupled-inductor in a multiple-output forward-type converter
- General analytical solution found for DCM/CCM boundaries, and for conversion ratio when one of the outputs operates in DCM
- Three coupled-inductor design approaches evaluated and compared:
  - near-ideal coupling
  - moderate coupling
  - “zero-ripple” approach
- Effects of possible DCM operation in the auxiliary outputs on the main contol loop were pointed out and explained