MODELING OF CROSS-REGULATION IN CONVERTERS CONTAINING COUPLED INDUCTORS

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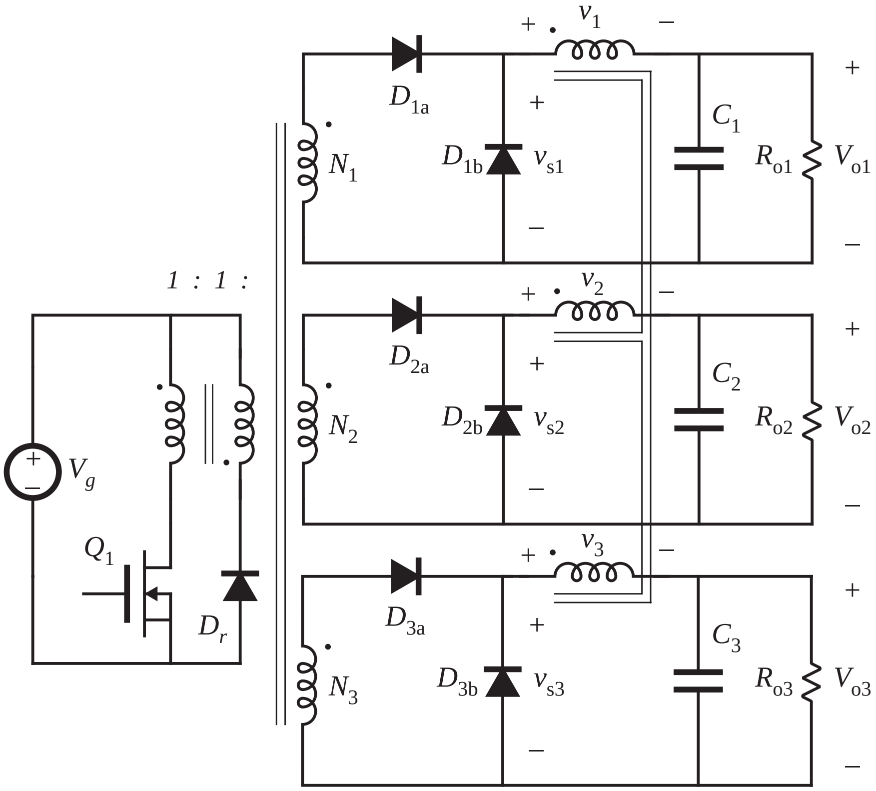
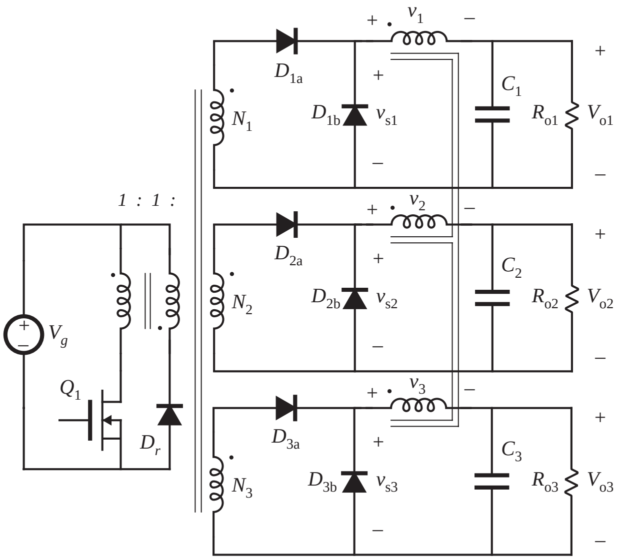
INTRODUCTION



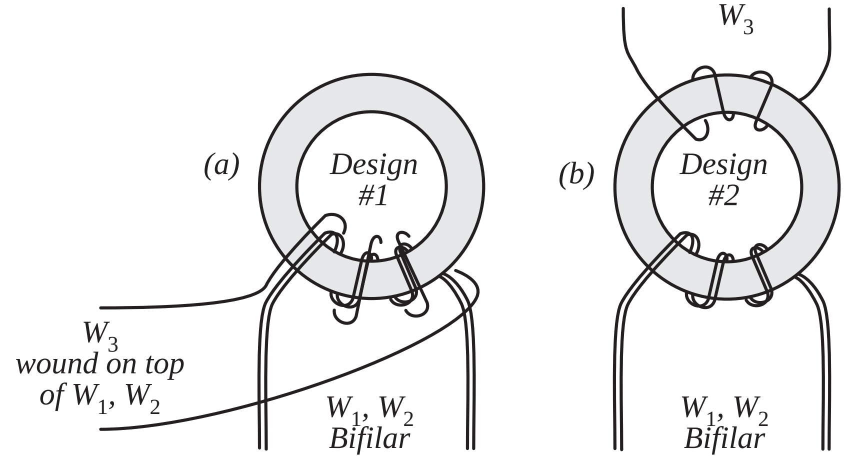
Example: a three-output forward converter with coupled inductors

Static Cross Regulation



- **Conduction losses (diodes, magnetics windings, capacitor esr)**
- **Transformer leakage inductances, unequal diode conduction times**
- **<u>Discontinuous conduction modes</u>**
- **Suitable multiple-winding magnetics models ?**

Multiple-Winding Magnetics Models



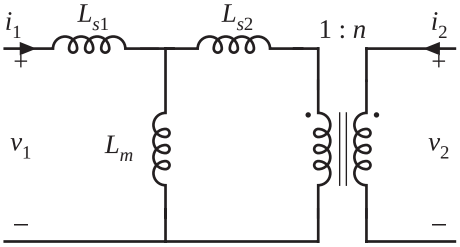
- **General case:**
  n **windings** $\Rightarrow$ $n(n+1)/2$ **model parameters**
- **Simple model with ideal transformer, magnetizing inductance and leakage in series with each winding:**
  - $2n$ parameters
  - In general, not valid for $n > 3$
- **Reduced-order models based on physical and geometrical arguments [5,6,...]**
  - **Model validity ?**
  - **May be difficult to derive (toroidal geometries, distributed air-gaps)**

- **Model with self (L_{jj}) and mutual ($L_{ij} = L_{ji}$) inductances:**
  - **General**
  - **Parameters can be measured easily**
  - **Supported by simulation tools**

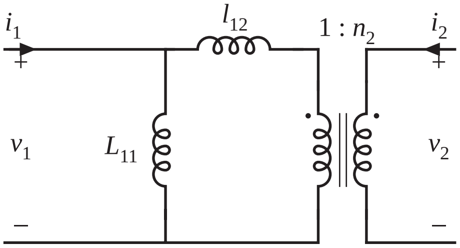
  Not well-suited for magnetics with tightly-coupled windings because:
  - **calculations of leakage inductances are ill-conditioned**
  - **small errors in L_{jj}, L_{ij} may result in completely wrong values of leakage inductances**

A Different Approach

- **T-model of a 2-winding transformer:**

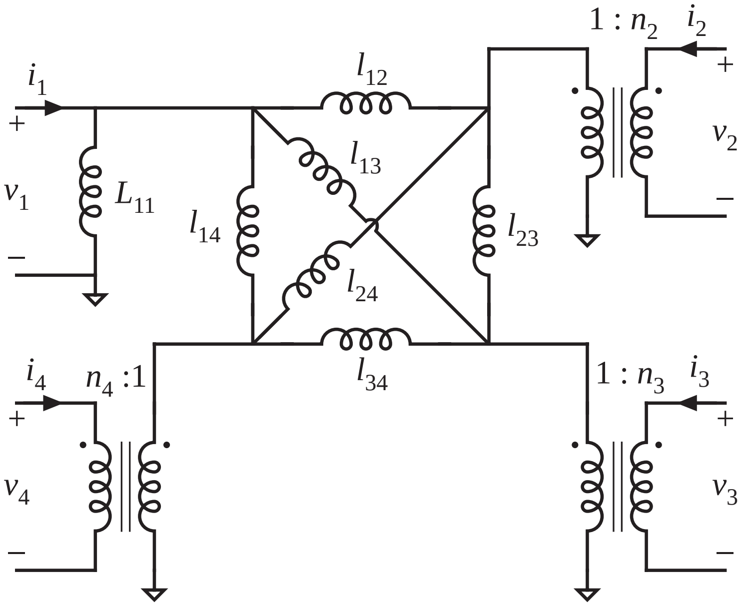


- **Cantilever model of a 2-winding transformer:**



- **Approach:**
  extend the cantilever model to n windings

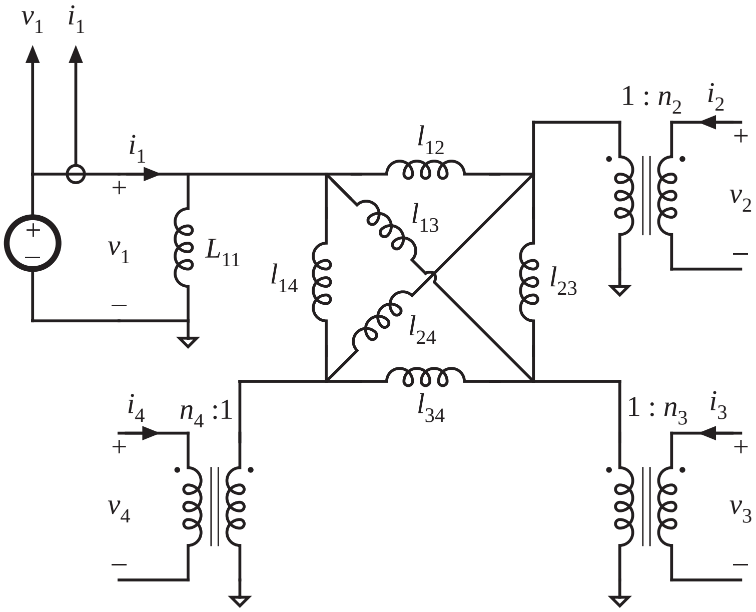
Extended Cantilever Model (4-winding example)



Parameters:

- **Shunt inductance:** L_{11}
- **Effective turns ratios:** n_j
- **Leakage inductances:** l_{ij}

Extended Cantilever Model
(4-winding example)



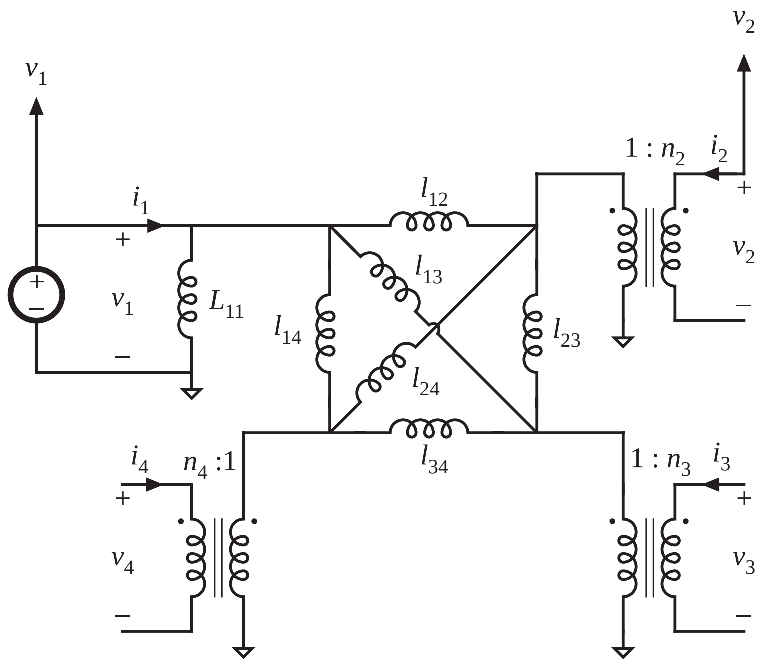
Parameters:

- **Shunt inductance, measurement:**

$$L_{11} = \frac{1}{\omega} \left\| \frac{v_1}{i_1} \right\|$$

- **Effective turns ratios:** n_j
- **Leakage inductances:** l_{ij}

Extended Cantilever Model
(4-winding example)



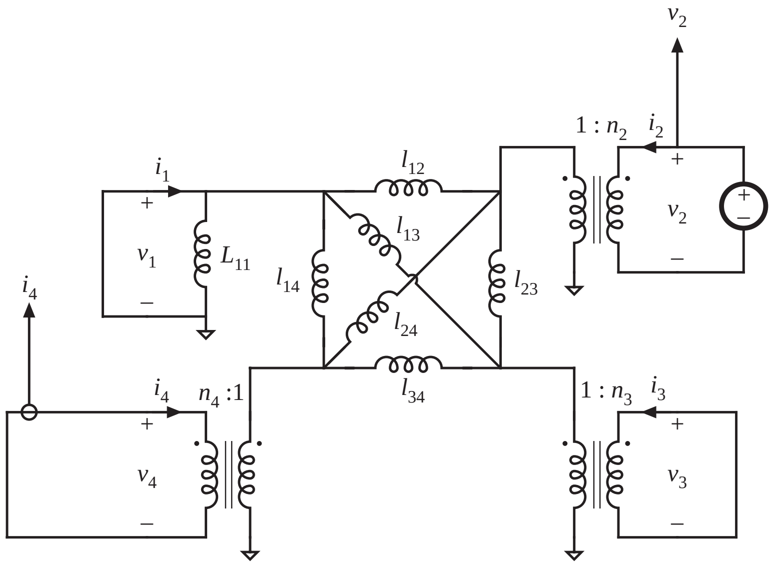
Parameters:

- **Shunt inductance** L_{11}
- **Effective turns ratios:** n_j, **measurement example:**

$$n_2 = \left\| \frac{v_2}{v_1} \right\|$$

- **Leakage inductances:** l_{ij}

Extended Cantilever Model
(4-winding example)



Parameters:

- **Shunt inductance**
- **Effective turns ratios:** n_j
- **Leakage inductances:** l_{ij}**, measurement example:**

$$l_{24} = \frac{1}{\omega}\frac{1}{n_2 n_4}\left\|\frac{v_2}{i_4}\right\|$$

(In general, in same cases not frequently found in practice, l_{ij} can have negative values, so the phase information should be observed)

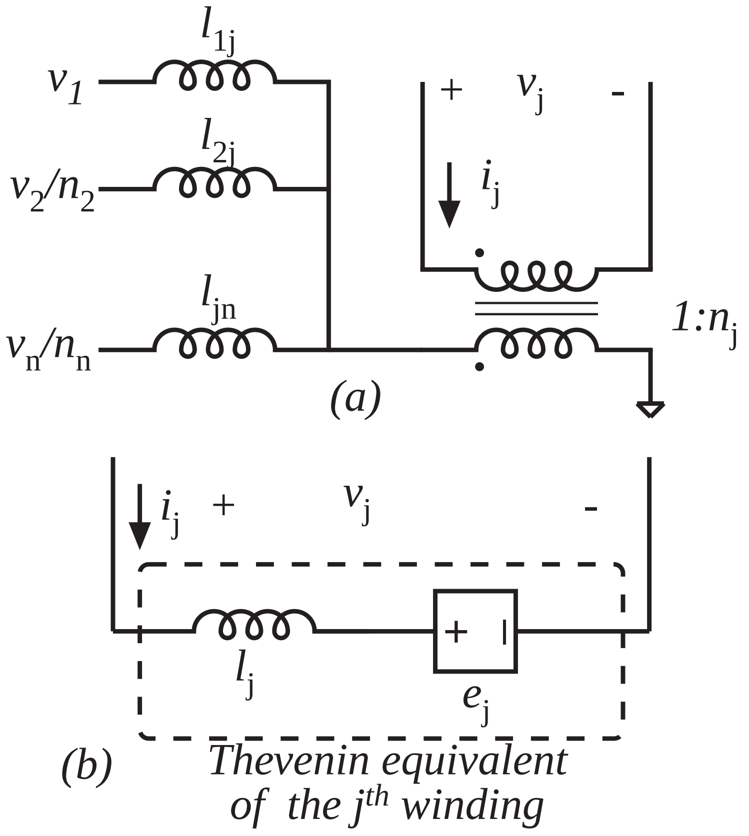
Extended Cantilever Model

- **General**
- **Parameters can be measured directly and easily**
- **Parameters are useful for analysis and modeling of power converters**

Model application:

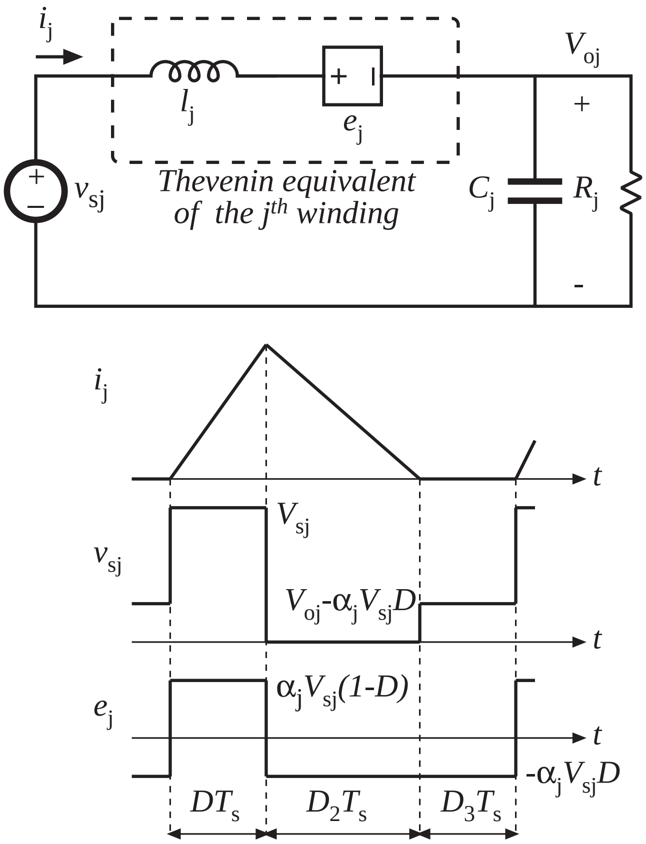
- **Discontinuous-mode analysis and modeling of cross-regulation in multiple-output converters**

DCM Analysis Using the Extended Cantilever Model



Thevenin equivalent of the j^{th} winding

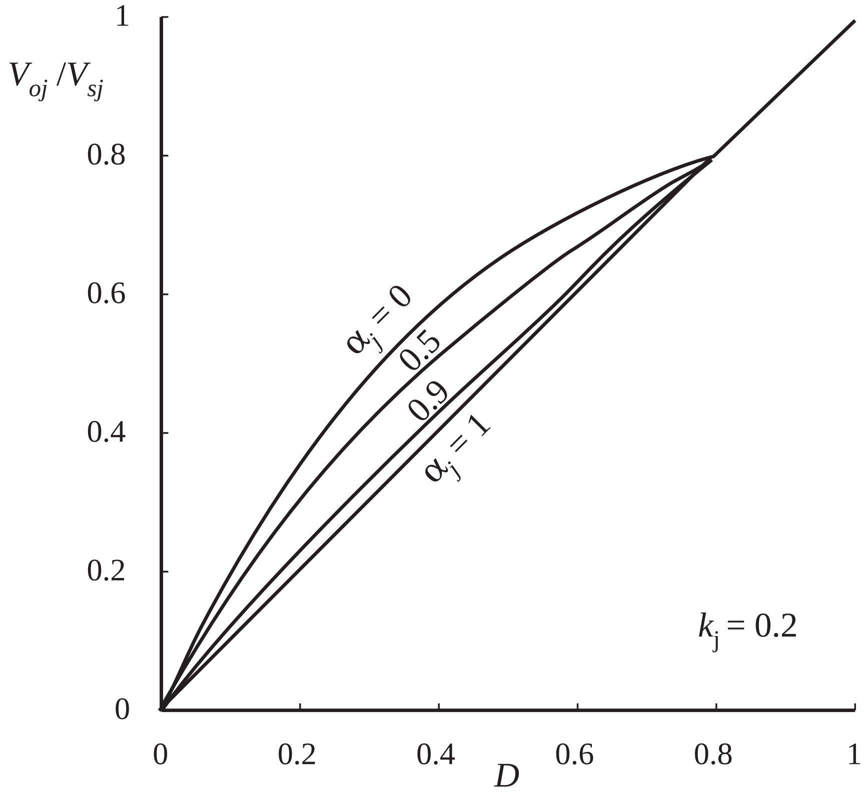
DCM Analysis Using the Extended Cantilever Model



CCM condition:

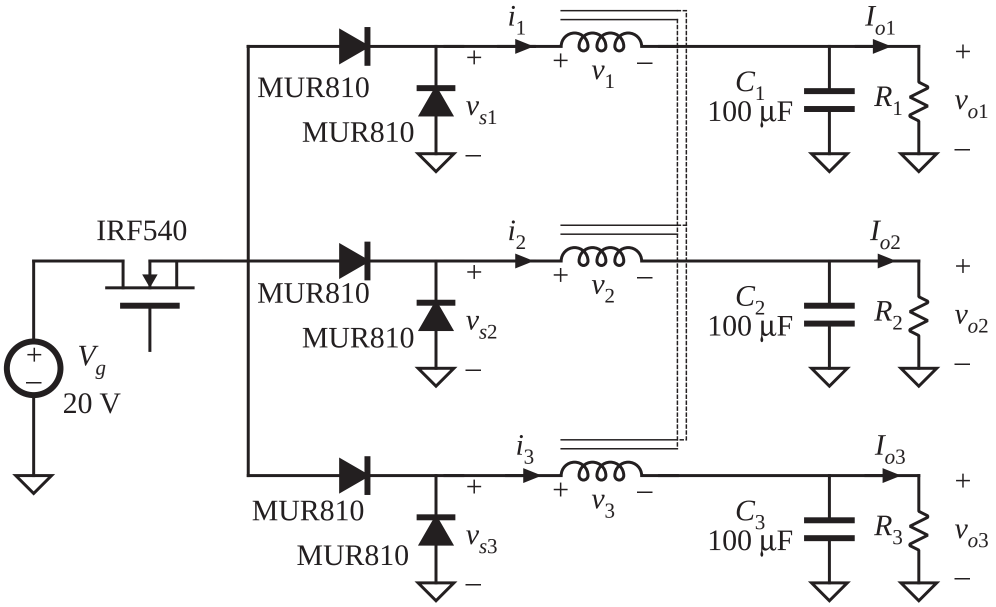
$$k_j = \frac{2l_j f_s}{R_j} \frac{1}{|1-\alpha_j|} \geq 1 - D$$

DC Conversion Ratio



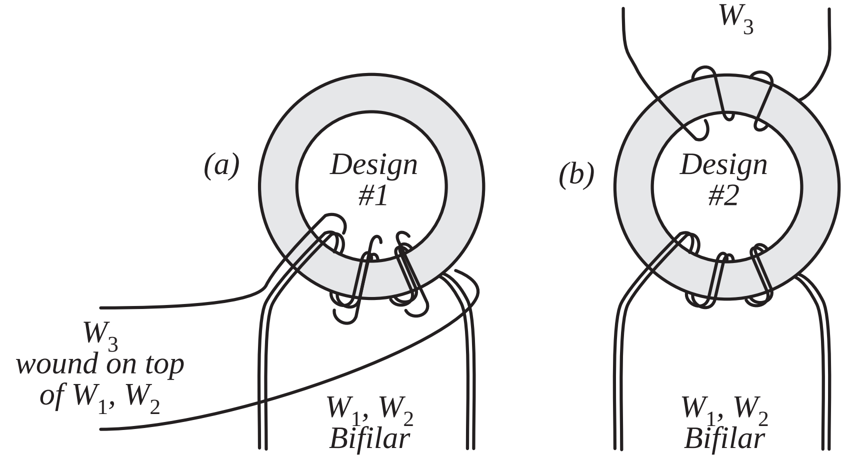
- $\alpha_j = 0$: **no coupling**
- $\alpha_j = 1$: **zero-ripple approach, CCM for all loads**

Experimental Example



- V_{o1} regulated at 5V
- $V_g = 20\text{V}$, $f_s = 50\text{kHz}$

Magnetics In the Experimental Example



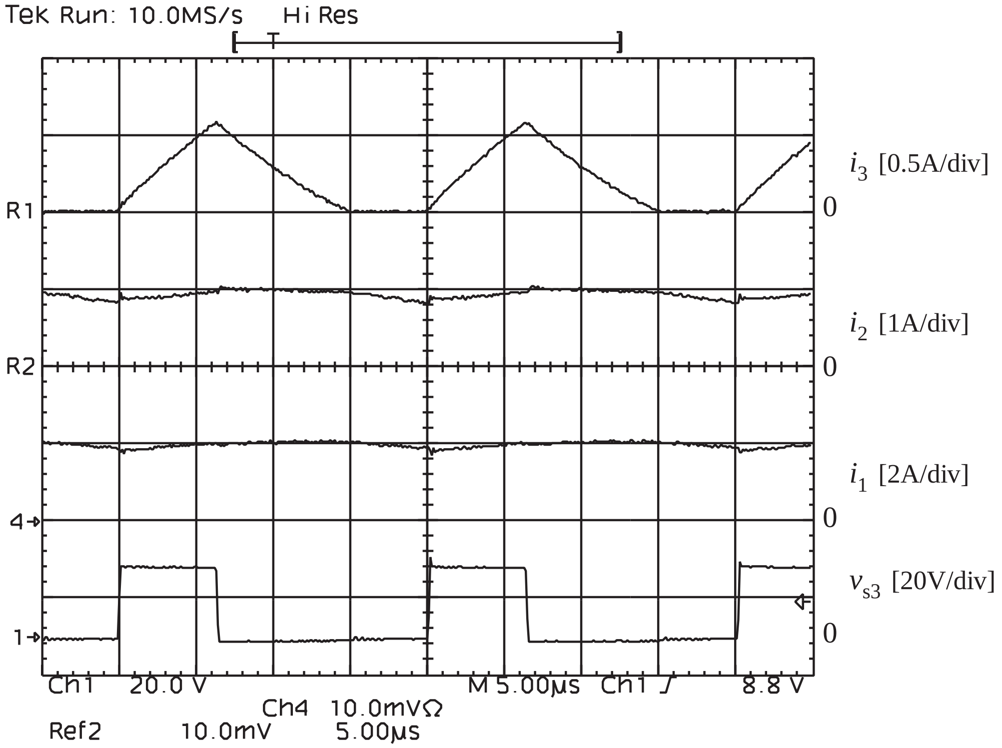
	W1 turns	W2 turns	W3 turns
design #1	24	24	24
design #2	24	24	28

	n_2	n_3	l_{12}	l_{13}	l_{23}
design #1	1.004	0.919	0.36μH	21.3μH	16.4μH
design #2	0.997	0.994	0.36μH	96.2μH	84.1μH

	l_2	α_2	l_3	α_3
design #1	0.358μH	1.006	7.81μH	0.919
design #2	0.384μH	0.990	44.3μH	0.994

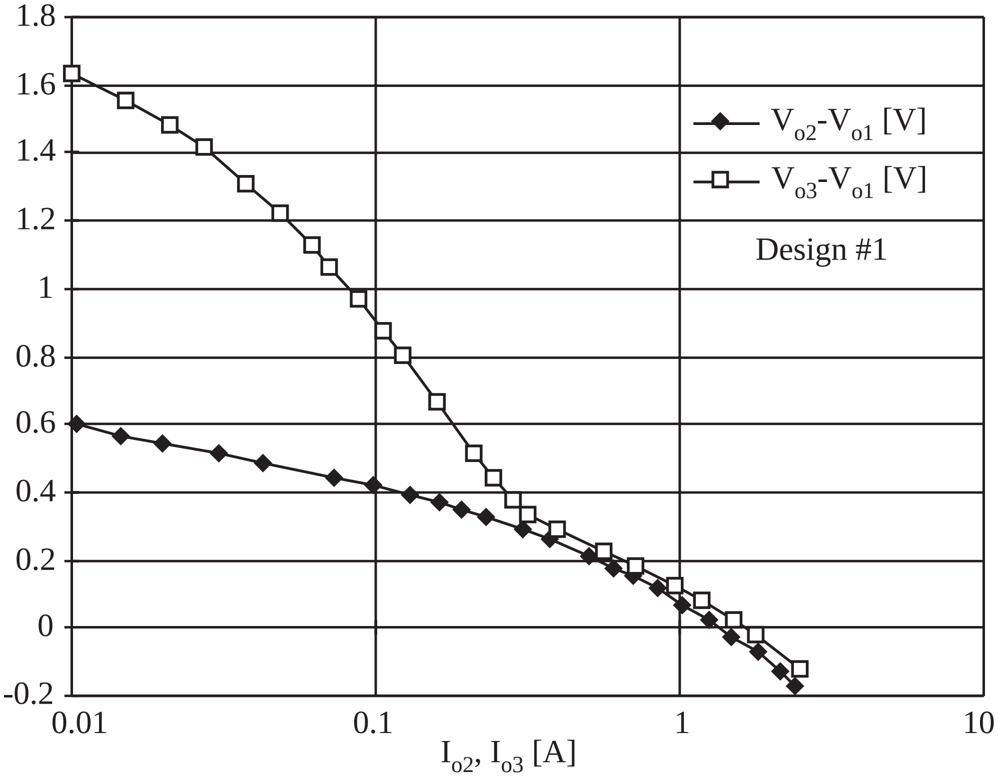
Magnetics Inc. 58254 powdered iron core

Design Approaches: Near-Ideal Coupling vs. Moderate Coupling (design #1)



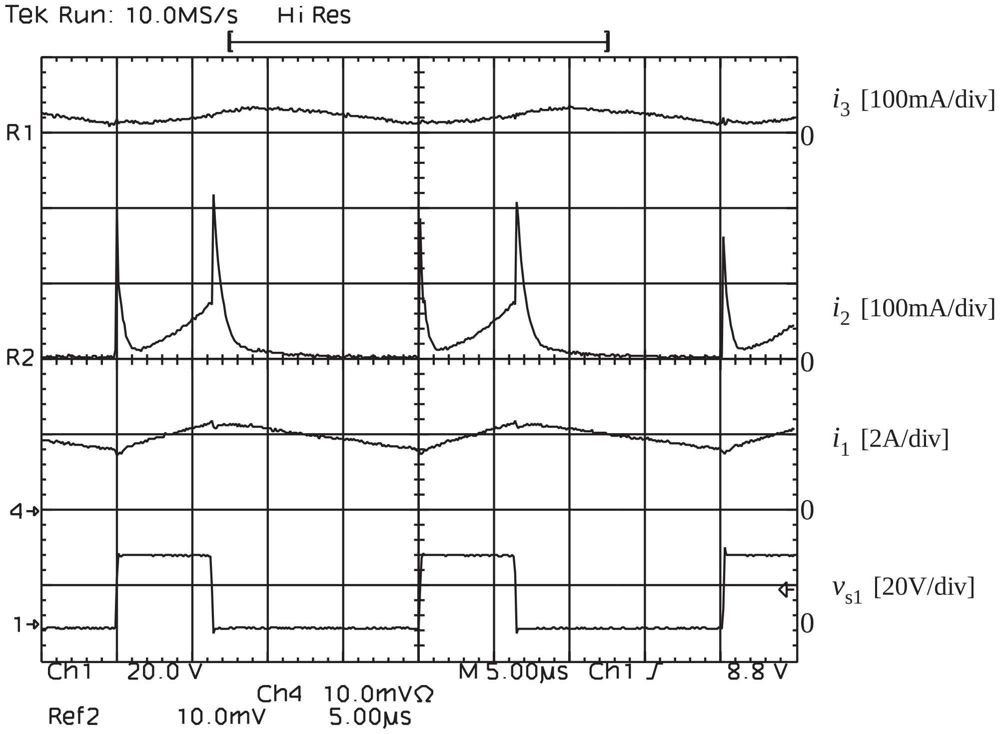
Operating conditions: $V_{o1} = 5.1\text{V}$, $I_{o1} = 2\text{A}$, $V_{o2} = 5.3\text{V}$, $I_{o2} = 0.97\text{A}$, $V_{o3} = 5.6\text{V}$, $I_{o3} = 0.2\text{A}$, $V_g = 20\text{V}$.

Design Approaches: Near-Ideal Coupling vs. Moderate Coupling (design #1) Cross Regulation



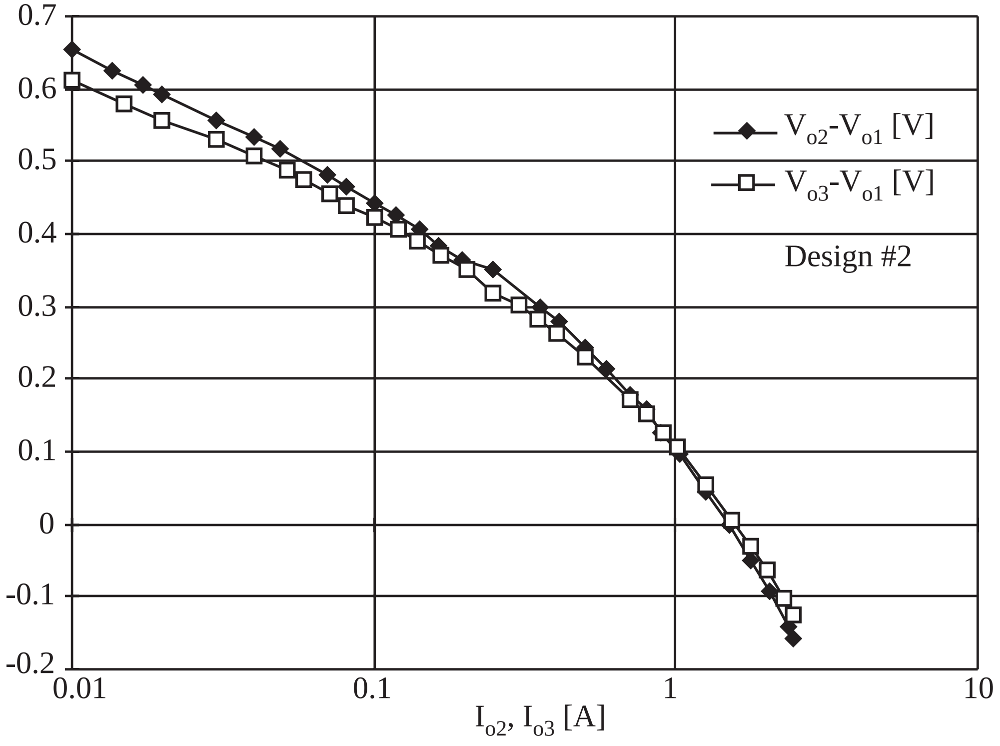
$I_{o1} = 2$**A**, $I_{o2} = 1$**A, coupled-inductor design #1.**

Design Approaches: Near-Ideal Coupling vs. Zero-Ripple Approach (design #2)



Operating conditions: $V_{o1} = 5.1\text{V}$, $I_{o1} = 2\text{A}$, $V_{o2} = 5.7\text{V}$, $I_{o2} = 20\text{mA}$, $V_{o3} = 5.66\text{V}$, $I_{o3} = 20\text{mA}$.

Design Approaches: Near-Ideal Coupling vs. Zero-Ripple Approach (design #2) Cross Regulation



$I_{o1} = 2$A, $I_{o2} = 1$A, coupled-inductor design #2.

Design Approaches: Summary

- **Near-ideal coupling (very small l_j, $\alpha_j \approx 1$)**
  - **Good cross-regulation even in DCM**
  - **Exact matching of turns-ratios is necessary**
  - **Significant current spikes, larger ripples**
  - **May be difficult to achieve in practice**
- **Moderate coupling (small l_j, $\alpha_j \neq 1$)**
  - **Degraded cross-regulation in DCM**
- **Zero-ripple approach (moderate l_j, $\alpha_1 \approx 1$)**
  - **Effective turns ratios match the ratios of imposed voltages**
  - **Effective turns ratios differ from turns ratios of physical windings**
  - **CCM operation down to almost zero load**
  - **Very small ripples**
  - **Best (static) cross regulation (if the non-zero-ripple output operates always in CCM)**

CONCLUSIONS

- Extended cantilever magnetics model:
  - General
  - Model parameters can be measured directly
  - Parameters are useful for converter modeling
- Extended cantilever model applied to DCM analysis of a multiple-output converter with coupled inductors
- Design approaches compared:
  - Near-ideal coupling
  - Moderate coupling
  - Zero-ripple approach

  using the model predictions and an experimental example.