

Reference Design Section

Magnetic Core Properties

Lloyd Dixon

Summary:

A brief tutorial on magnetic fundamentals leads into a discussion of magnetic core properties. A modified version of Intusoft's magnetic core model is presented. Low-frequency hysteresis is added to the model, making it suitable for magnetic amplifier applications.

Magnetic Fundamentals:

Units commonly used in magnetics design are given in Table I, along with conversion factors from the older CGS system to the SI system (*système international* – rationalized MKS). SI units are used almost universally throughout the world. Equations used for magnetics design in the SI system are much simpler and therefore more intuitive than their CGS equivalents. Unfortunately, much of the published magnetics data is in the CGS system, especially in the United States, requiring conversion to use the SI equations.

Table I - CONVERSION FACTORS, CGS to SI

		SI	CGS	CGS to SI
FLUX DENSITY	B	Tesla	Gauss	10^{-4}
FIELD INTENSITY	H	A-T/m	Oersted	$1000/4\pi$
PERMEABILITY (space)	μ_0	$4\pi \cdot 10^{-7}$	1	$4\pi \cdot 10^{-7}$
PERMEABILITY (relative)	μ_r			1
AREA (Core, Window)	A	m ²	cm ²	10^{-4}
LENGTH (Core, Gap)	l	m	cm	10^{-2}
TOTAL FLUX = $\int B dA$	ϕ	Weber	Maxwell	10^{-8}
TOTAL FIELD = $\int H d\ell$	F,MMF	A-T	Gilbert	$10/4\pi$
RELUCTANCE = F/ϕ	R			$10^9/4\pi$
PERMEANCE = $1/R = L/N^2$	P			$4\pi \cdot 10^{-9}$
INDUCTANCE = $P \cdot N^2$	L	Henry	(Henry)	1
ENERGY	W	Joule	Erg	10^{-7}

Figure 1 is the B-H characteristic of a magnetic core material – flux density (Tesla) vs. magnetic field intensity (A-T/m). The slope of a line on this set of axes is permeability ($\mu = B/H$). Area on the

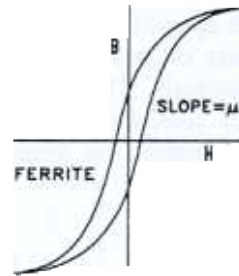


Fig 1. - Magnetic Core B-H Characteristic

surface of Fig. 1 represents energy per unit volume. The area enclosed by the hysteresis loop is unrecoverable energy (loss). The area between the hysteresis loop and the vertical axis is recoverable stored energy:

$$W/m^3 = \int H dB$$

In Figure 2, the shape is the same as Fig. 1, but the axis labels and values have been changed. Figure 2 shows the characteristic of a specific core made from the material of Figure 1. The flux density axis

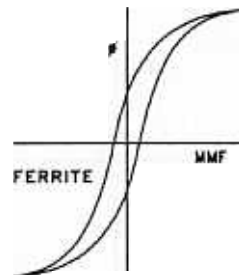


Fig 2. - Core Flux vs. Magnetic Force

is transformed into the total flux, ϕ , through the entire cross-sectional area of the core, while field intensity is transformed into total magnetic force around the entire magnetic path length of the core:

$$\phi = B \cdot A_e ; \quad F = H \cdot \ell_e$$

The slope of a line on these axes is *permeance* ($P = \phi/F = \mu A_e / \ell_e$). Permeance is the inductance of one turn wound on the core.

Area in Fig. 2 represents total energy — hysteresis loss or recoverable energy.

Changing the operating point in Fig. 1 or Fig. 2 requires a change of energy, therefore it can not change instantaneously. When a winding is coupled to the magnetic core, the electrical to magnetic relationship is governed by Faraday's Law and Ampere's Law.

Faraday's Law:

$$\frac{d\phi}{dt} = -\frac{E}{N} ; \quad \phi = \frac{1}{N} \int E dt$$

Ampere's Law:

$$NI = \int H d\ell = H\ell$$

These laws operate bi-directionally. According to Faraday's Law, flux change is governed by the voltage applied to the winding (or voltage induced in the winding is proportional to $d\phi/dt$). Thus, electrical energy is transformed into energy lost or stored in the magnetic system (or stored magnetic energy is transformed into electrical energy).

Applying Faraday's and Ampere's Laws, the axes can be transformed again into the equivalent

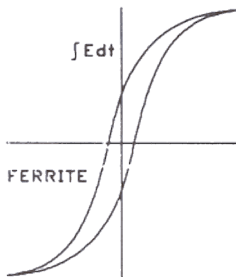


Fig 3. - Equivalent Electrical Characteristic

electrical characteristic of the magnetic core wound with a specific number of turns, N , as shown in Figure 3.

$$\int E dt = NBA_e \quad I = \frac{H\ell_e}{N}$$

Area in Fig. 3 again represents energy, this time in electrical terms: $W = \int E I dt$. The slope of a line in Fig 3 is *inductance*.

$$L = E \frac{dt}{di} = \mu N^2 \frac{A_e}{\ell_e}$$

Low-Frequency Core Characteristics:

Ferromagnetic core materials include: Crystalline metal alloys, amorphous metal alloys, and ferrites (ferrimagnetic oxides).^[1]

Figure 4 shows the low frequency electrical characteristics of an inductor with an ungapped toroidal core of an idealized ferromagnetic metal alloy. This homogeneous core becomes magnetized at a specific field intensity H (corresponding to specific current through the winding $I = H\ell_e/N$). At this magnetizing current level, all of the magnetic dipoles (domains) within the core gradually align, causing the flux to increase toward saturation. The domains cannot align and the flux cannot change instantaneously because energy is required. The changes occur at a rate governed by the voltage applied to the winding, according to Faraday's Law.

Thus, to magnetize this core, a specific magnetizing current, I_M , is required, and the time to accomplish the flux change is a function of the voltage applied to the winding. These factors combined —

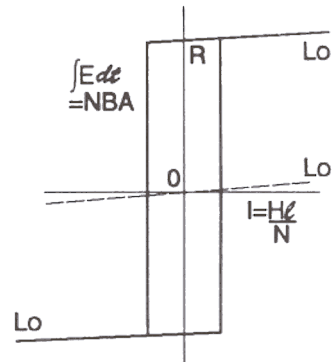


Fig 4. - Ideal Magnetic Core

current, voltage, and time — constitute energy put into the core. The amount of energy put in is the area between the core characteristic and the vertical axis. In this case, none of this energy is recoverable — it is all loss, incurred *immediately* while the current and voltage is being applied.

The vertical slope of the characteristic represents an apparent infinite inductance. However, there is no real inductance — no recoverable stored energy — the characteristic is actually resistive. (A resistor driven by a square wave, plotted on the same axes, has the same vertical slope.)

When all of the domains have been aligned, the material is saturated, at the flux level corresponding to complete alignment. A further increase in current produces little change in flux, and very little voltage can exist across the winding as the operating point moves out on the saturation characteristic. The small slope in this region is true inductance — recoverable energy is being stored. With this ideal core, inductance L_0 is the same as if there were no core present, as shown by the dash line through the origin. The small amount of stored energy is represented by a thin triangular area *above* the saturation characteristic from the vertical axis to the operating point (not shown).

If the current is now interrupted, the flux will decrease to the *residual* flux level (point R) on the vertical axis. The small flux reduction requires reverse Volt- μ seconds to remove the small amount of energy previously stored. (If the current is interrupted rapidly, the short voltage spike will be quite large in amplitude.) As long as the current remains at zero (open circuit), the flux will remain — forever — at point R.

If a negative magnetizing current is now applied to the winding, the domains start to realign in the opposite direction. The flux decreases at a rate determined by the negative voltage across the winding, causing the operating point to move down the characteristic at the left of the vertical axis. As the operating point moves down, the cumulative area between the characteristic and the vertical axis represents energy lost in this process. When the horizontal axis is reached, the net flux is zero — half the domains are oriented in the old direction, half in the new direction.

The Effect of Core Thickness: Fig. 1 applies only to a very thin toroidal core with Inner Diameter almost equal to Outer Diameter resulting in a single valued magnetic path length ($\pi \cdot D$). Thus the same field intensity exists throughout the core, and the entire core is magnetized at the same current level.

A practical toroidal core has an O.D. substantially greater than its I.D., causing magnetic path length to increase and field intensity to decrease with increasing diameter. The electrical result is shown in Figure 5. As current increases, the critical field intensity, H, required to align the domains is achieved first at the inner diameter. According to Ampere:

$$H = \frac{NI}{l} = \frac{NI}{\pi D}$$

Hold on to your hats!! At first, the domains realign and the flux changes in the new direction *only at the inner diameter*. The entire outer portion of the core is as yet unaffected, because the field intensity has not reached the critical level except at the inside diameter. The outer domains remain fully aligned in the *old* direction and the outer flux density remains saturated in the *old* direction. In fact, *the core saturates completely in the new direction at its inner diameter yet the remainder of the core remains saturated in the old direction*. Thus, complete flux reversal always takes place starting from the core inside diameter and progressing toward the outside.

In a switching power supply, magnetic devices are usually driven at the switching frequency by a

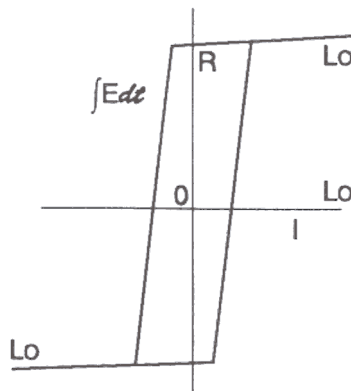


Fig 5. - Effect of Core Thickness

voltage source. A voltage across the winding causes the flux to change at a fixed rate. What actually happens is the flux change starts at the core inner diameter and progresses outward, at a rate equal to the applied volts/turn, E/N . The entire core is always saturated, but the inner portion is saturated in the new direction, while the outer portions remain saturated in the old direction. (This is the lowest energy state — lower than if the core were completely demagnetized.) There is in effect a boundary at the specific diameter where the field intensity is at the critical level required for domain realignment. Flux does *not* change gradually and uniformly throughout the core!

When the operating point reaches the horizontal axis, the net flux is zero, but this is achieved with the inner half of the core saturated in the new direction, while the outer half is simultaneously saturated in the old direction.

When voltage is applied to the winding, the net flux changes by moving the reversal boundary outward. The magnetizing current increases to provide the required field intensity at the larger diameter. If the O.D. of the core is twice the I.D., the magnetizing current must vary by 2:1 as the net flux traverses from minus to plus saturation. This accounts for the finite slope or “inductance” in the characteristic of Fig. 5. The apparent inductance is an illusion. The energy involved is *not* stored — it is all loss, incurred while the operating point moves along the characteristic — the energy involved is unrecoverable.

Non-Magnetic Inclusions: Figure 6 goes another step further away from the ideal, with

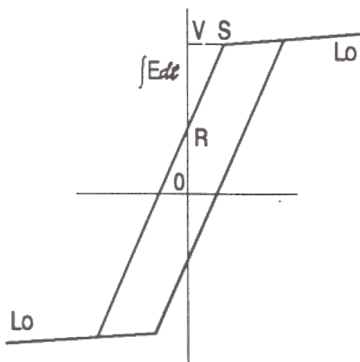


Fig 6. - Non-Magnetic Inclusions

considerable additional skewing of the characteristic. This slope arises from the inclusion of small non-magnetic regions in series with the magnetic core material. For example, such regions could be the non-magnetic binder that holds the particles together in a metal powder core, or tiny gaps at the imperfect mating surfaces of two core halves. Additional magnetic force is required, proportional to the amount of flux, to push the flux across these small gaps. The resulting energy stored in these gaps is theoretically recoverable. To find out how much energy is lost and how much is recoverable look at Figure 6. If the core is saturated, the energy within triangle S-V-R is recoverable because it is between the operating point S and the vertical axis, and *outside the hysteresis loop*. That doesn't ensure the energy *will* be recovered — it could end up dumped into a dissipative snubber.

Another important aspect of the skewing resulting from the non-magnetic inclusions is that the residual flux (point R) becomes much less than the saturation flux level. To remain saturated, the core must now be driven by sufficient magnetizing current. When the circuit is opened, forcing the magnetizing current to zero, the core will reset itself to the lower residual flux level at R.

Reviewing some Principles:

- Ideal magnetic materials do not store energy, but they do dissipate the energy contained within the hysteresis loop. (Think of this loss as a result of “friction” in rotating the magnetic dipoles.)
- Energy is stored, not dissipated, in non-magnetic regions.
- Magnetic materials *do* provide an easy path for flux, thus they serve as “magnetic bus bars” to link several coils to each other (in a transformer) or link a coil to a gap for storing energy (an inductor).
- High inductance does *not* equate to high energy storage. Flux swing is always limited by saturation or by core losses. High inductance requires less magnetizing current to reach the flux limit, hence *less* energy is stored. Referring to Figure 6, if the gap is made larger, further skewing the characteristic and *lowering* the inductance, triangle S-V-R gets bigger, indicating *more* stored energy.

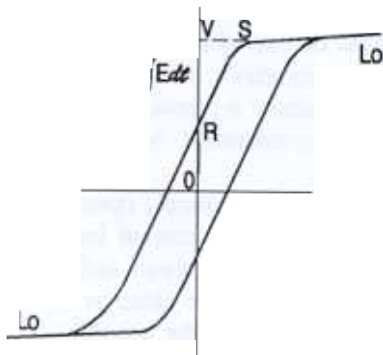


Fig 7. - Non-Homogeneous Effects

Non-Homogeneous Aspects: Figure 7 is the same as Figure 6 with the sharp corners rounded off, thereby approaching the observed shape of actual magnetic cores. The rounding is due to non-homogeneous aspects of the core material and core shape.

Material anomalies that can skew and round the characteristic include such things as variability in ease of magnetizing the grains or particles that make up the material, contaminants, precipitation of metallic constituents, etc.

Core shapes which have sharp corners will paradoxically contribute to rounded corners in the magnetic characteristic. Field intensity and flux density are considerably crowded around inside corners. As a result, these areas will saturate before the rest of the core, causing the flux to shift to a longer path as saturation is approached. Toroidal core shapes are relatively free of these effects.

Adding a Large Air Gap: The cores depicted in Figures 4 - 7 have little or no stored recoverable energy. This is a desirable characteristic for Mag-amps and conventional transformers. But filter inductors and flyback transformers require a great deal of stored energy, and the characteristics of Figures 4 - 7 are unsuitable.

Figure 8 is the same core as in Fig. 7 with much larger gap(s) — a few millimeters total. This causes a much more radical skewing of the characteristic. The horizontal axis scale (magnetizing current) is perhaps 50 times greater than in Figure 7. Thus the stored, recoverable energy in triangle S-V-R,

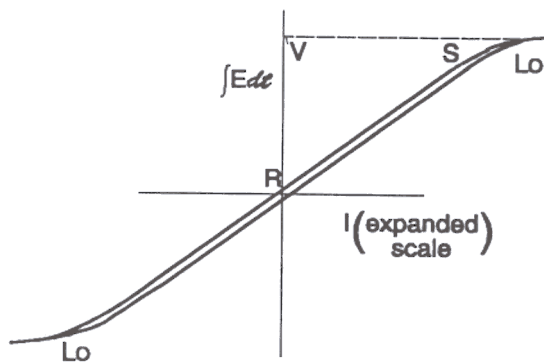


Fig 8. - Large Air Gap

outside of the hysteresis loop, is relatively huge. The recoverable energy is almost all stored in the added gap. A little energy is stored in the non-magnetic inclusions within the core. Almost zero energy is stored in the magnetic core material itself.

With powdered metal cores, such as Mo-Permalloy, the large gap is distributed between the metal particles, in the non-magnetic binder which holds the core together. The amount of binder determines the effective total non-magnetic gap. This is usually translated into an equivalent permeability value for the composite core.

Core Eddy Current Losses:

Up to this point, the low frequency characteristics of magnetic cores have been considered. The most important distinction at high frequencies is that the core eddy currents become significant and eventually become the dominant factor in core losses. Eddy currents also exist in the windings of magnetic devices, causing increased copper losses at high frequencies, but this is a separate topic, not discussed in this paper.

Eddy currents arise because voltage is induced within the magnetic core, just as it is induced in the windings overlaying the core. Since all magnetic core materials have finite resistivity, the induced voltage causes an eddy current to circulate within the core. The resulting core loss is in addition to the low frequency hysteresis loss.

Ferrite cores have relatively high resistivity. This reduces loss, making them well suited for high frequency power applications. Further improvements in

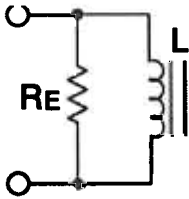


Fig 9. - Core Eddy Current Model

high frequency power ferrite materials focus on achieving higher resistivity. Amorphous metal cores and especially crystalline metal cores have much lower resistivity and therefore higher losses. These cores are built-up with very thin laminations. This drastically reduces the voltages induced within the core because of the small cross section area of each lamination.

The core can be considered to be a single-turn winding which couples the eddy current loss resistance into any actual winding. Thus, as shown in Figure 9, the high frequency eddy current loss resistance can be modeled as a resistor R_E in parallel with a winding which represents all of the low frequency properties of the device.

In Figure 10, the solid line shows the low frequency characteristic of a magnetic core, with dash lines labeled f_1 and f_2 showing how the hysteresis loop effectively widens at successively higher frequencies. Curves like this frequently appear on manufacturer's data sheets. They are not very useful for switching power supply design, because they are based on frequency, assuming symmetrical drive waveforms, which is not the case in switching power supplies.

In fact, it is really not appropriate to think of

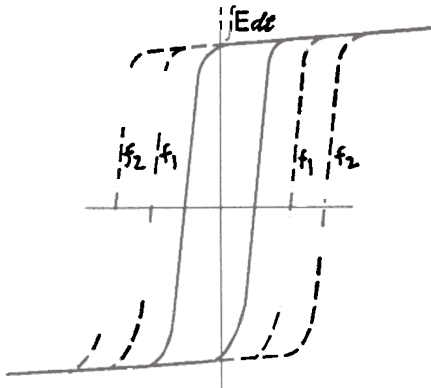


Fig 10. - L.F. Hysteresis plus Eddy Current

eddy current losses as *frequency* dependent. Losses really depend on rate of flux change, and therefore according to Faraday's Law, upon the applied volts/turn. Frequency is relevant only in the case of sinusoidal or symmetrical square wave voltage waveforms.

In a switching power supply operating at a fixed frequency, f_s , core eddy current losses vary with pulse voltage amplitude *squared*, and inversely with pulse width — exactly the same as for a discrete resistor connected across the winding:

$$\text{Loss} = \frac{V_P^2 t_P}{R_E T}$$

If the pulse voltage is doubled and pulse width halved, the same flux swing occurs, but at twice the rate. V_P^2 is quadrupled, t_P is halved — losses double.

If the flux swing and the duty cycle is maintained constant, eddy current loss varies with f_s^2 (but usually the flux swing is reduced at higher frequency to avoid excessive loss).

Forward Converter Illustration:

Figure 11 provides an analysis of transformer operation in a typical forward converter. Accompanying waveforms are in Fig. 12. The solid line in Fig. 11 is the low-frequency characteristic of the ferrite core. The dash lines show the actual path of the operating point, including core eddy currents reflected into the winding. Line X-Y is the mid-point of the low frequency hysteresis curve. Hysteresis loss will be incurred to the right of this line as the flux increases, to the left of this line when the flux decreases.

Just before the power pulse is applied to the winding, the operating point is at point R, the residual flux level. When the positive (forward) pulse is applied, the current rises rapidly from R to D (there is no time constraint along this axis). The current at D includes a low-frequency magnetizing component plus an eddy current component proportional to the applied forward voltage. The flux increases in the positive direction at a rate equal to the applied volts/turn.

As the flux progresses upward, some of the energy taken from the source is stored, some is loss. Point E is reached at the end of the positive

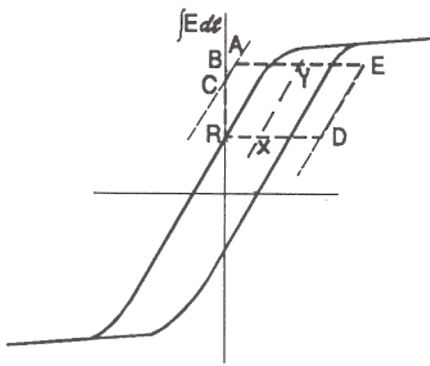


Fig 11. - Forward Converter Core Flux, I_M

pulse. The energy enclosed by X-D-E-Y-X has been dissipated in the core, about half as hysteresis loss, half eddy current loss as shown. The energy enclosed by R-X-Y-B-R is stored (temporarily).

When the power switch turns off, removing the forward voltage, the stored energy causes the voltage to rapidly swing negative to reset the core, and the operating point moves rapidly from E to A. Assuming the reverse voltage is clamped at the same level as the forward voltage, the eddy current magnitude is the same in both directions, and the flux will decrease at the same rate that it increased during the forward interval.

As the operating point moves from A to C, the current delivered into the clamp is small. During this interval, a little energy is delivered to the source, none is received from the source. Most of the energy that had been temporarily stored at point E is turned into hysteresis and eddy current loss as the flux moves from A to C to R. The only energy recovered is the area of the small triangle A-B-C.

Note that as the flux diminishes, the current into the clamp reaches zero at point C. The clamp diode prevents the current from going negative, so the winding disconnects from the clamp. The voltage tails off toward zero, while previously stored energy continues to supply the remaining hysteresis and eddy current losses. Because the voltage is diminishing, the flux slows down as it moves from C to D. Therefore the eddy current also diminishes. The total eddy current loss on the way down through the trapezoidal region A-C-R is therefore slightly less than on the way up through D and E.

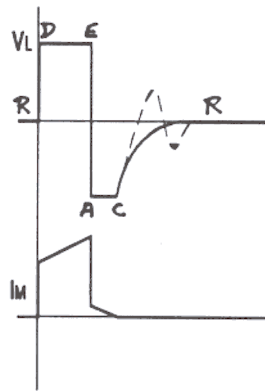


Fig 12. - Forward Converter Waveforms

In a forward converter operating at a fixed switching frequency, a specific V- μ s forward pulse is required to obtain the desired V_{OUT} . When V_{IN} changes, pulse width changes inversely. The transformer flux will always change the same amount, from D to E, but with higher V_{IN} the flux changes more rapidly. Since the higher V_{IN} is also across RE , the equivalent eddy current resistance, the eddy current and associated loss will be proportional to V_{IN} . Worst case for eddy current loss is at high line.

- [1] T.G. Wilson, Sr., *Fundamentals of Magnetic Materials*, APEC Tutorial Seminar 1, 1987

Eddy Current Losses in Transformer Windings and Circuit Wiring

Lloyd H. Dixon, Jr.

Introduction

As switching power supply operating frequencies increase, eddy current losses and parasitic inductances can greatly impair circuit performance. These high frequency effects are caused by the magnetic field resulting from current flow in transformer windings and circuit wiring.

This paper is intended to provide insight into these phenomena so that improved high frequency performance can be achieved. Among other things, it explains (1) why eddy current losses increase so dramatically with more winding layers, (2) why parallelling thin strips doesn't work, (3) how passive conductors (Faraday shields and C.T. windings) have high losses, and (4) why increasing conductor surface area will actually worsen losses and parasitic inductance if the configuration is not correct.

Basic Principles

The following principles are used in the development of this topic and are presented here as a review of basic magnetics.

1. **Ampere's Law:** The total magneto-motive force along *any* closed path is equal to the total current enclosed by that path:

$$F = \oint H d\ell = I_t = NI \quad \text{Amps} \quad (1)$$

where F is the total magneto-motive force (in Amperes) along a path of length ℓ (m), H is field intensity (A/m), and I_t is the total current through all turns enclosed by the path.

2. **Conservation of energy:** At any moment of time, the current within the conductors and the magnetic field are distributed so as to minimize the energy taken from the source.

3. **Energy content of the field:** The magnetic field *is* energy. The energy density at any point in the field is:

$$w = \int H dB \quad \text{Joules/m}^3$$

where B is the flux density (Tesla). In switching power supplies, almost all magnetic

energy is stored in air gaps, insulation between conductors, and within the conductors, where relative permeability μ_r is essentially 1.0 and constant. The energy density then becomes:

$$w = \frac{1}{2}BH = \frac{1}{2}\mu_0H^2 \quad \text{J/m}^3$$

where μ_0 is the absolute permeability of free space ($=4\pi \cdot 10^{-7}$ in S.I. units). Total energy W (Joules) is obtained by integrating the energy density over the entire volume, v , of the field:

$$W = \frac{1}{2}\mu_0 \int H^2 dv \quad \text{Joules}$$

Within typical transformers and inductors, the magnetic energy is almost always confined to regions where the field intensity H is relatively constant and quite predictable. This often occurs in circuit wiring, as well. In these cases:

$$W = \frac{1}{2}\mu_0 H^2 A \cdot \ell \quad \text{Joules} \quad (2)$$

and from (1), $H\ell = NI$. Substituting for H :

$$W = \frac{1}{2}\mu_0 N^2 I^2 A / \ell \quad \text{Joules} \quad (3)$$

where A is the cross-section area (m^2) of the region normal to the flux, and ℓ is the length of the region in meters (and the effective length of the field).

4. **Circuit inductance:** Inductance is a measure of an electrical circuit's ability to store magnetic energy. Equating the energy stored in the field from (3) with the same energy in circuit terms:

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 N^2 I^2 A / \ell$$
$$L = \mu_0 N^2 A / \ell \quad (4)$$

Skin Effect

Figure 1 shows the magnetic field (flux lines) in and around a conductor carrying dc or low frequency current I . The field is radially symmetrical, as shown, only if the return current with its associated field is at a great distance.

At low frequency, the energy in the magnetic field is trivial compared to the energy loss in the resistance of the wire. Hence the current

IMPORTANT NOTICE

Texas Instruments and its subsidiaries (TI) reserve the right to make changes to their products or to discontinue any product or service without notice, and advise customers to obtain the latest version of relevant information to verify, before placing orders, that information being relied on is current and complete. All products are sold subject to the terms and conditions of sale supplied at the time of order acknowledgment, including those pertaining to warranty, patent infringement, and limitation of liability.

TI warrants performance of its products to the specifications applicable at the time of sale in accordance with TI's standard warranty. Testing and other quality control techniques are utilized to the extent TI deems necessary to support this warranty. Specific testing of all parameters of each device is not necessarily performed, except those mandated by government requirements.

Customers are responsible for their applications using TI components.

In order to minimize risks associated with the customer's applications, adequate design and operating safeguards must be provided by the customer to minimize inherent or procedural hazards.

TI assumes no liability for applications assistance or customer product design. TI does not warrant or represent that any license, either express or implied, is granted under any patent right, copyright, mask work right, or other intellectual property right of TI covering or relating to any combination, machine, or process in which such products or services might be or are used. TI's publication of information regarding any third party's products or services does not constitute TI's approval, license, warranty or endorsement thereof.

Reproduction of information in TI data books or data sheets is permissible only if reproduction is without alteration and is accompanied by all associated warranties, conditions, limitations and notices. Representation or reproduction of this information with alteration voids all warranties provided for an associated TI product or service, is an unfair and deceptive business practice, and TI is not responsible nor liable for any such use.

Resale of TI's products or services with statements different from or beyond the parameters stated by TI for that product or service voids all express and any implied warranties for the associated TI product or service, is an unfair and deceptive business practice, and TI is not responsible nor liable for any such use.

Also see: [Standard Terms and Conditions of Sale for Semiconductor Products](http://www.ti.com/sc/docs/stdterms.htm). www.ti.com/sc/docs/stdterms.htm

Mailing Address:

Texas Instruments
Post Office Box 655303
Dallas, Texas 75265