

A Computational Small-Signal Modeling Technique for Switch Mode Converters

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Abstract

A computational method for modeling a switch mode converter, including power stage and pulse-width modulator is presented. The agreement between the dynamic characteristics obtained by the proposed approach and the state-space averaging approach is shown. The only analytical work involved is the derivation of the equivalent circuits for a given converter in state-space representation. The developed computer simulation program then provides a good approximated small-signal model of the switch mode converter at a given operating point for use in the feedback loop design process.

1 Introduction

1.1 Discussion of available modeling techniques.

In obtaining a small signal model, many researchers have tried to make a compromise between the simplicity of the model and the accuracy of the result of the analysis. Among the researched methods are state-space averaging [1] and circuit-averaging [2] techniques, which have been developed to represent the dynamic characteristics of the power stage of the switch mode converter.

The first technique is simple but numerous analytical steps must be followed to obtain the final resulting state-space averaged model. One of the drawbacks is that the state-space averaging technique requires the effective output filter corner frequency, f_c , to be much smaller than the switching frequency, f_s . This means that as the response frequency increases, the state-space averaged model gives erroneous results. On the other hand, the technique using a circuit-averaging method needs a number of equivalent circuit manipulations for a single equivalent linear circuit model of the power stage for the switch mode converter.

1.2 Proposed computational approach

The proposed computational small-signal modeling technique uses an "approximation approach of the sinusoidal response" to derive dynamic characteristics of a switch mode converter.

Figure 1 presents a general description of voltage mode switch mode converter. The objective of the state-space averaging method is to derive a transfer function from duty ratio, d , to the output voltage, v_o , at a particular steady state operating point. This requires a small signal perturbation around the steady state duty ratio, D at a selected frequency, f .

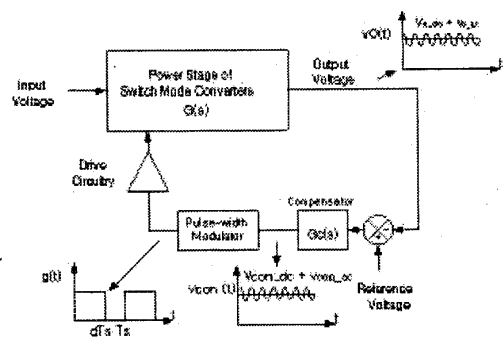


Figure 1. A simple switch mode converter with feedback circuit.

In practice, the duty ratio does not modulate within a switching cycle, thus the resulting model does not reflect the actual hardware behavior. Thus, a small-signal perturbation should be employed on a control signal, V_{con} as shown in figure 1. The new objective becomes to solve for a transfer function from the control signal, v_{con} to the output signal, v_o .

As shown in (2), (3), (4), and (5), non-ideal and parasitic effects, such as ESR of the capacitor, series resistance of the inductor, and on-voltage drop of the semiconductor switches are included to improve the accuracy of the resulting model.

A simulation program is written in C language to determine the output voltage by discrete samples. To aid in understanding the simulation program, the detail of the simulation program is discussed as below:

Main engine

The program is designed so that the state variables at the end of the ON (or OFF) time simulation are used as the initial conditions for the OFF (or ON) time simulation and updated with the output voltage at each sampling index and collected at the end of each switching period.

Allowable amplitude of the actuator signal, $v_{con}(t)$

To extract the dynamic information in the frequency domain, the control variable, $v_{con}(t)$ is programmed as:

$$v_{con}(kT_{sam}) = V_{con_dc} + \tilde{v}_{con_ac} \quad (6)$$

$$\tilde{v}_{con_ac} = V_m \sin(2\pi f k T_{sam}) \quad (7)$$

where k is the sampling index, 0, 1, 2,.....; T_{sam} is the sampling period; \tilde{v}_{con_ac} is the perturbation in V_{con_dc} ; V_m and f are the amplitude and frequency of the sinusoidal perturbation of the control variable respectively and determined V_m by inspecting the shape of the output voltage, v_o .

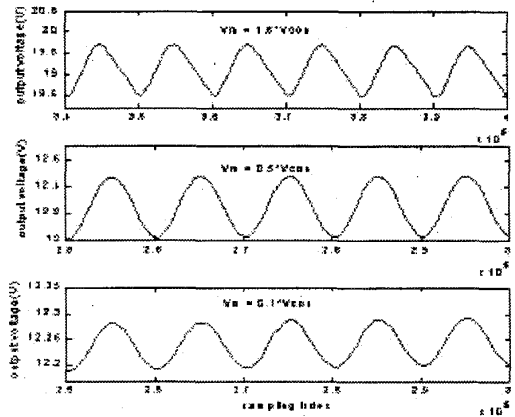


Figure 4. Plots of the output voltage at various perturbation amplitudes

As can be seen in figure 4, the output plots at various amplitude perturbations deviate significantly from that of a sinusoid when the amplitude of V_m is unreasonably large. In addition, it should be noted that the output voltage reaches the steady state at a higher voltage than the expected value 12.0V. With V_m properly set, the step response can be analyzed with the step input duty ratio. A step response curve $v_o(t)$ with step input duty ratio, $d = 0.25$ is shown in figure 5.

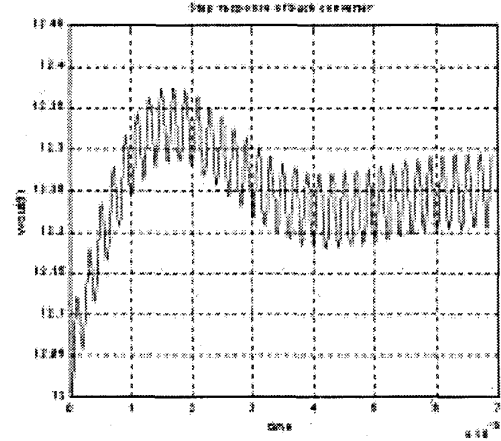


Figure 5. Simulated result of step response of the buck converter

Since the frequency response is calculated after waiting until steady-state conditions are reached, the step response curve in figure 5 illustrates that the output voltage, v_o reaches steady state after 6mS. By taking the amplitude ratio of the output and input sinusoids and the phase shift between the input sinusoid and the output sinusoid at each frequency sweep, the steady state response of the buck converter is calculated by:

$$|M| = \frac{M_{out}}{M_{input}} \quad (8)$$

$$\phi = \frac{\phi_{delay}}{P} \times 360 \quad (9)$$

where M_{out} and M_{inp} are the amplitudes of output and input sinusoids, ϕ_{delay} is the phase shift between the input and output sinusoids, and P is the period of the sinusoid.

Frequency sweep

A frequency sweep of the interested frequency range can be performed for the computation of a frequency response of the converter under investigation. At each frequency step, the gain and phase shift is calculated and plotted in as figure 6.

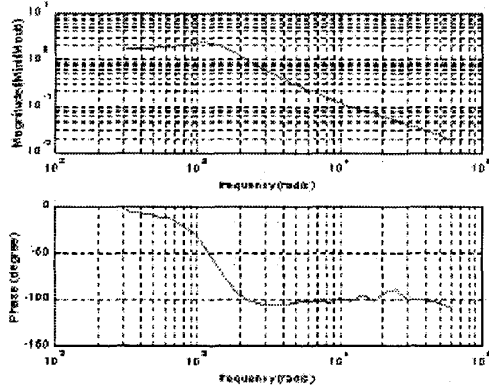


Figure 6 Frequency response of buck converter

Open loop plant transfer function

Using Matlab and its control toolboxes, the collected simulation data is further processed to return the plant transfer function. In figure 7, Bode plots are shown for the buck converter based on: simulation, estimated transfer function by matlab [3], and the transfer function made to be strictly proper.

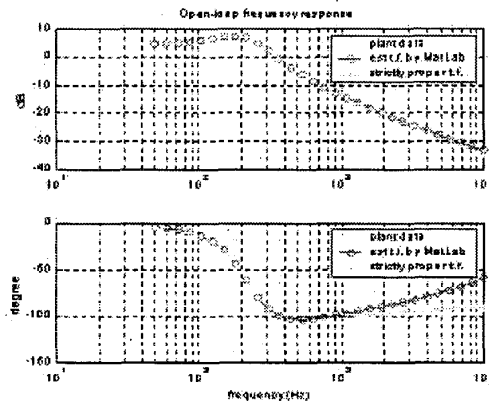


Figure 7. Frequency response plots of the buck converter

Matlab return a transfer function in the form of a rational polynomial:

$$T.F.(s) = \frac{N(s)}{D(s)} \quad (10)$$

that is always proper, but should be made to strictly proper transfer function in the form:

$$T.F.(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0}{s^n + a_{n-1} s^{n-1} + \dots + a_0} \quad (11)$$

where the degree m of the numerator must be less than the degree n of the denominator.

As shown in figure 7, the “+” marked plot represents the frequency plot of the strictly proper transfer function of the buck converter which agrees well with the data obtained by the simulation.

2.3 Frequency response plot of the buck converter using the state-space averaging model approach

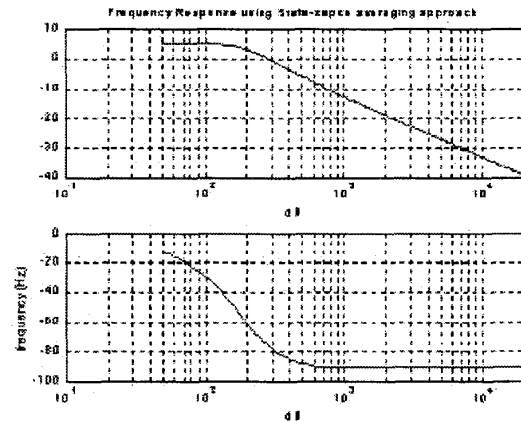


Figure 8. Frequency response plot using state-space averaging approach.

As observed in figure 7 and figure 8, a good agreement is obtained between the developed computational modeling approach and the state-space averaging approach. A few comments are made about these plots.

- Uncertainties at high frequency characteristics can be observed with the computational approach.
- The limitation of the state-space averaged model can be easily visualized.

3. Conclusion

The state-space averaging method employs a perturbation and linearization process required to include the duty ratio modulation effect. In practice, the duty ratio does not modulate within a switching cycle, thus the resulting model does not reflect the actual hardware behavior. The computational small-signal modeling technique uses the control signal as the control variable not the duty ratio, thus it provides the dynamic characteristics of a switch mode converter more closely resembling the actual converter behavior. It should also be noted that no linearization and averaging steps are performed explicitly.

4. References

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