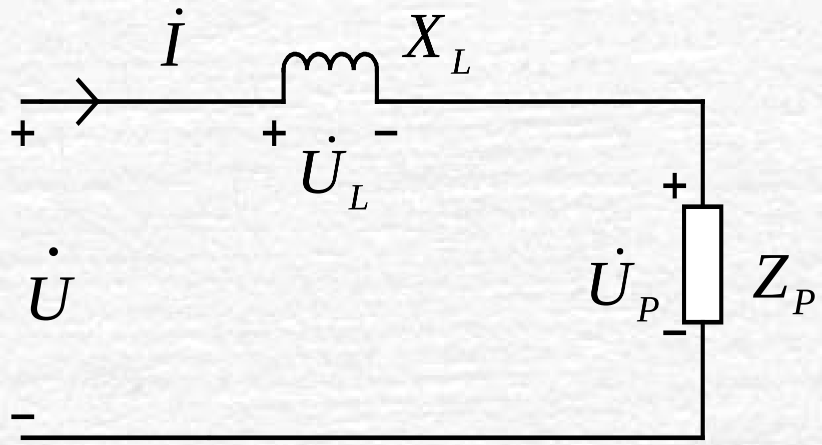
3-10 简单正弦交流电路的计算

1.已知电路结构参数，求各支路电流、电压。

例1：如图电路，已知：U=380V，X_L=22Ω，Z_P 为感性负载，其阻抗角为30^0，且U_P=U_L 。

求：I、U_P 的有效值



解：

$$Z_P = \frac{\dot{U}_P}{\dot{I}} = \frac{U_P}{I}\angle\varphi$$

$$= \frac{U_L}{I}\angle\varphi$$

$$= X_L\angle\varphi$$

$$= 22\angle 30^\circ$$

$$\dot{U} = \dot{U}_L + \dot{U}_P = jX_L\dot{I} + Z_P\dot{I}$$

$$Z = jX_L + Z_P$$

$$= j22 + 22\angle 30^\circ = 38\angle 60^\circ$$

令：$\dot{U} = 380\angle 0^\circ V$

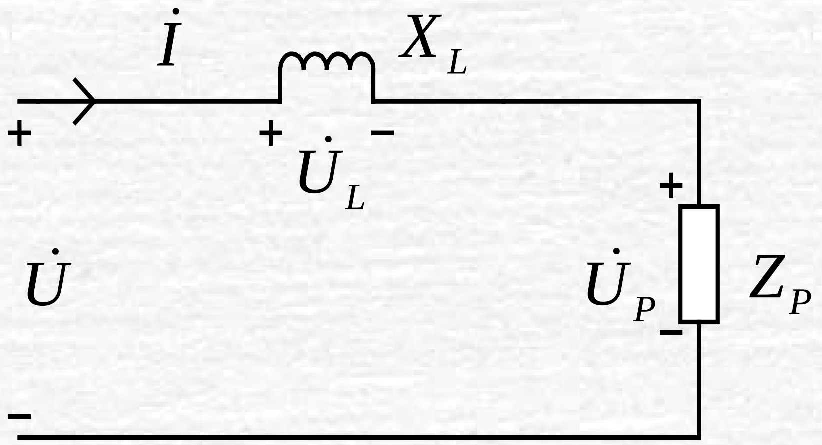
$$\dot{I} = \frac{\dot{U}}{Z} = \frac{380\angle 0^\circ}{38\angle 60^\circ} = 10\angle -60^\circ$$

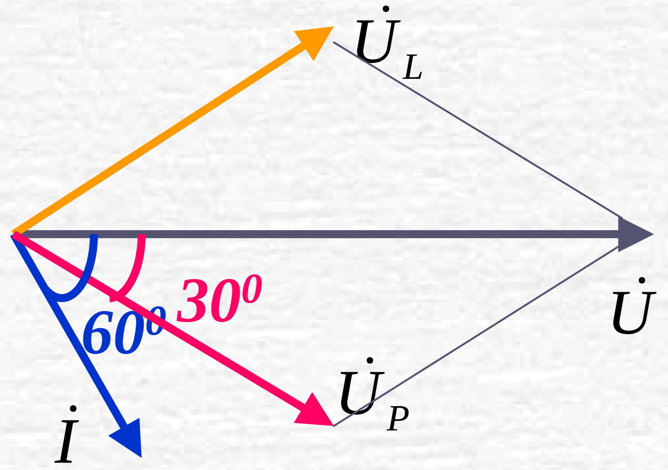
$$I = 10A$$

$$\dot{U}_P = \dot{I}Z_P$$

$$= 10\angle -60^\circ \cdot 22\angle 30^\circ = 220\angle -30^\circ$$

$$U_P = 220V$$





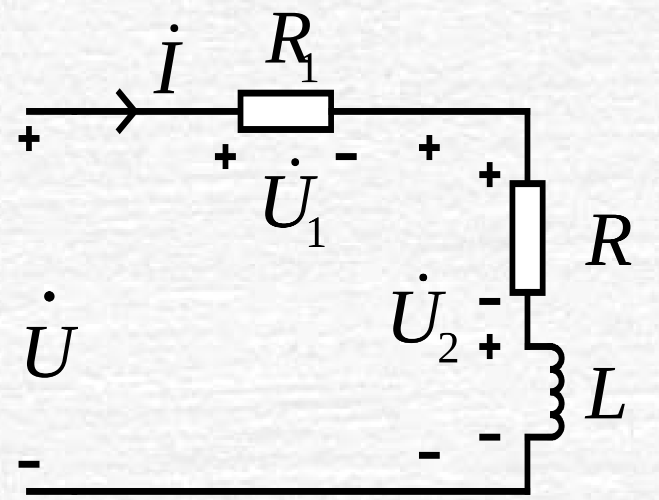
2.已知电路结构及某些条件，推求电路参数。

一般均要借助相量图求解

例2：已知：$U=65V$，$U_1=30V$，$U_2=50V$

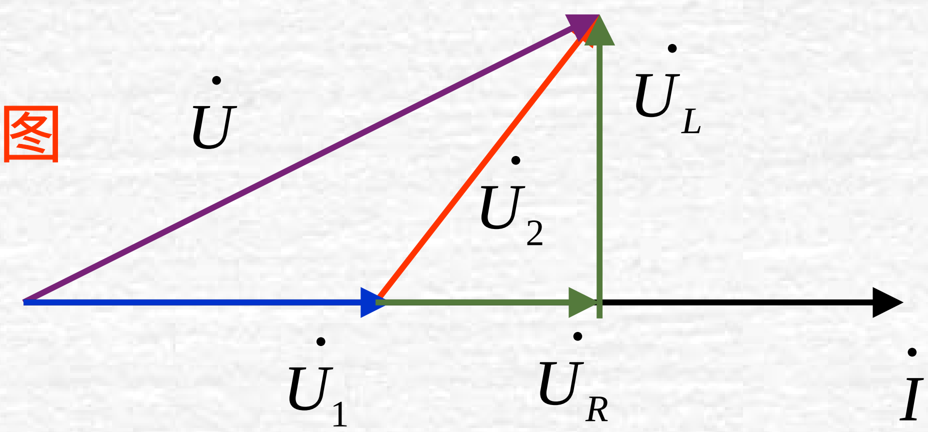
$R_1=2\Omega$

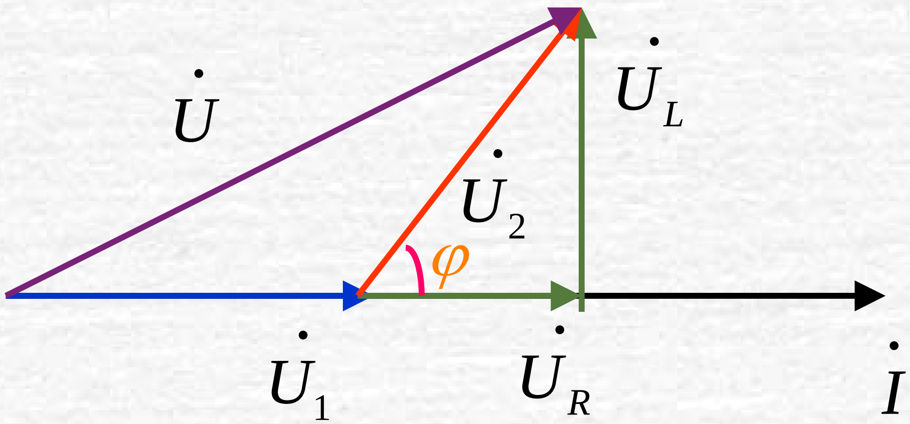
求：X_L，R



解：各相量的参考方向如图，

选择电流为参考相量，画相量图





$$\cos\varphi = \frac{U^2 - U_1^2 - U_2^2}{2U_1U_2} = 0.275$$

$$\varphi = 82.26^\circ$$

$$U_R = U_2\cos\varphi = 50\cos 82.26^\circ = 13.75$$

$$U_L = U_2\sin\varphi = 50\sin 82.26^\circ = 48.07$$

$$I = \frac{U_1}{R_1} = \frac{30}{2} = 15A$$

$$R = U_R / I = 0.917\Omega$$

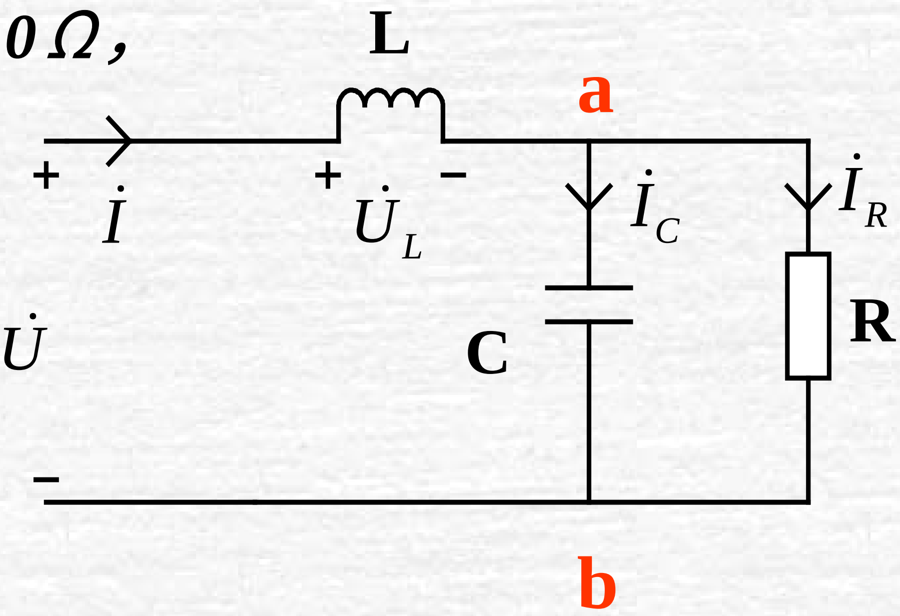
$$f = 50Hz$$

$$X_L = U_L / I = 3.205\Omega$$

$$L = \frac{X}{\omega} = \frac{3.205}{314} = 10.2mH$$

例3：已知：$I_C=6A$，$I_R=8A$，$X_L=10\,\Omega$，

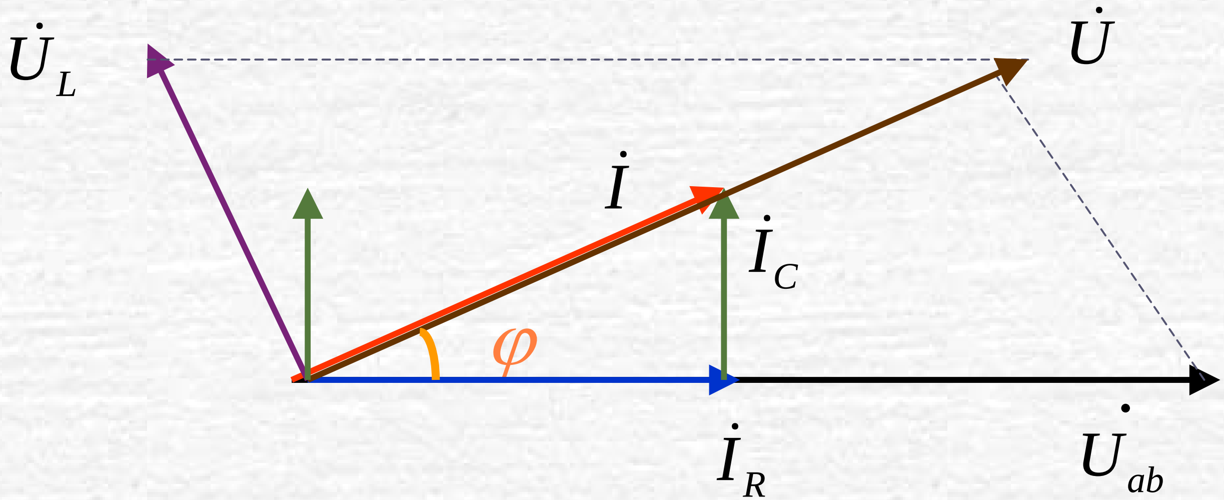
$\dot{U}$、$\dot{I}$ 同相　求：R、X_C ？

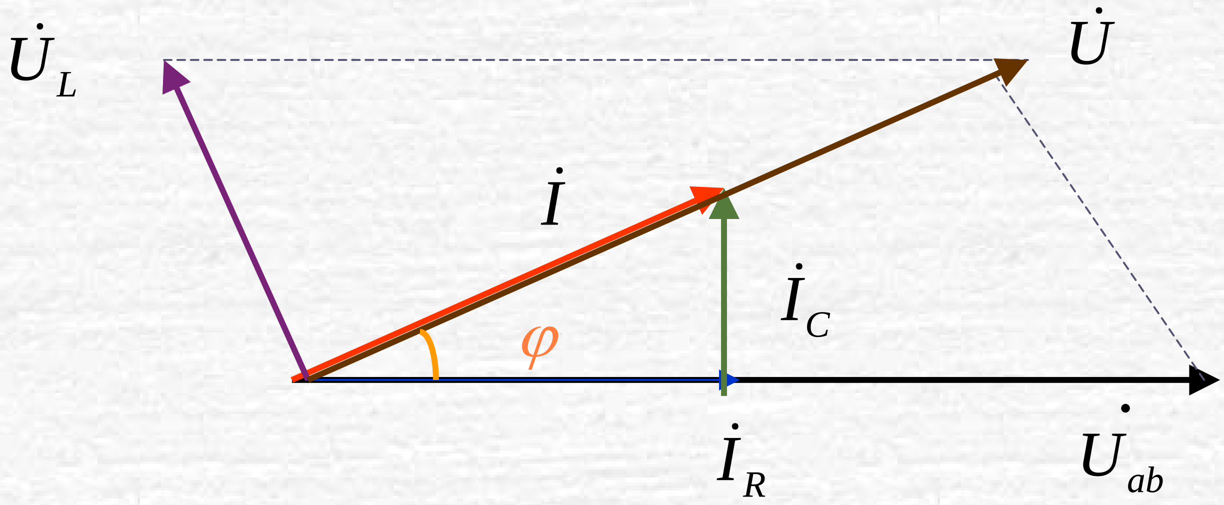


解：各相量的参考方向如图，

选择参考相量，画相量图

$$\dot{U}_{ab} = U_{ab}\angle 0^{\circ}$$





$$\varphi = tg^{-1}\frac{I_C}{I_R} = 36.9^\circ$$

$$I = \sqrt{I_R^2 + I_C^2} = 10A$$

$$U_L = I \cdot X_L = 100V$$

$$U_L = U_{ab} \cdot \sin\varphi \qquad U_{ab} = U_L / \sin\varphi = 500/3\,(V)$$

$$R = U_{ab} / I_R = 20.83\Omega$$

$$X_C = U_{ab} / I_C = 27.78\Omega$$

3-12 复杂正弦交流电路的计算

当用相量表示正弦交流电路中的电压、电流后，正弦交流电路中基本定律与直流电路中相应的定律具有相同的形式。

比较如下：

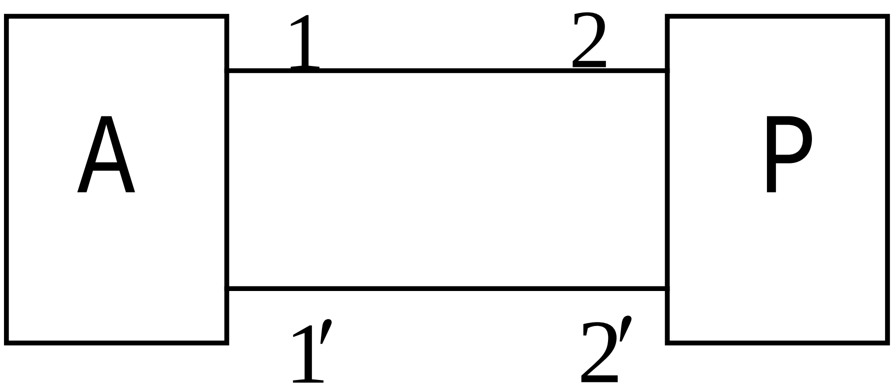
	直流电路	正弦交流电路
欧姆定律	$U = R\ I$	$\dot{U} = Z\ \dot{I}$
	$I = G\,U$	$\dot{I} = Y\,\dot{U}$
元件参数关系	$R = \dfrac{1}{G}$	$Z = \dfrac{1}{Y}$
KCL	$\sum I = 0$	$\sum \dot{I} = 0$
KVL	$\sum U = 0$	$\sum \dot{U} = 0$

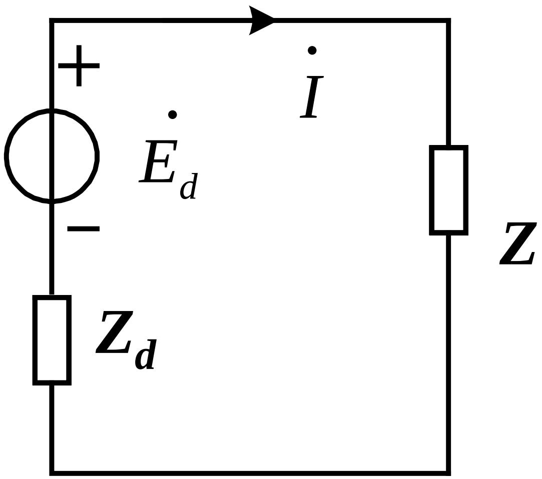
$R - Z$ $\qquad G - Y$ $\qquad U - \dot{U}$ $\qquad I - \dot{I}$

因此，根据基尔霍夫定律和欧姆定理所推导出的关于直流电路的所有计算方法和定理，都可以推广应用到正弦交流电路。直流电路中解题方法均可在交流电路中套用(无互感时)。

不同点：最大功率传输、平衡电桥、 交流电路会出现互感、谐振

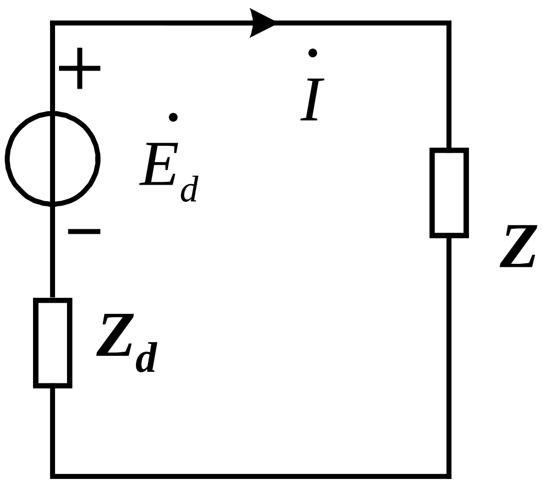
最大功率传输





条件：E_d Z_d 固定，负载 $Z=R+jX$ 的R、X均独立可调

问题：当Z、Z_d 满足什么关系时，负载Z获得最大功率？



$$\dot{I} = \frac{\dot{E}_d}{Z_d + Z}$$

$$= \frac{\dot{E}_d}{R_d + jX_d + R + jX}$$

$$= \frac{\dot{E}_d}{(R_d + R) + j(X_d + X)}$$

$$I = \frac{E_d}{\sqrt{(R_d + R)^2 + (X_d + X)^2}}$$

$$P = I^2 R = \frac{E_d{}^2 R}{(R_d + R)^2 + (X_d + X)^2}$$

$$P = I^2 R = \frac{E_d^{\,2} R}{(R_d + R)^2 + (X_d + X)^2}$$

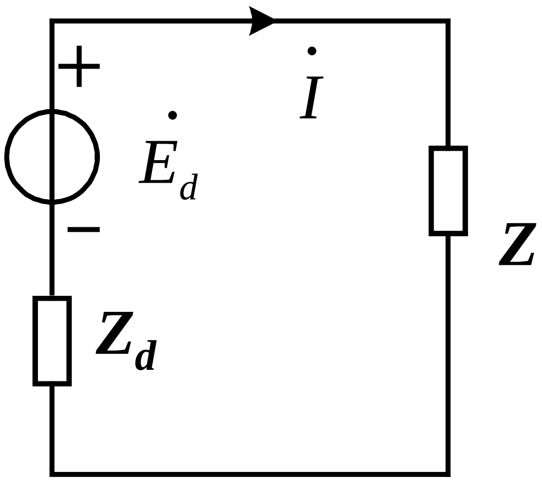
R.X可以单独调节，意味着X.R为两个独立变量，求偏导：

$$\frac{\partial P}{\partial X} = \frac{-E_d^{\,2} \cdot R \cdot \left[2(X_d + X)\right]}{\left[(R_d + R)^2 + (X_d + X)^2\right]^2} = 0 \qquad X = -X_d$$

$$\frac{\partial P}{\partial R} = \frac{E_d^{\,2}\left\{\left[(R_d + R)^2 + (X_d + X)^2\right] - R \cdot 2(R_d + R)\right\}}{\left[(R_d + R)^2 + (X_d + X)^2\right]^2} \qquad R = R_d$$

$$= E_d^{\,2} \cdot \frac{(R_d + R)(R_d - R) + (X_d + X)^2}{\left[(R_d + R)^2 + (X_d + X)^2\right]^2} = 0$$

$$Z = R + jX = R_d - jX_d = \overset{*}{Z}_d$$

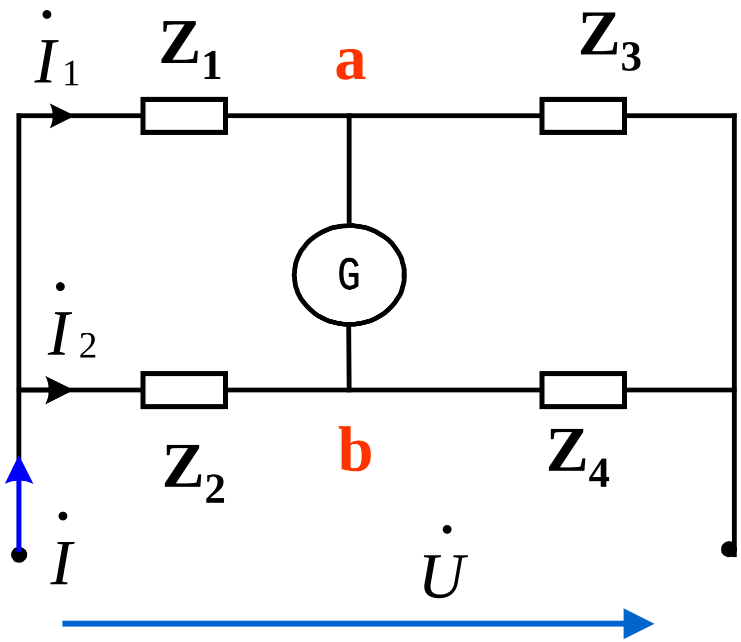


$$P = I^2 R = \frac{E_d^{\,2} R}{(R_d + R)^2 + (X_d + X)^2} = \frac{E_d^{\,2}}{4R}$$

$$Z = \overset{*}{Z}_d \qquad P_{\max} = \frac{E_d^{\,2}}{4R}$$

可直接套用结果

交流电桥



电桥平衡，a,b为自然等位点

a,b断开：

$$\dot{I}_1 = \frac{\dot{U}}{Z_1 + Z_3} \qquad \dot{I}_2 = \frac{\dot{U}}{Z_2 + Z_4}$$

a,b短路:

$$\dot{I} = \frac{\dot{U}}{\frac{Z_1 Z_2}{Z_1 + Z_2} + \frac{Z_3 Z_4}{Z_3 + Z_4}}$$

$$\dot{I}_1 = \dot{I} \times \frac{Z_2}{Z_1 + Z_2} \qquad \dot{I}_2 = \dot{I} \times \frac{Z_1}{Z_1 + Z_2}$$

$$\Rightarrow \quad Z_1 \times Z_4 = Z_2 \times Z_3$$

$$Z_1 \times Z_4 = Z_2 \times Z_3$$

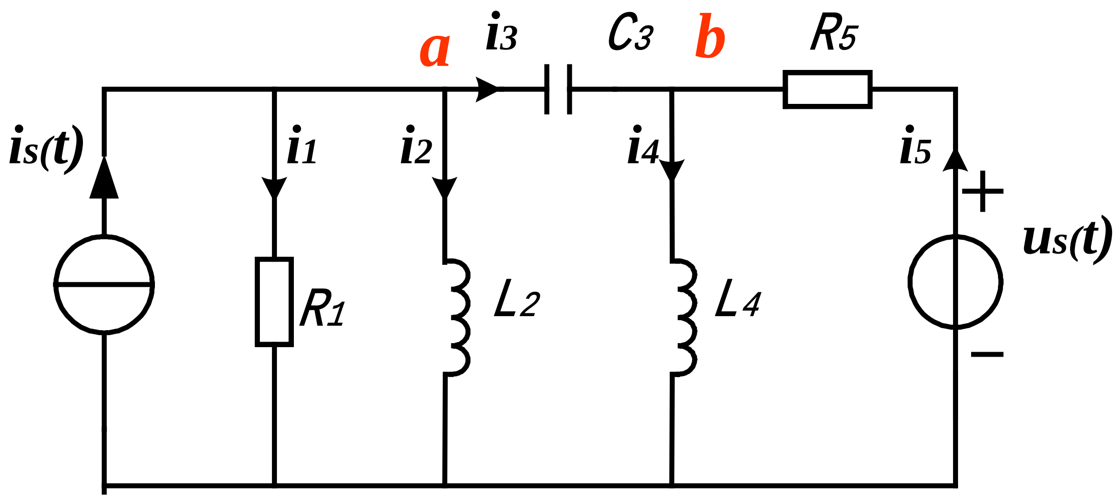
$$(R_1 + jX_1)(R_4 + jX_4) = (R_2 + jX_2)(R_3 + jX_3)$$

$$\Rightarrow \begin{cases} R_1R_4 - X_1X_4 = R_2R_3 - X_2X_3 \\ R_1X_4 + R_4X_1 = R_2X_3 + R_3X_2 \end{cases}$$

一个复数方程 → 两个实数方程

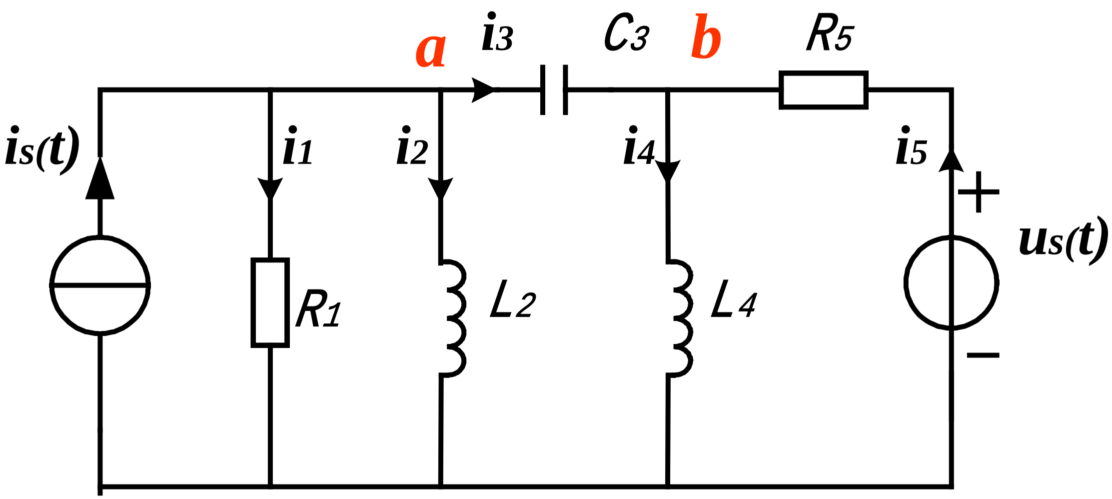
例1.已知：R_1， L_2， C_3， L_4， R_5， $i_S(t)$ ，$u_s(t)$

求：i_1, i_2, i_3, i_4, i_5？



解：　1.从时域分析

$$\begin{cases} i_1 + i_2 + i_3 = i_S \\ i_3 + i_5 = i_4 \end{cases}$$



$$\begin{cases} i_1 + i_2 + i_3 = i_S \\ i_3 + i_5 = i_4 \end{cases}$$

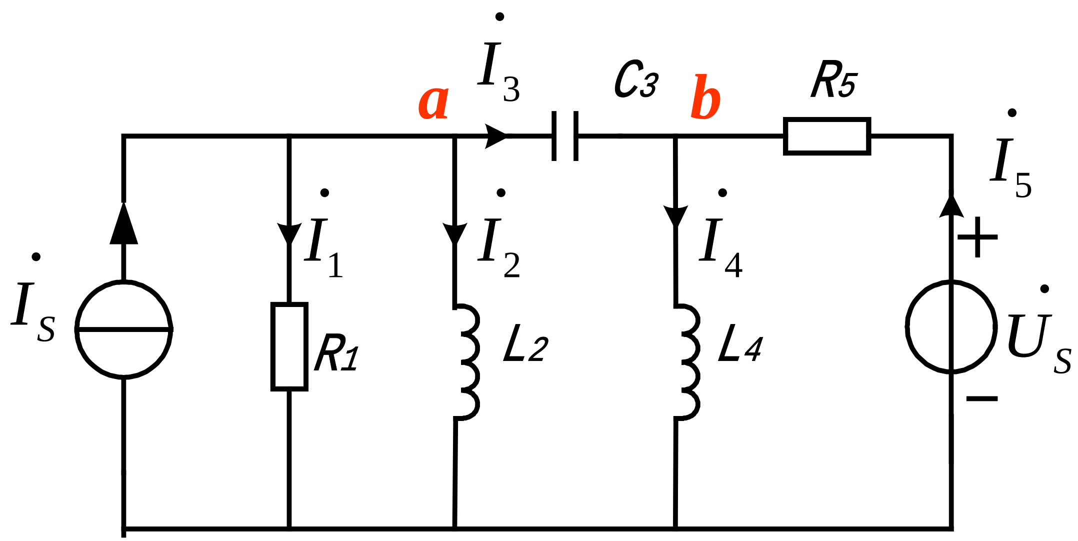
$$\frac{u_a}{R_1} + \frac{1}{L_2}\int_{-\infty}^{t} u_a d\xi + c_3 \frac{d\left(u_a - u_b\right)}{dt} = i_S\left(t\right)$$

$$c_3 \frac{d\left(u_a - u_b\right)}{dt} + \frac{u_S\left(t\right) - u_b\left(t\right)}{R_5} = \frac{1}{L_4}\int_{-\infty}^{t} u_b d\xi$$

$$\begin{cases} c_3 \dfrac{d^2\left(u_a - u_b\right)}{dt^2} + \dfrac{1}{R_1}\dfrac{du_a}{dt} + \dfrac{1}{L_2}u_a = i_S{}'\left(t\right) \\ c_3 \dfrac{d^2\left(u_a - u_b\right)}{dt^2} + \dfrac{u_S{}'\left(t\right) - u_b{}'\left(t\right)}{R_5} = \dfrac{1}{L_4}u_b \end{cases}$$

求解微分方程的特解

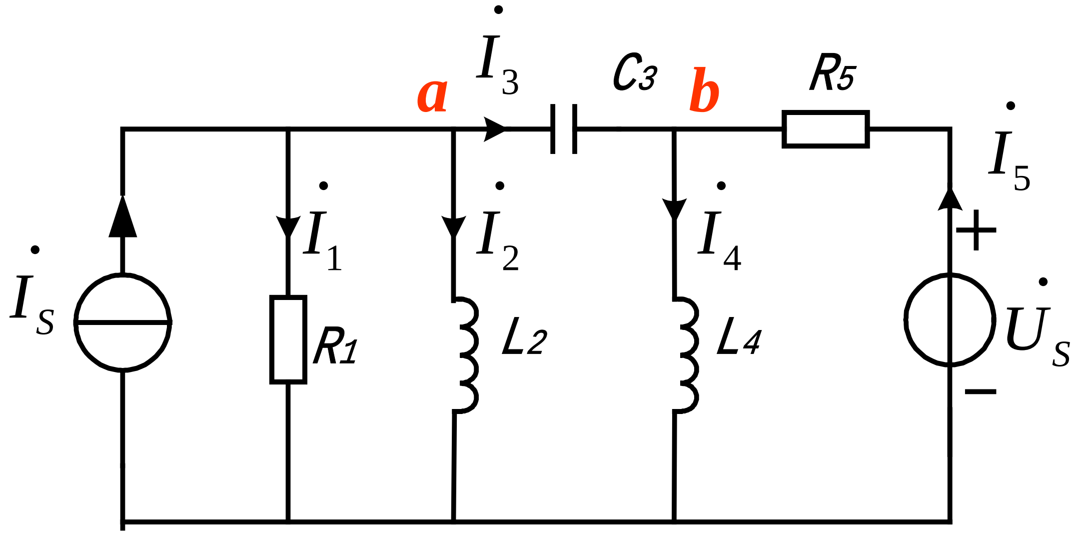
2.相量电路模型.



$$\begin{cases} \dot{I}_1 + \dot{I}_2 + \dot{I}_3 = \dot{I}_S \\ \dot{I}_3 + \dot{I}_5 = \dot{I}_4 \end{cases}$$

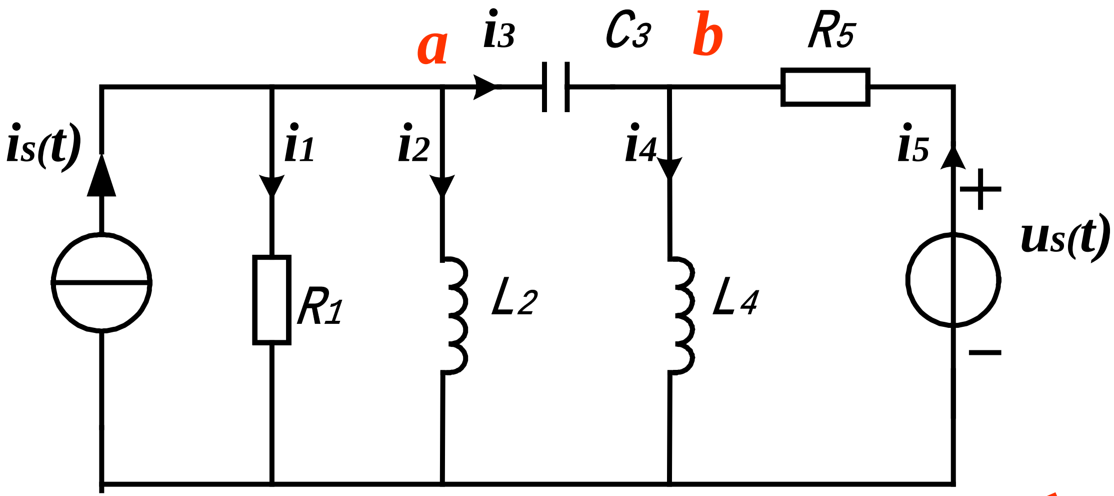
$$\frac{U_a}{R_1} + \frac{\dot{U}_a}{j\omega L_2} + \left(\dot{U}_a - \dot{U}_b\right) \cdot j\omega C_3 = \dot{I}_S$$

$$\left(\dot{U}_a - \dot{U}_b\right) \cdot j\omega C_3 + \frac{\dot{U}_S - \dot{U}_b}{R_5} = \frac{\dot{U}_b}{j\omega L_4}$$



$$\begin{cases} \dot{U}_a\left(\dfrac{1}{R_1}+\dfrac{1}{j\omega L_2}+j\omega C_3\right)-\dot{U}_b\cdot j\omega C_3=\dot{I}_S \\ \dot{U}_b\left(j\omega C_3+\dfrac{1}{j\omega L_4}+\dfrac{1}{R_5}\right)-\dot{U}_a\cdot j\omega C_3=\dfrac{\dot{U}_S}{R_5} \end{cases}$$

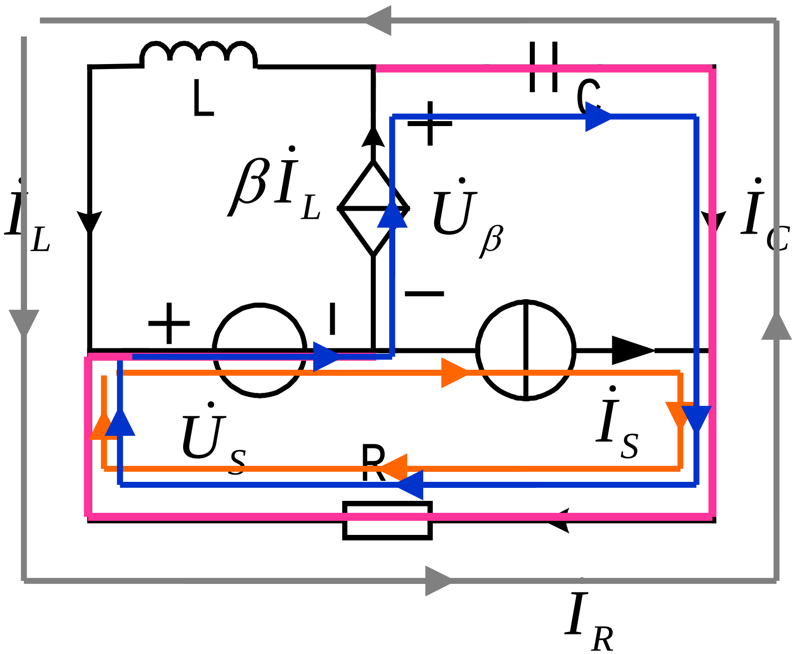
相量形式的节点电压方程------频域 的节点电压方程



$$u_a\left(\frac{1}{R_1}+\frac{1}{j\omega L_2}+j\omega C_3\right)-u_b\cdot j\omega C_3=i_S(t)$$

$$u_b\left(j\omega C_3+\frac{1}{j\omega L_4}+\frac{1}{R_5}\right)-u_a\cdot j\omega C_3=\frac{u_S(t)}{R_5}$$

例2：已知：$L\ \ C\ \ R\ \ \dot{U}_S\ \ \dot{I}_S\ \ \omega\ \ \beta$ 求：$\dot{I}_L\ \ \dot{I}_C\ \ \dot{I}_R\ \ \dot{U}_\beta$



解：如图选树.

$$\dot{I}_C = \beta\dot{I}_L - \dot{I}_L$$

$$\dot{I}_R = \dot{I}_S + \dot{I}_C$$

$$\dot{U}_\beta = \dot{I}_L\left(jX_L\right) + \dot{U}_S$$

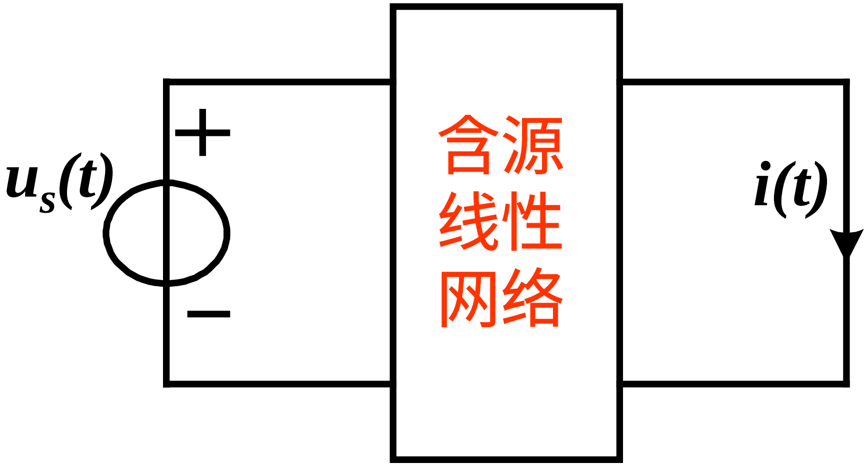
$$\dot{I}_L\left(j\omega L + R - j\frac{1}{\omega C}\right) - \beta\dot{I}_L\left(R - j\frac{1}{\omega C}\right) - \dot{I}_S R = 0$$

$$\dot{I}_L = \frac{\dot{I}_S R}{R + j\omega L - j\frac{1}{\omega C} - \beta R + j\beta\frac{1}{\omega C}}$$

例3 已知： $u_S(t)=0 \qquad i(t)=3\sin\omega t$

$u_S(t)=3\sin\left(\omega t+30^\circ\right) \qquad i(t)=3\sqrt{2}\sin\left(\omega t+45^\circ\right)$

求： $u_S(t)=4\sin\left(\omega t+30^\circ\right)\Rightarrow i(t)=?$



解： $\dot{I}=G\dot{U}_S+\dot{I}_0$

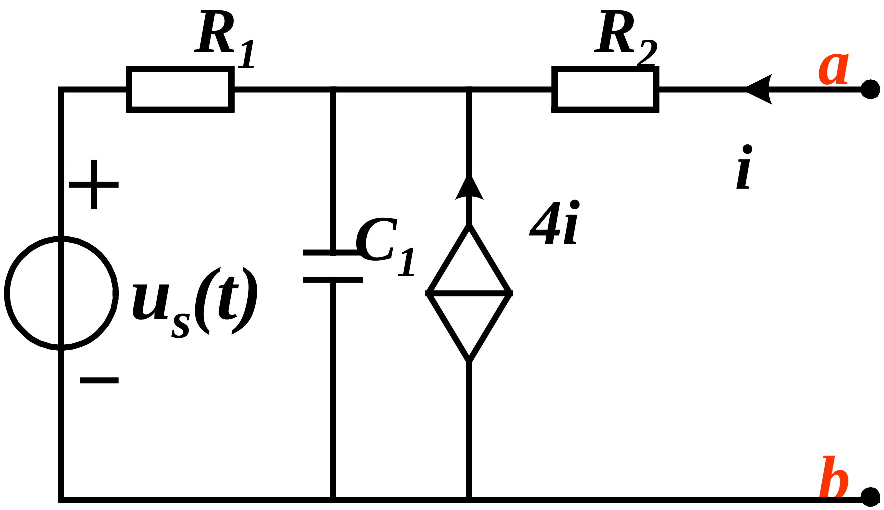
$$\begin{cases}\dfrac{3}{\sqrt{2}}=\dot{I}_0 \\ 3\angle 45^\circ=G\cdot\dfrac{3}{\sqrt{2}}\angle 30+\dot{I}_0\end{cases}\Rightarrow \begin{array}{l}\dot{I}_0=\dfrac{3}{\sqrt{2}} \\ G=1\angle 60^\circ\end{array}$$

$$\dot{I}=1\angle 60^\circ\cdot\frac{4}{\sqrt{2}}\angle 30^\circ+\frac{3}{\sqrt{2}}=\frac{5\angle 53.1^\circ}{\sqrt{2}}$$

$$i(t)=5\sin\left(\omega t+53.1^\circ\right)\quad (A)$$

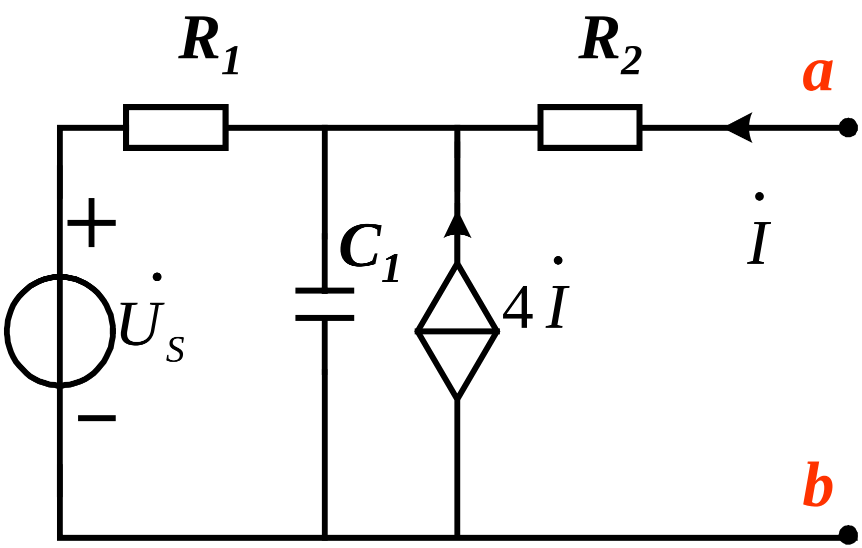
例4 已知：$u_S(t)=10\sqrt{2}\sin\left(10^6 t+15^\circ\right)V \quad R_1=R_2=2\Omega \quad C_1=0.5\mu F$

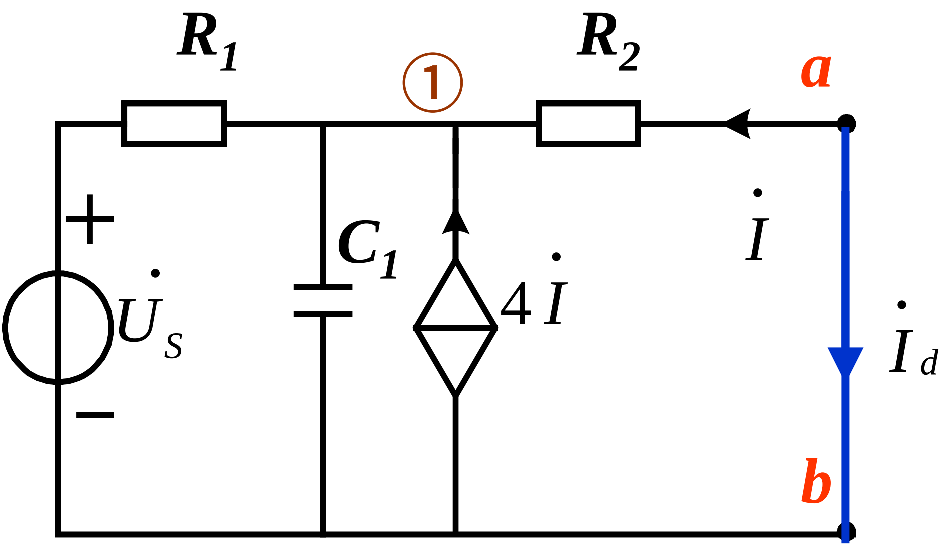
求：ab端口的戴维南等效电路



解：左图为时域电路

画出相量电路模型





相量电路模型---频域电路

当ab端口开路时

$$\dot U_{ab0} = \frac{\dot U_S}{R_1 - j\frac{1}{\omega C_1}}\cdot\left(-j\frac{1}{\omega C_1}\right)$$

$$= \frac{10\angle 15^\circ}{2-j2}\cdot(-j2) = 5\sqrt{2}\angle -30^\circ\ (V)$$

当ab端口短路时

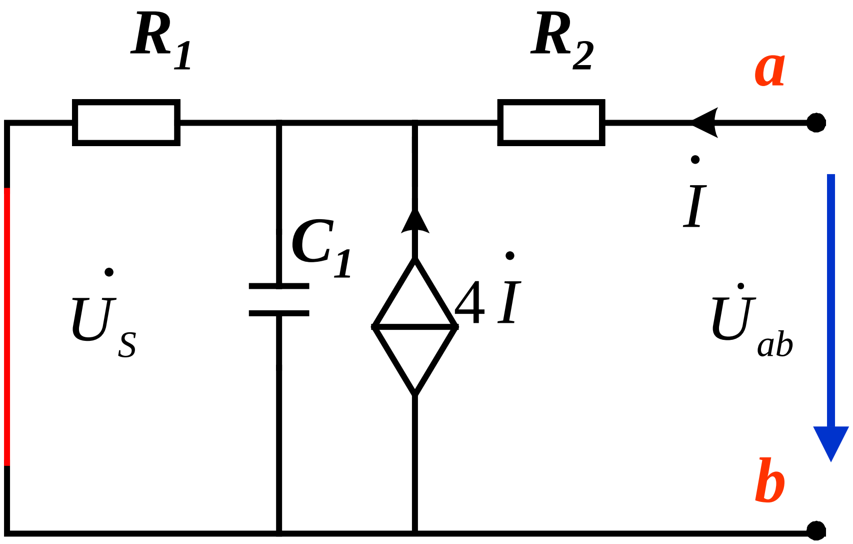
$$\dot U_1\left(\frac{1}{R_1}+j\omega c+\frac{1}{R_2}\right)=\frac{\dot U_S}{R_1}+4\dot I$$

$$\dot U_1 = \frac{5\angle 15^\circ}{3+j0.5}(V)$$

$$\dot I = -\frac{\dot U_1}{R_2}$$

$$\dot I_d = -\dot I = \frac{\dot U_1}{R_2} = 0.822\angle 5.5^\circ\ (A)$$

$$Z_d = \dot U_{ab0}\Big/\dot I_d = 7-j5\,(\Omega)$$

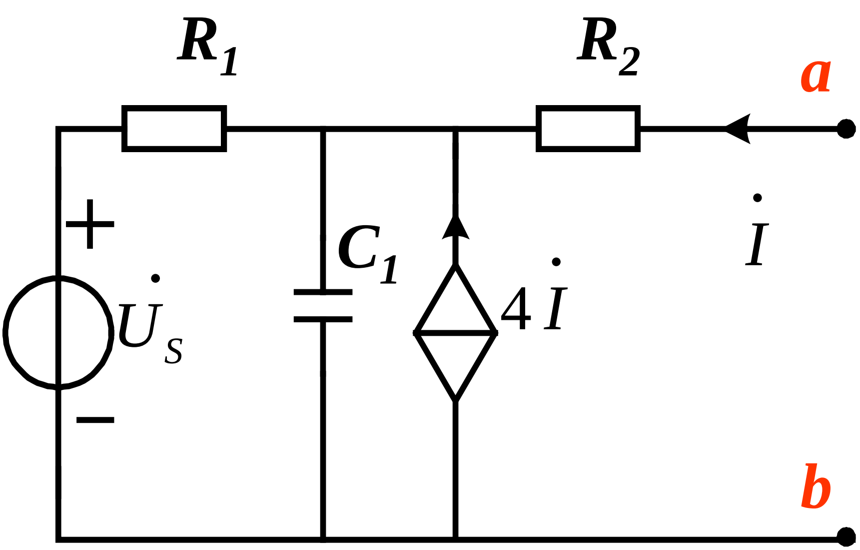


加压法：令 $\dot{U}_S = 0$

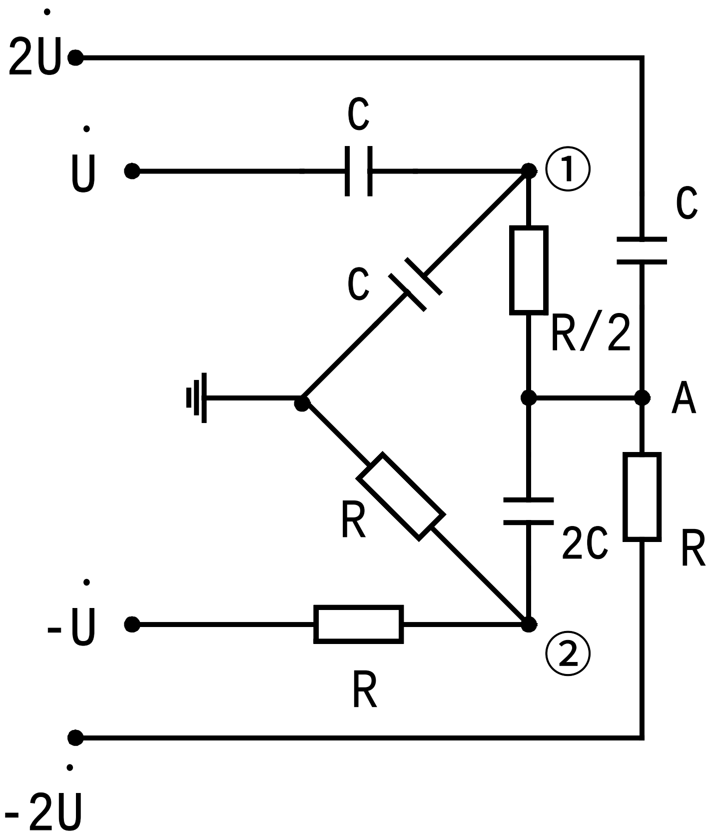
$$\dot{U}_{ab} = \dot{I}\ R_2 + \left(\dot{I} + 4\dot{I}\right) R_1 // (-j\frac{1}{\omega C_1})$$

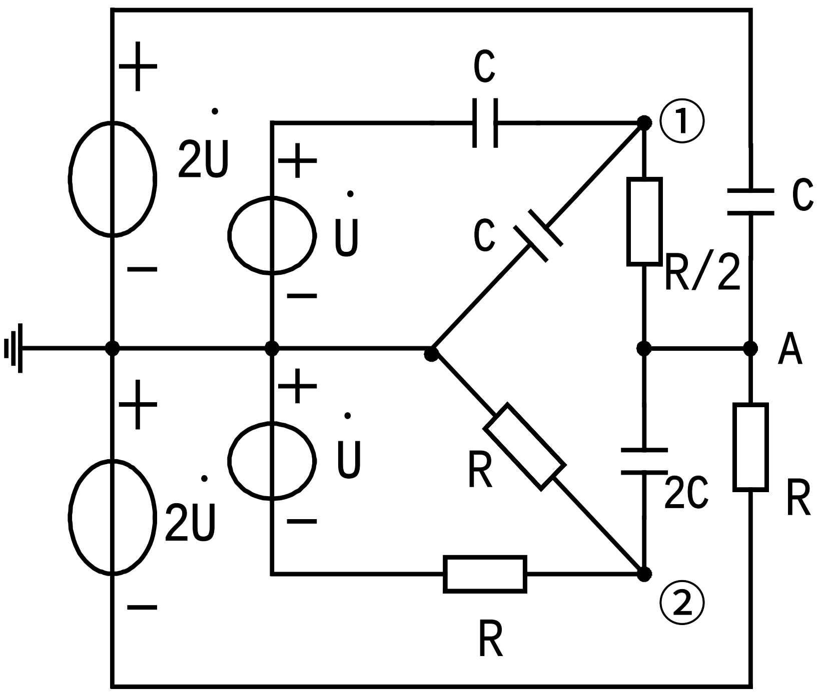
$$\Rightarrow Z_d = \frac{\dot{U}_{ab}}{\dot{I}} = R_2 + 5 \cdot R_1 // (-j\frac{1}{\omega C_1})$$

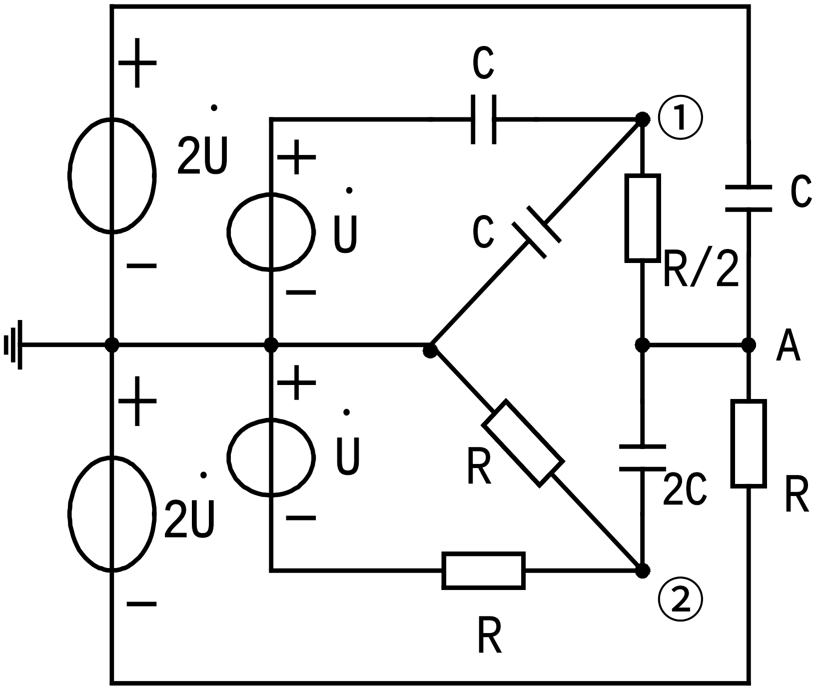
$$= 7 - j5(\Omega)$$

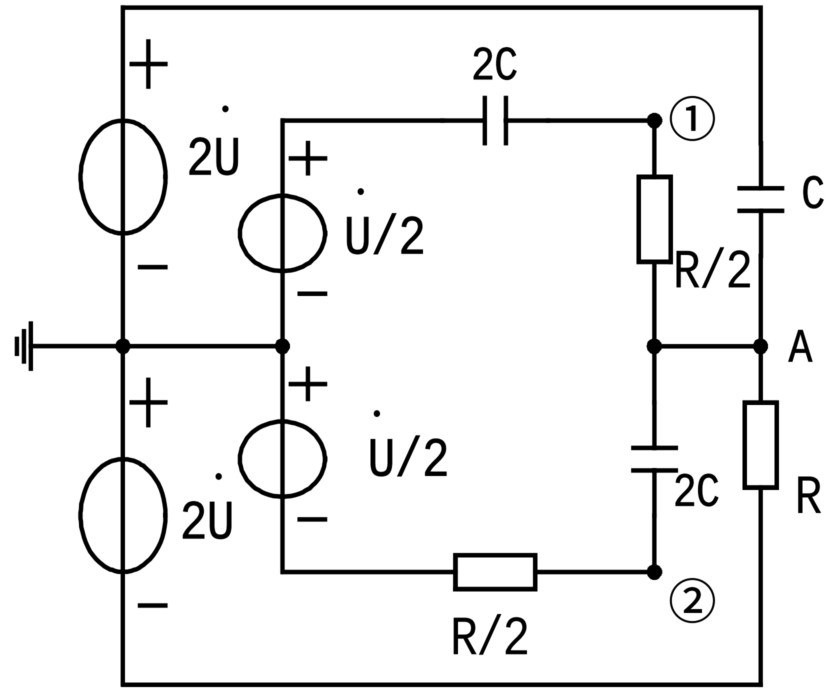


例5：已知 RωC=1，求A点的电压。









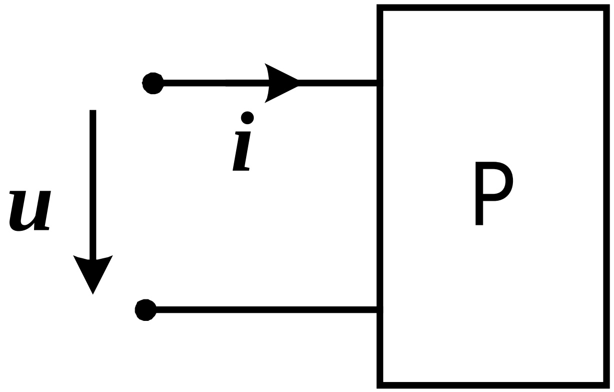
$$\dot{U}_A = \frac{2\dot{U}(j\omega C) - 2\dot{U}/R}{j\omega C + \dfrac{2}{R/2 - j/2\omega C} + \dfrac{1}{R}} = j\frac{2}{3}\dot{U}$$

§3–12 正弦交流电路的功率

实现电路的目的之一是为了实现电能的传输与分配。所以功率计算是电路分析的一项重要任务。在正弦交流电路中除了阻性负载消耗能量还有储能元件L，C储存的电磁能量与外界交换的现象，故而功率种类较多。

瞬时功率和平均功率

当一端口网络的端口电压和端口电流的参考方向一致时，



$$p = ui \qquad \psi_u - \psi_i = \varphi$$

设

$$u = \sqrt{2}U \sin \omega t$$

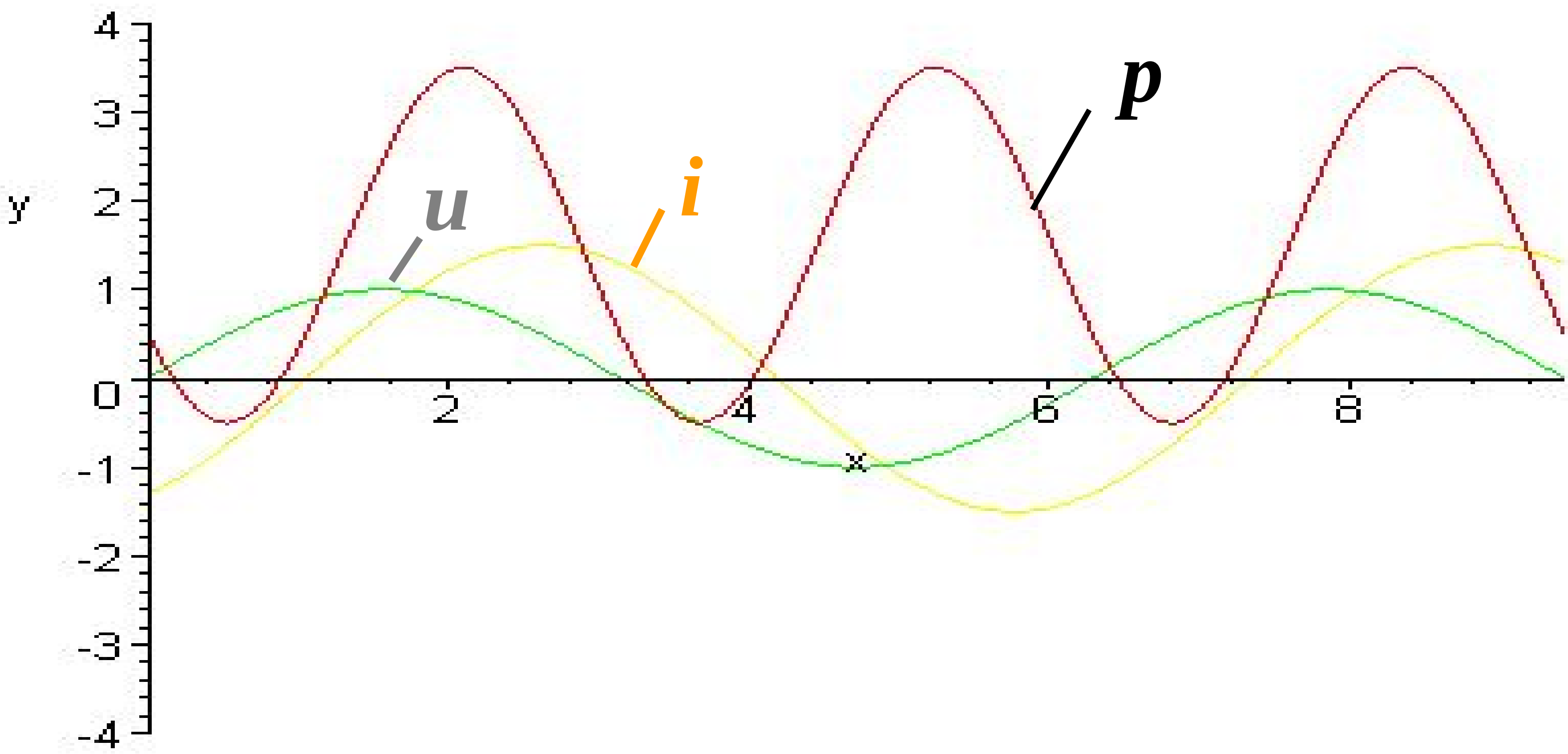
$$i = \sqrt{2}I \sin\left(\omega t - \varphi\right)$$

1）瞬时功率

$$p(t) = 2UI \sin \omega t \sin(\omega t - \varphi)$$

$$= UI \cos \varphi - UI \cos(2\omega t - \varphi)$$

波形图：



2）平均功率 $$P=\frac{1}{T}\int_0^T p(t)\,dt$$

$$=\frac{1}{T}\int_0^T \left[UI\cos\varphi - UI\cos(2\omega t-\varphi)\right]dt \quad = UI\cos\varphi$$

有功功率.无功功率和视在功率

1.有功功率：即平均功率

①定义式 $$P=\frac{1}{T}\int_0^T p(t)\,dt \quad = UI\cos\varphi$$

② $\cos\varphi$ —— 功率因数

φ —— 阻抗角也称功率因数角

- u，i参考方向一致时 $|\varphi|\le\frac{\pi}{2}$ $P\ge 0$

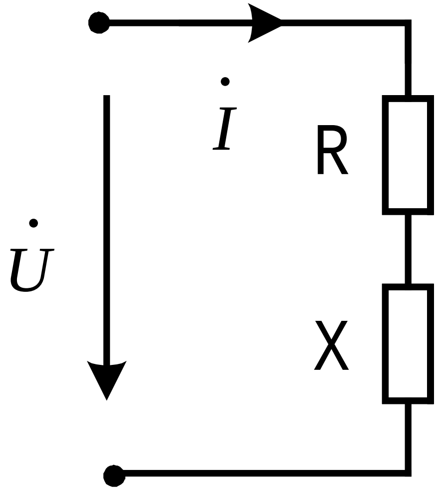
$$p(t) = UI\cos\varphi - UI\cos(2\omega t - \varphi)$$

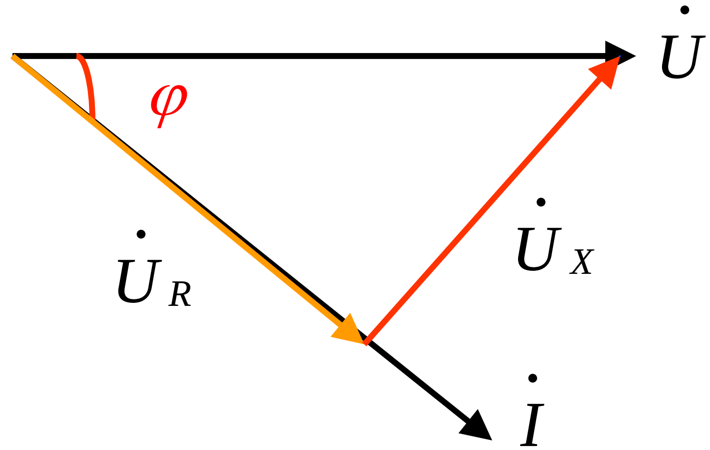
$$= UI\cos\varphi - UI\cos(2\omega t - 2\varphi + \varphi)$$

$$= UI\cos\varphi - UI\cos(2\omega t - 2\varphi)\cos\varphi + UI\sin(2\omega t - 2\varphi)\sin\varphi$$

$$= UI\cos\varphi[1 - \cos(2\omega t - 2\varphi)] + UI\sin\varphi\sin(2\omega t - 2\varphi)$$

$$= p_R(t) + p_X(t)$$





$$u = \sqrt{2}U \sin \omega t$$

$$i = \sqrt{2}I \sin\left(\omega t - \varphi\right)$$

$$\dot{U}_R = U \cos\varphi \angle -\varphi$$

$$u_R(t) = \sqrt{2}U \cos\varphi \sin(\omega t - \varphi)$$

$$\dot{U}_X = U \sin\varphi \angle 90^\circ - \varphi$$

$$u_X(t) = \sqrt{2}U \sin\varphi \sin(\omega t - \varphi + 90^\circ)$$

$$p_R(t) = u_R(t)i = \sqrt{2}U \cos\varphi \sin(\omega t - \varphi) \times \sqrt{2}I \sin(\omega t - \varphi)$$

$$= 2UI \cos\varphi \sin^2(\omega t - \varphi)$$

$$= UI \cos\varphi [1 - \cos(2\omega t - 2\varphi)]$$

$$p_X(t) = u_X(t)i = \sqrt{2}U \sin\varphi \sin(\omega t - \varphi + 90^\circ) \times \sqrt{2}I \sin(\omega t - \varphi)$$

$$= 2UI \sin\varphi \sin(\omega t - \varphi + 90^\circ) \sin(\omega t - \varphi)$$

$$= UI \sin\varphi \sin(2\omega t - 2\varphi)$$

2、无功功率：

① 定义： $Q = UI\sin\varphi$

② u, i 参考方向一致时 $|\varphi| \le \frac{\pi}{2}$

$\varphi > 0$ $Q > 0$ 电路呈感性

$\varphi < 0$ $Q < 0$ 电路呈容性

$\varphi = 0$ $Q = 0$ 电路呈阻性

③ 并不表示单位时间能作的功或转化的能量，无源一端口网络的无功功率表示该一端口网络与外电路功率交换的大小，也就是该无源一端口网络等效电抗(电纳)元件与外电路能量(功率)交换的幅值。

④ 单位：乏 VAR

具有功率的量纲，但区别于有功功率

3.视在功率：

1）定义： $S = UI$

实际为电气设备的容量

2）单位 ：伏安 VA

3)P、Q、S关系

$$P = UI\cos\varphi = S\cos\varphi$$

$$Q = UI\sin\varphi = S\sin\varphi$$

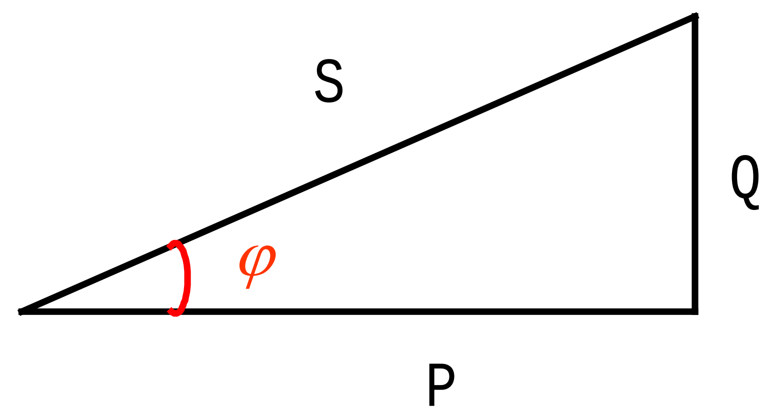
$$S = \sqrt{P^2 + Q^2}$$

$$\varphi = tg^{-1}\frac{Q}{P}$$

1000VA电动机

$\cos\varphi = 1$ P=1000W

$\cos\varphi = 0.5$ P=500W



功率三角形

复数功率

为了分析方便，引入复数功率概念，使一端口网络有功功率和无功功率用一个统一表达式来表示

1)定义.

$$\tilde{S} = \dot{U} \overset{*}{\dot{I}}$$

$$\dot{U} = U\angle\psi_u \qquad \dot{I} = I\angle\psi_i \qquad \overset{*}{\dot{I}} = I\angle-\psi_i$$

$$\tilde{S} = UI\angle\psi_u - \psi_i = S\angle\varphi$$

$$= S\cos\varphi + jS\sin\varphi$$

$$= P + jQ$$

模：视在功率

幅角：功率因数角

实部：有功功率

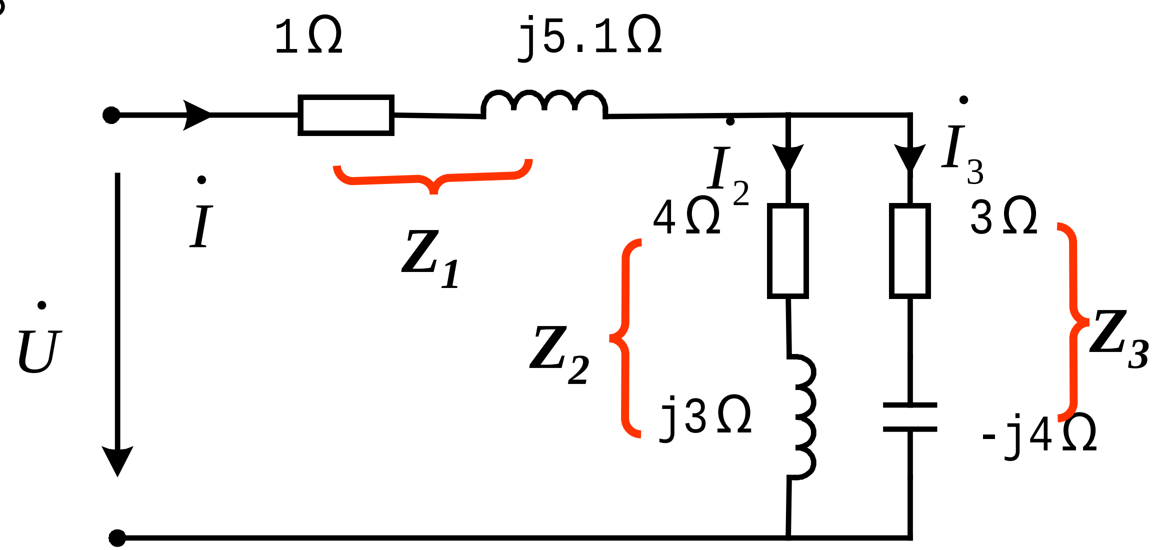
虚部：无功功率

说明：① $\tilde{S}$ 纯粹是一个复数，不是相量

② $$\tilde{S} = \dot{U}\overset{*}{\dot{I}} = Z\dot{I}\overset{*}{\dot{I}} = I^2 Z$$

$$\tilde{S} = \dot{U}\overset{*}{\dot{I}} = \dot{U}\left(\overset{*}{\dot{U}Y}\right) = U^2\overset{*}{Y}$$

例：如图所示，U=220V.求该网络吸收的复数功率，并校验功率平衡。



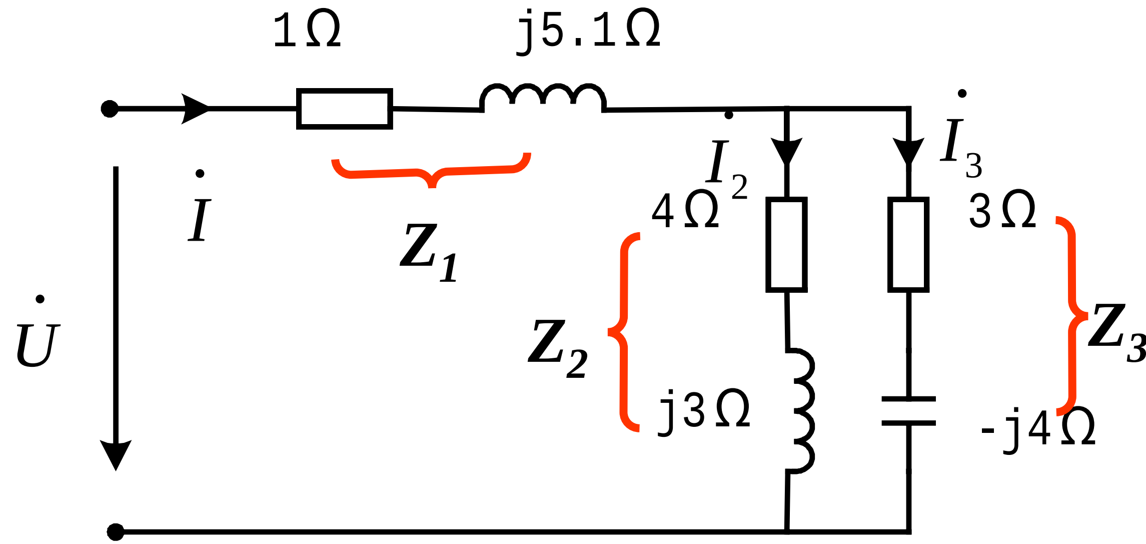
解：

$$Z = Z_1 + Z_2 // Z_3$$

$$= 1 + j5.1 + \frac{(4 - j3)(3 - j4)}{4 - j3 + 3 - j4}$$

$$= 4.51 + j4.51 \quad = 6.38\angle 45^\circ \ (\Omega)$$

设：$\dot{U} = 220\angle 0^\circ$

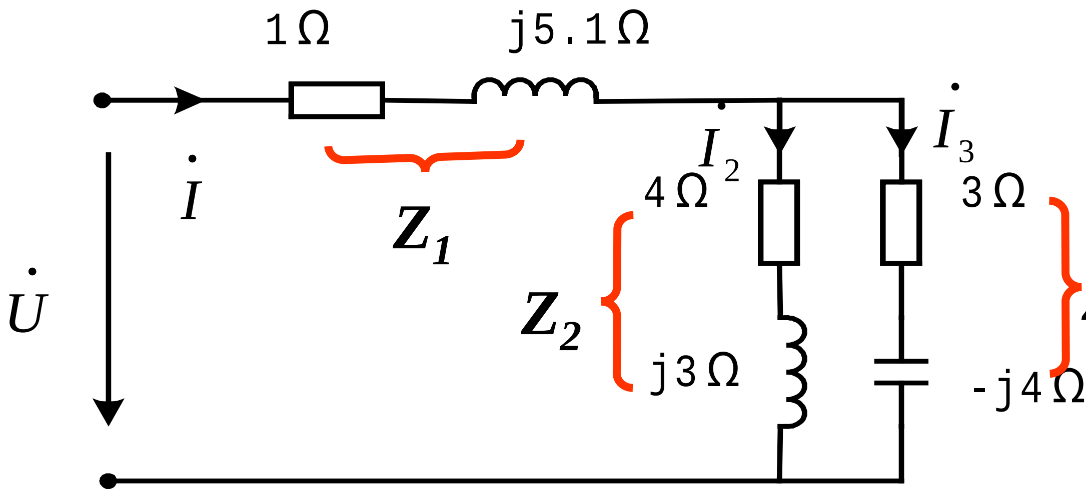


$$\dot{I} = \frac{\dot{U}}{Z} = \frac{220\angle 0^\circ}{6.38\angle 45^\circ} = 34.49\angle -45^\circ$$

$$\tilde{S} = \dot{U}\overset{*}{\dot{I}} = 220\angle 0^\circ \cdot 34.49\angle 45^\circ = 5365.4 + j5365.4$$

$$P = 5365.4(W) \qquad Q = 5365.4(VAR)$$

$$\dot{I}_2 = \dot{I} \cdot \frac{Z_3}{Z_2 + Z_3} = 24.38\angle -90^\circ$$



$$\dot{I}_3 = \dot{I} - \dot{I}_2 = 24.38\angle 0^\circ$$

$$\tilde{S}_1 = I^2 Z_1 = 34.49^2 (1 + j5.1)$$

$$= 1188.87 + j6063.24$$

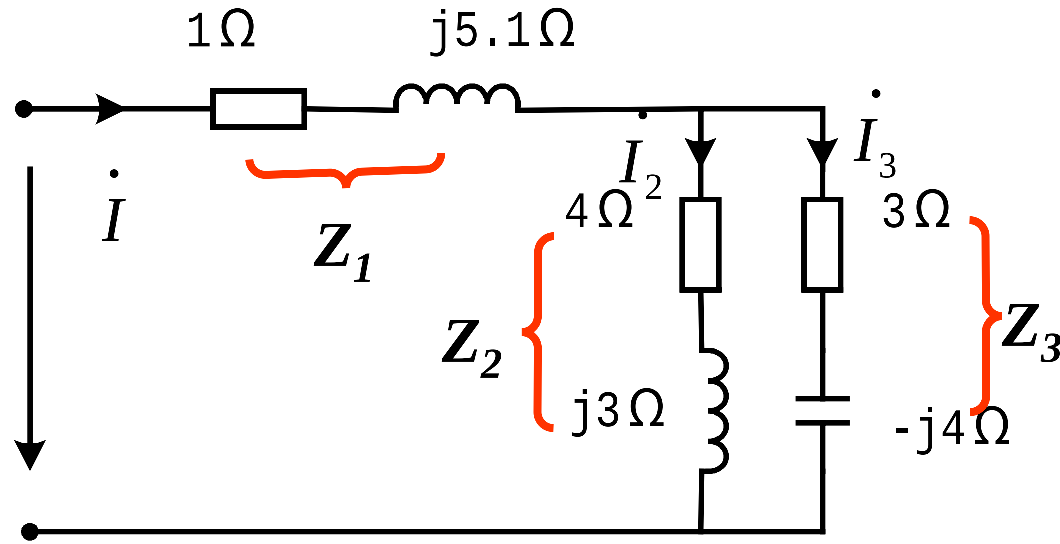
$$\tilde{S}_2 = I_2^{\ 2} Z_2 = 24.38^2 (4 + j3) = 2377.54 + j1783.15$$

$$\tilde{S}_3 = I_3^{\ 2} Z_3 = 24.38^2 (3 - j4) = 1783.15 - j2377.54$$

(1) $\tilde{S}_1 + \tilde{S}_2 + \tilde{S}_3 = \tilde{S}$

$$P_1 + P_2 + P_3 = P$$

$$Q_1 + Q_2 + Q_3 = Q$$

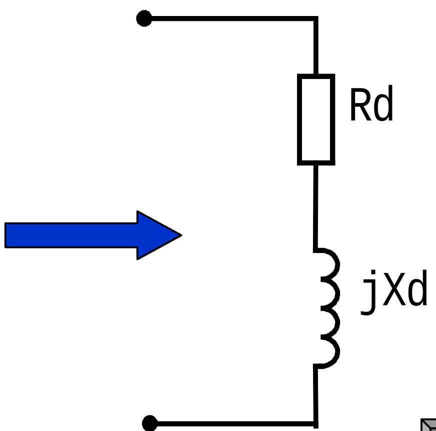


(2) $P = I^2 \cdot R_d$

$$= 34.49^2 \times 6.38 \cdot \cos 45$$

$$= 5366.5$$

$$Z = 4.51 + j4.51 = R_d + jX_d$$

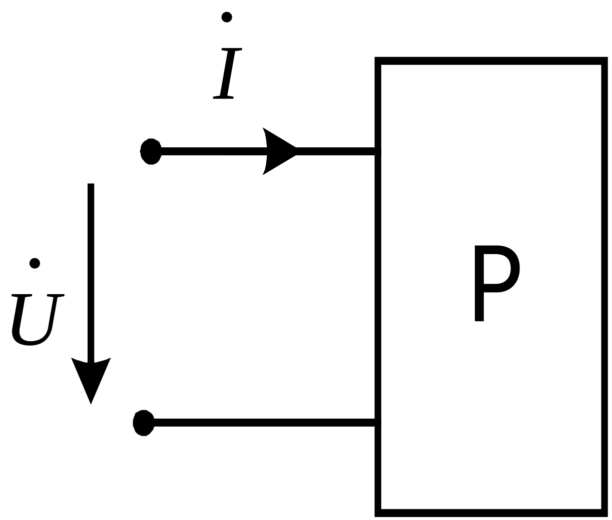


$Q = I^2 \cdot X_d$

$$= 34.49^2 \times 6.38 \cdot \sin 45$$

$$= 5366.5$$

功率判据



关联参考方向：

$P > 0$ 吸收有功功率

$P < 0$ 产生有功功率

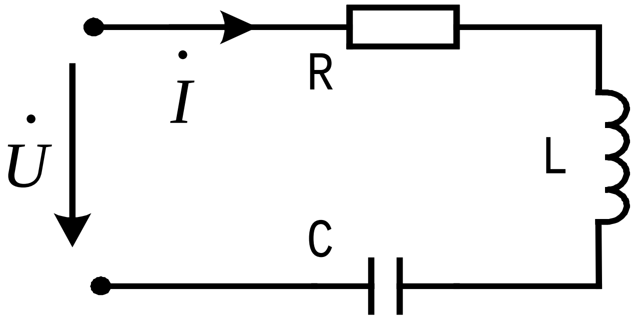
$Q > 0$ 吸收感性无功功率

$Q < 0$ 产生感性无功功率

吸收容性无功功率

以R-L-C为例：

假设 $|Q_L|>|Q_C|$



L 吸收感性无功功率， C 吸收容性无功功率，从外界吸收 $|Q_L|-|Q_C|$

$$\tilde{S}=\dot{U}\overset{*}{\dot{I}}=I^2Z=I^2\left(R+j\omega L-j\frac{1}{\omega C}\right)$$

$$P=I^2R$$

$$Q=I^2(\omega L-\frac{1}{\omega C})$$
$$=|Q_L|-|Q_C|$$

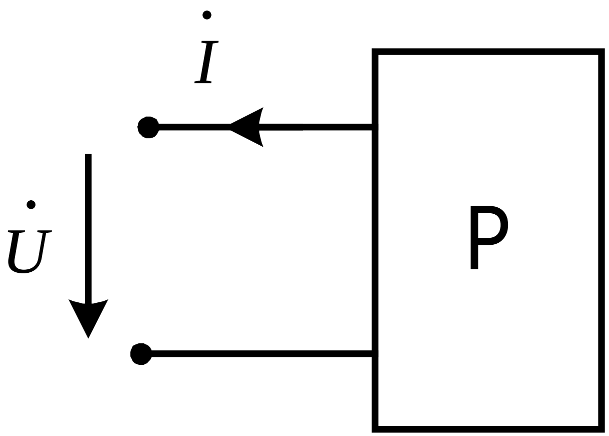
L吸收感性无功功率，

从外界吸收 $|Q_L|-|Q_C|$，

C 产生感性无功功率Q_C，

$|Q_L|=（|Q_L|-|Q_C|）+|Q_C|$

非关联参考方向



$P>0$ 产生有功功率

$Q>0$ 产生感性无功功率

(吸收容性)

$P<0$ 吸收有功功率

$Q<0$ 吸收感性无功功率

(产生容性)

有功功率的测量

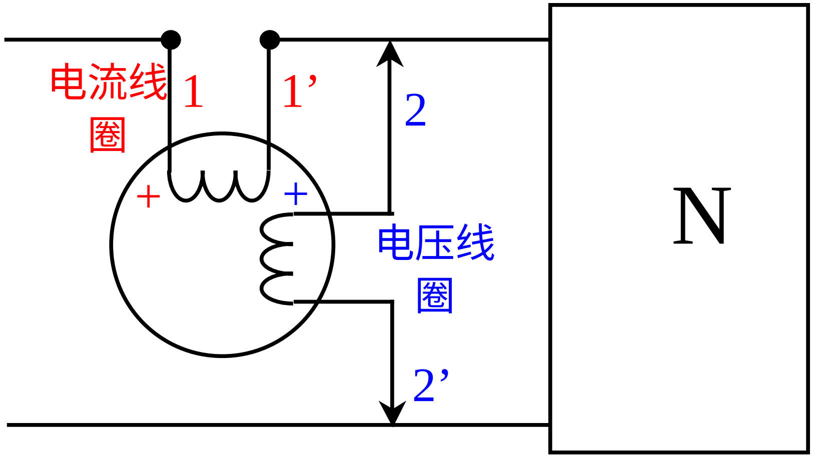
在正弦交流电路中，一般使用电功式的功率表(瓦特表)来测量平均功率.

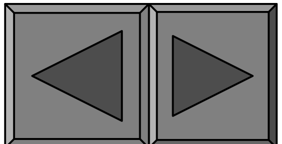
功率表有两对端钮

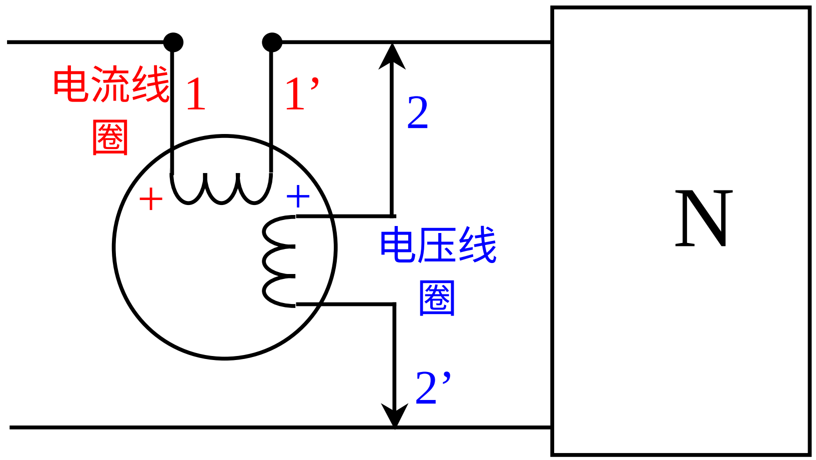
一对端钮—电流线圈端钮　　串接

一对端钮—电压线圈端钮　　并接

$$P = UI\cos\varphi \qquad \varphi = \psi_u - \psi_i$$



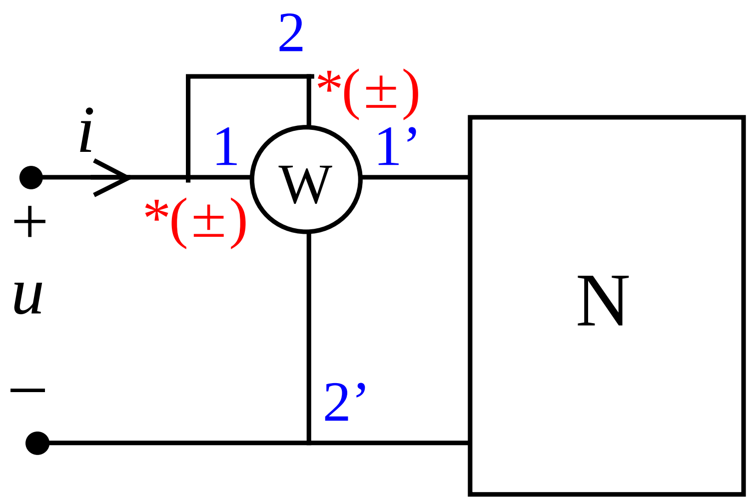




$$P = UI\cos\varphi \qquad \varphi = \psi_u - \psi_i$$

电流：从有∗ → 无∗

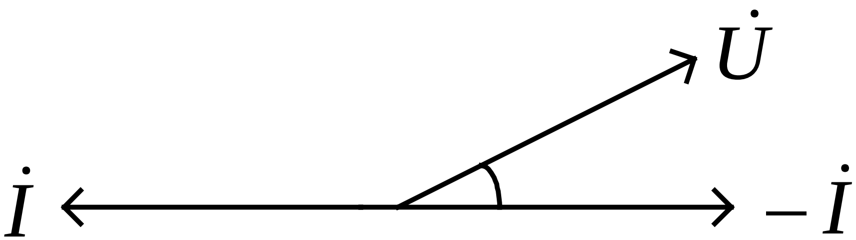
电压：从有∗ → 无∗



标明极性

$P = UI\cos\varphi \qquad \varphi = \psi_u - \psi_i$

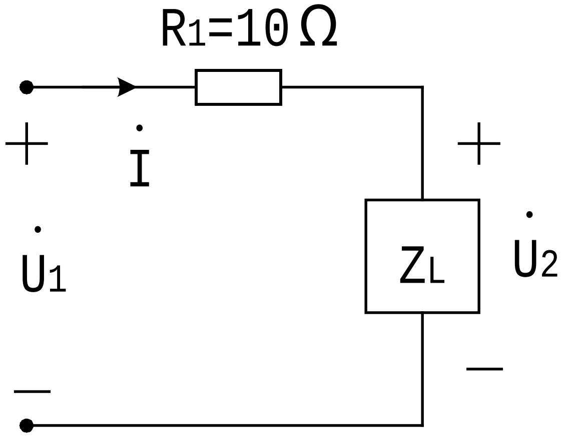
? 倘若： $P < 0$ 功率表无法读数

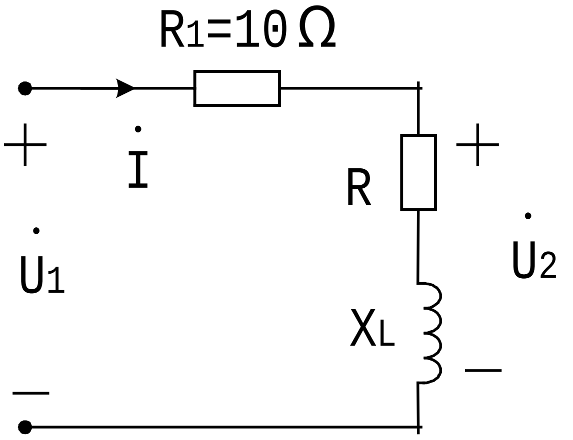


将电流线圈两端钮对调.测出P是网络发出的有功功率

所以说：从功率表指针偏转可判断功率传输方向

例：已知负载Z_L消耗的有功功率和无功功率分别为4W和12Var，且 $\dot{U}_1$和 $\dot{U}_2$的相位差为30^0，求Z_L和端口电流 I 。



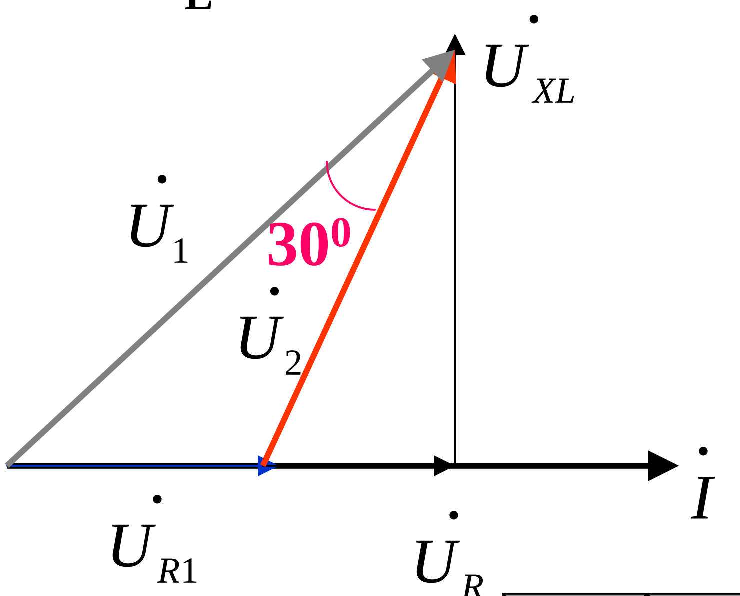


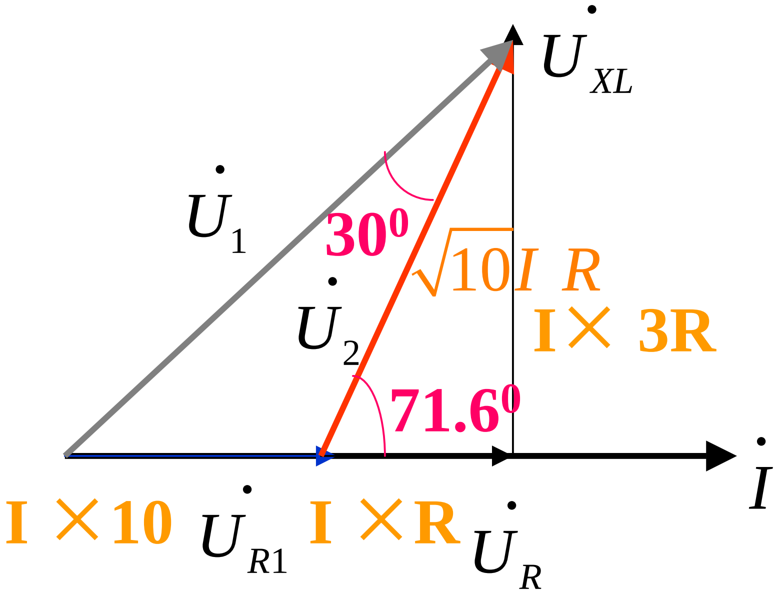
解：无功功率为 12Var， Z_L为感性负载

$Z_L=R+jX_L$ $X_L/R=3$

$I^2R=4$

$I^2X_L=12$





$$tg^{-1}3=71.6^0$$

正弦定理

$$\frac{I\times 10}{\sin 30^0}=\frac{\sqrt{10}I\ R}{\sin(71.6^0-30^0)}$$

R=4.2Ω

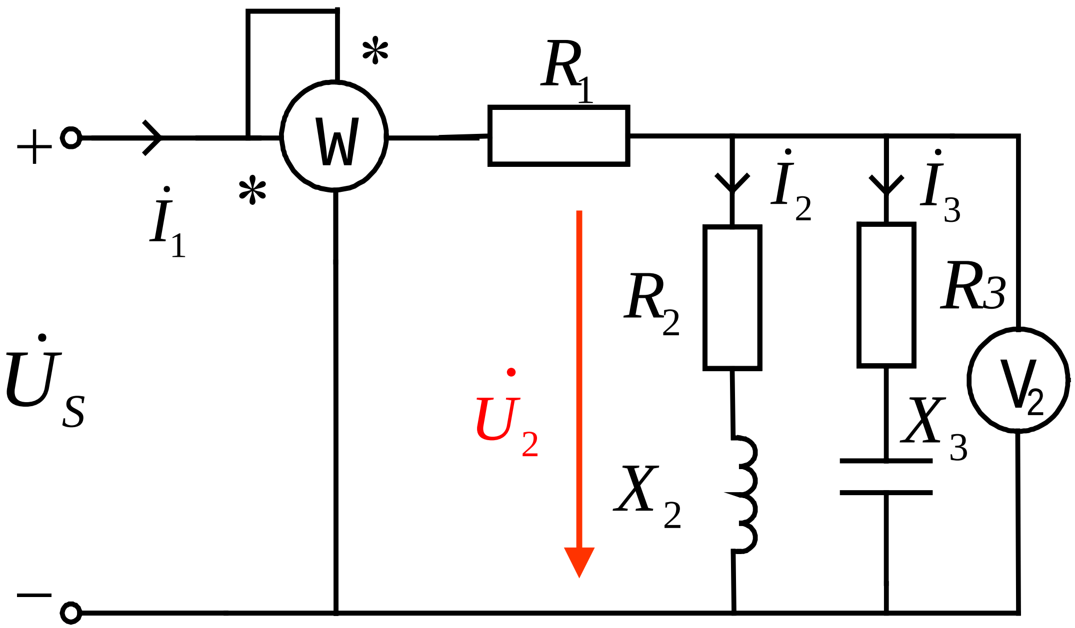
Z_L=4.2+j12.6

P=I^2R=4

I=0.976 A

例：如图.已知 $I_1=I_2=I_3$　$R_1=R_2=R_3$，$U_S=150V$，瓦特表读数1500W

求：R_1、R_2、R_3；X_2、X_3、电压表读数 U_2？



$$\frac{\dot{U}_2}{\dot{I}_2}=Z_2=R_2+jX_2$$

$$\frac{U_2}{I_2}=\sqrt{R_2^{\,2}+X_2^{\,2}}$$

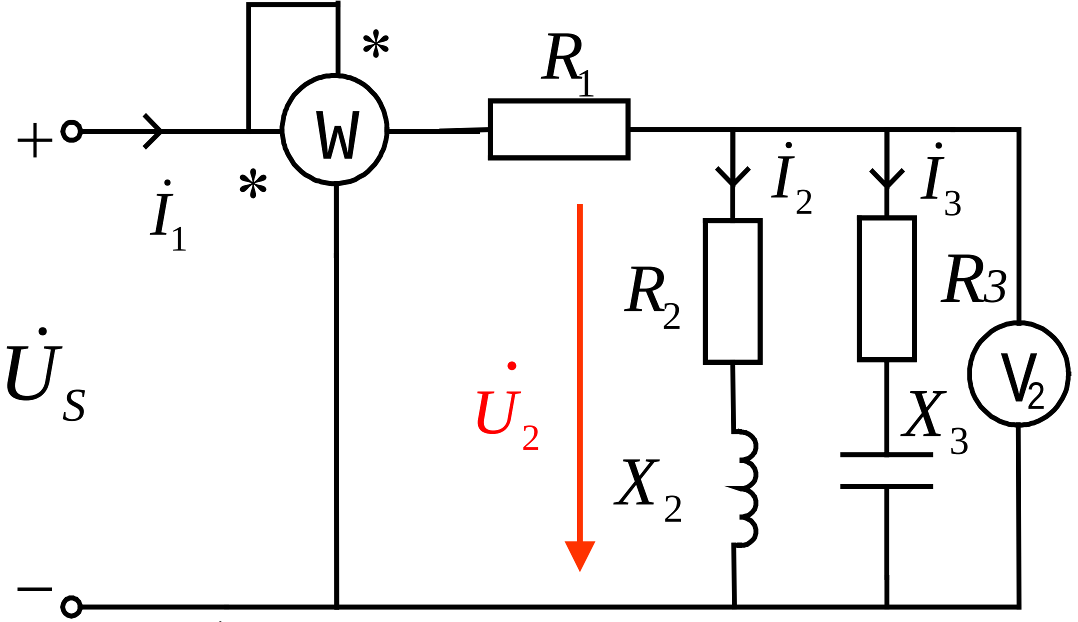
$$\frac{\dot{U}_2}{\dot{I}_3}=Z_3=R_3+jX_3$$

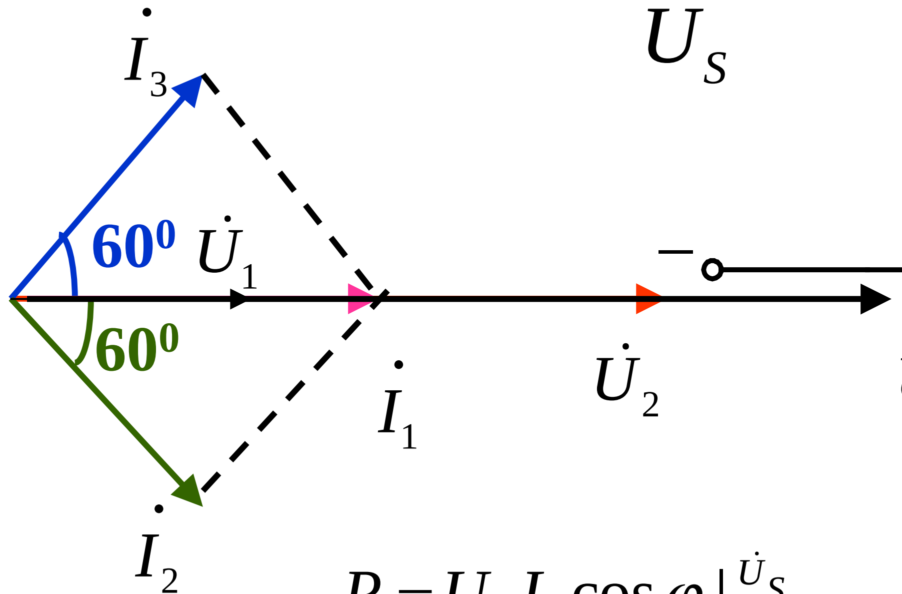
$$\frac{U_2}{I_3}=\sqrt{R_3^{\,2}+X_3^{\,2}}$$

解：以 $\dot{U}_2$ 作为参考相量

$\because I_2=I_3$　$R_2=R_3$　　$\therefore X_2=X_3$

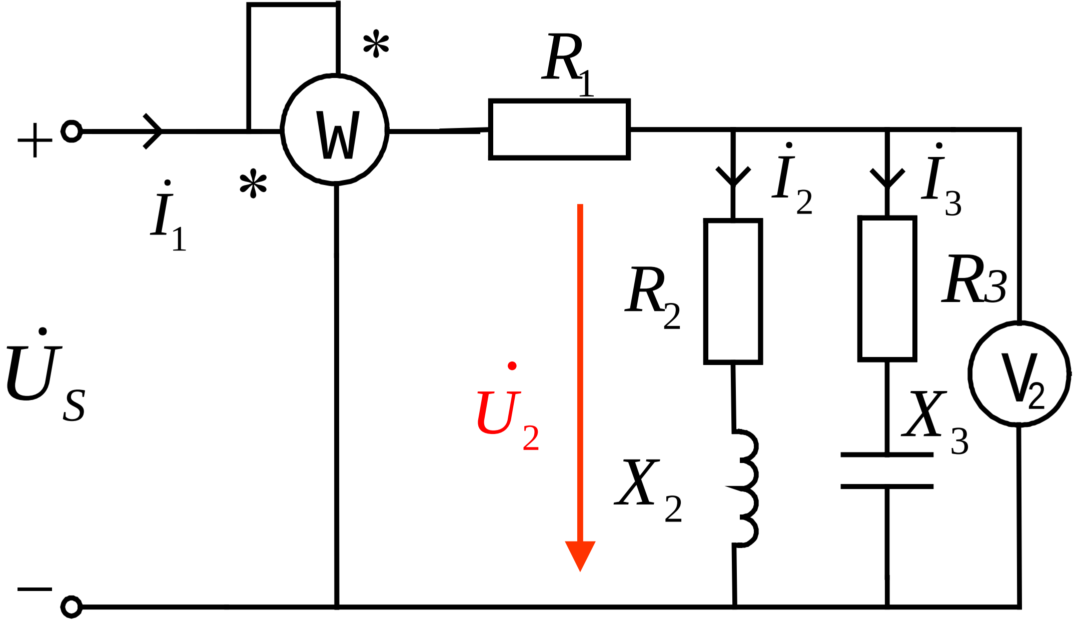
组成等边三角形





$$P = U_S I_1 \cos\varphi \,|^{\dot{U}_S}_{\dot{I}_1}$$

$$P = U_S I_1 \qquad I_1 = P/U_S = 10(A)$$



$$I_1 = I_2 = I_3 = 10A$$

$$I_1^{\,2}R_1 + I_2^{\,2}R_2 + I_3^{\,2}R_3 = P$$

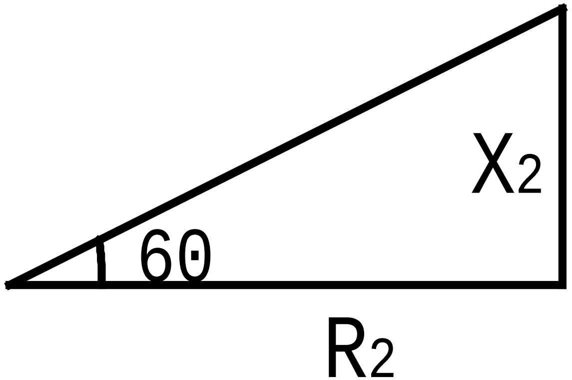
$$3I_1^{\,2}R_1 = P$$

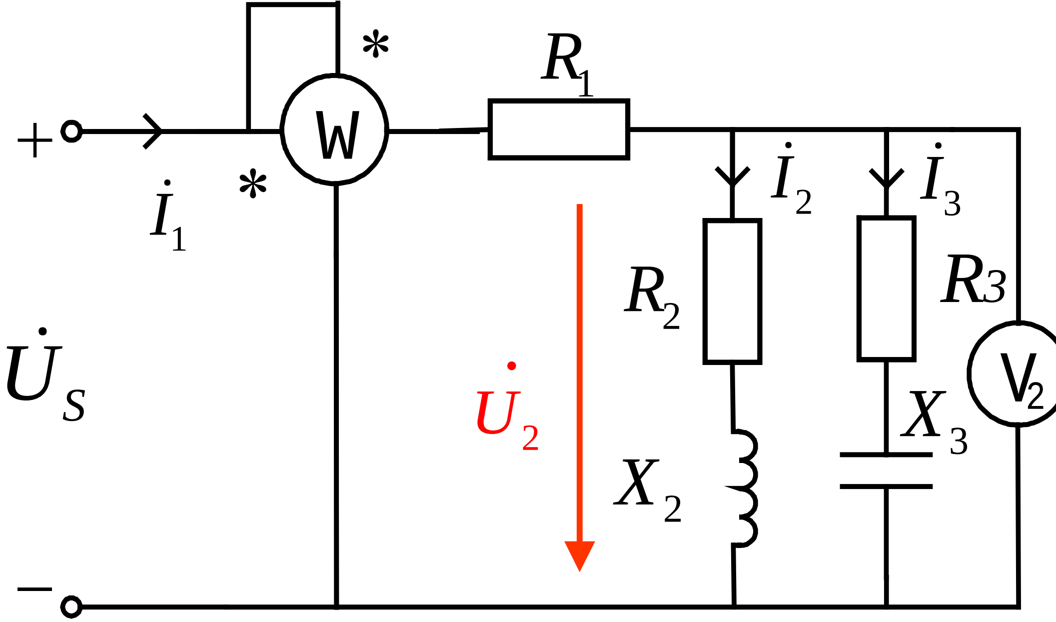
$$R_1 = P/3I_1^{\,2} = 5(\Omega) = R_2 = R_3$$

$$Z_2 = R_2 + jX_2 = Z_2\angle\varphi_2 \qquad Z_3 = R_3 - jX_3 = Z_3\angle\varphi_3$$

$$\varphi_2 = 60^0 \qquad \varphi_3 = -60^0$$

阻抗三角形



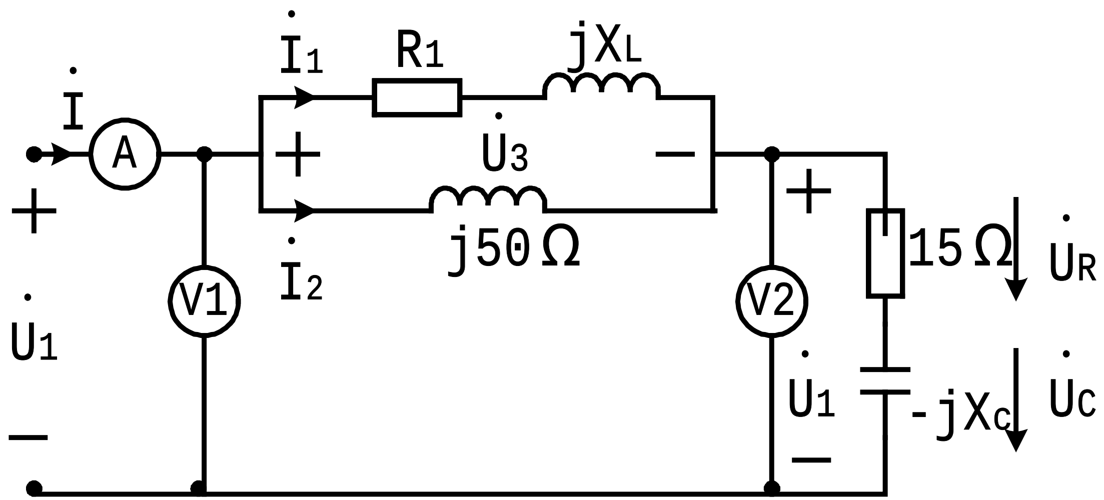


$$X_2 = R_2 tg\varphi_2 = 5\sqrt{3}\Omega$$

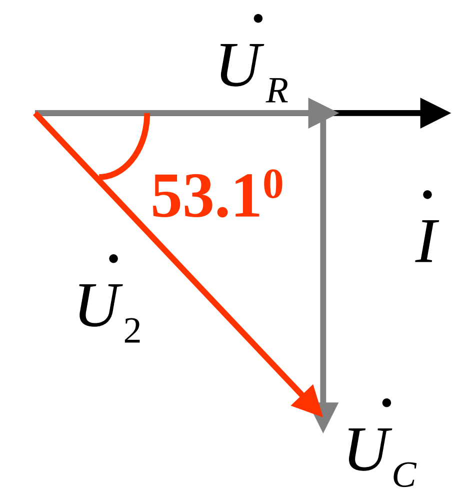
$$X_3 = -5\sqrt{3}\Omega$$

$$U_2 = I_2 \cdot Z_2 = 10 \times \sqrt{R_2^{\ 2} + X_2^{\ 2}} = 100\,(V)$$

例：感性二端网络中，电流表读数为10A，两电压表读数均为250V，已知二端网络消耗的功率为2000W，求X_C、R_1、X_L的值。



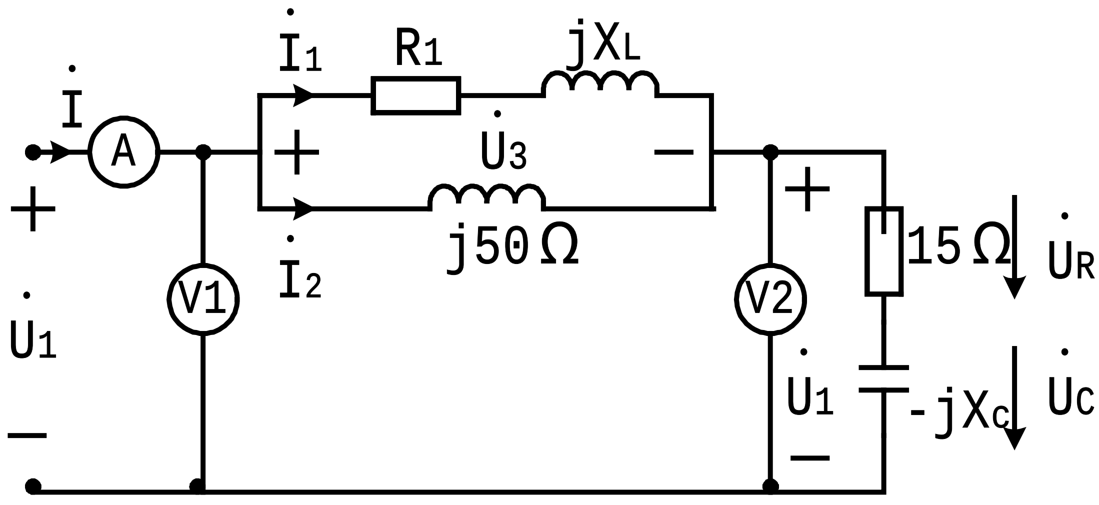
解：

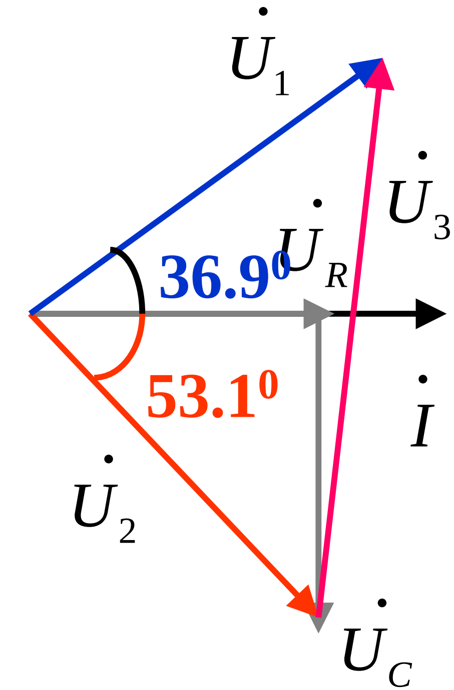


$$\dot{I}=10\angle 0^0\ A \qquad \dot{U}_R=15R\angle 0^0=150\angle 0^0\ V$$

$$U_C=\sqrt{U_2^{\,2}-U_R^{\,2}}=200(V) \qquad X_C=U_C/I=20\Omega$$

$$tg^{-1}\frac{200}{150}=53.1^0 \qquad P=U_1 I\cos\varphi \qquad \varphi=36.9^0$$





$$\dot{U}_3 = 250\sqrt{2}\angle(36.9^0 + 45^0) = 250\sqrt{2}\angle 81.9^0 \ (V)$$

$$\dot{I}_2 = \dot{U}_3 / j50 = 5\sqrt{2}\angle -8.1^0 \ (A)$$

$$\dot{I}_1 = \dot{I} - \dot{I}_2 = 10 - 5\sqrt{2}\angle -8.1^0 = 2.23\sqrt{2}\angle 18.37^0 \ (A)$$

$$R_1 + jX_L = \frac{\dot{U}_3}{\dot{I}_1} = 112\angle 63.53^0 = 50 + j100 \ (\Omega)$$

功率因数的提高

1.提高功率因数的意义 $S=UI$ $P=UI\cos\varphi$

1)充分利用设备 $S=1000VA$ $\cos\varphi=0.85$ $P=850W$

$\cos\varphi=0.95$ $p=950W$

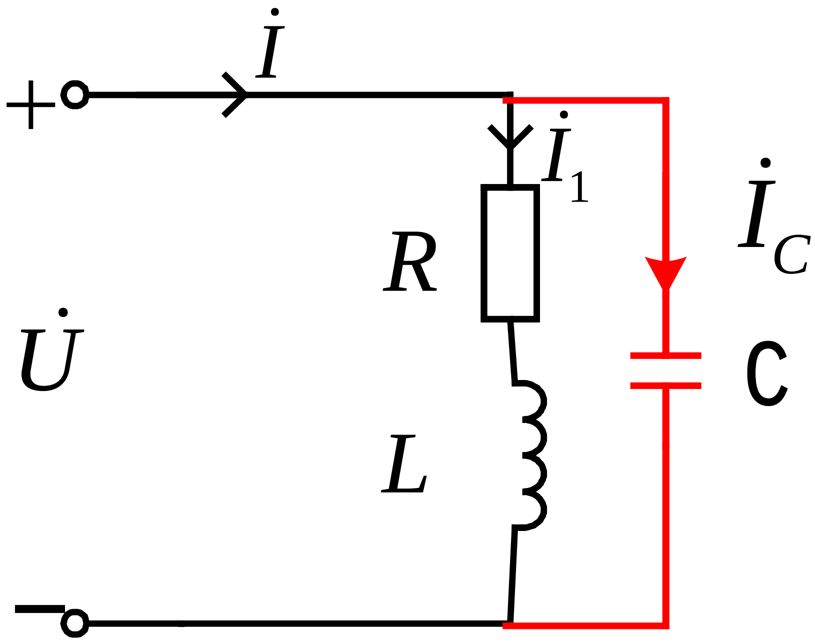
2)减少线路损耗 $P=UI\cos\varphi$ $I=\dfrac{P}{U\cos\varphi}$

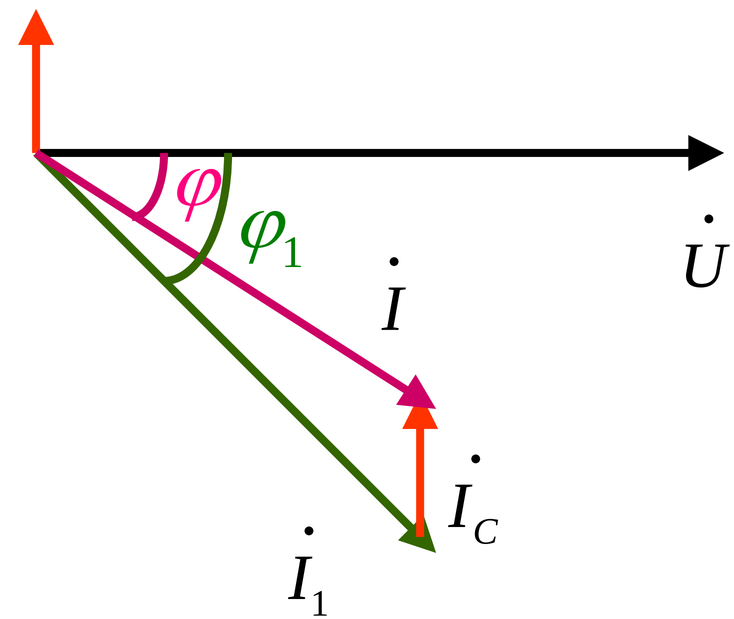
$\cos\varphi$小$\downarrow$ I大$\uparrow$ $\Delta P=I^2R\uparrow$

2.如何提高(措施)

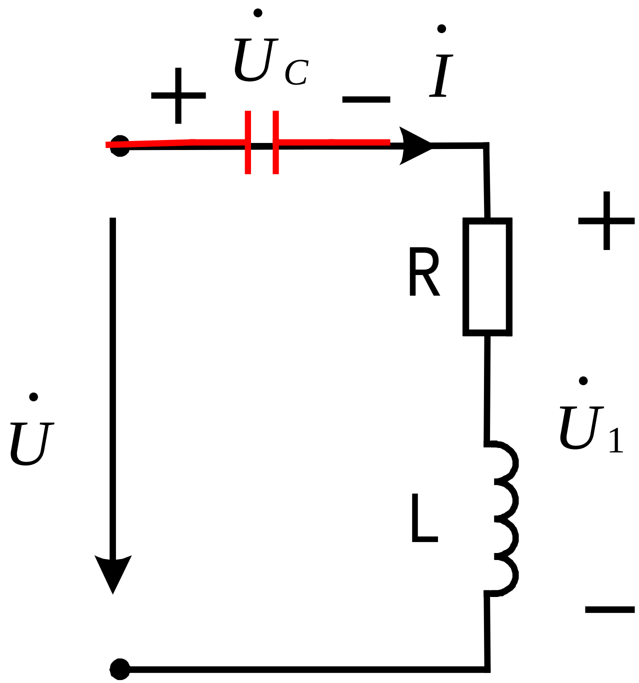
1）从无功电流补偿来看：

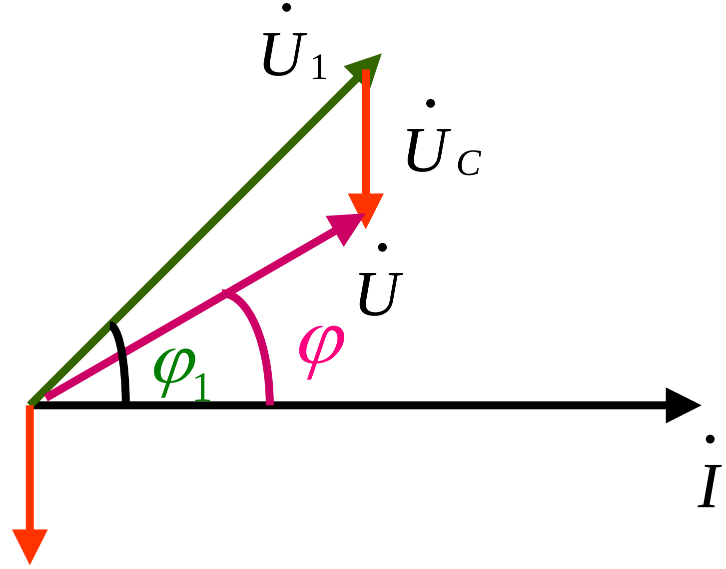
电力系统的负载主要是电动机，是感性负载.





$\varphi < \varphi_1$　　　　$\cos\varphi > \cos\varphi_1$





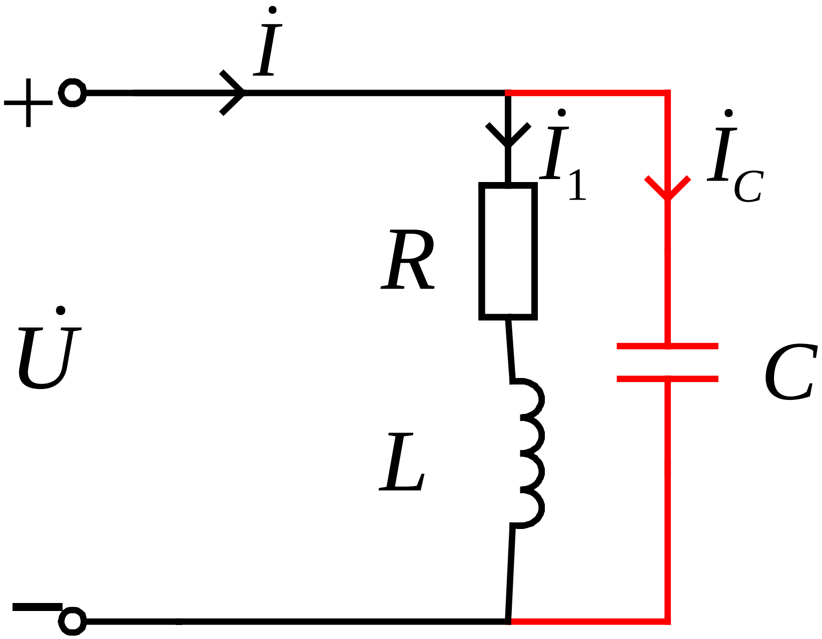
$$\varphi < \varphi_1$$

$$\cos\varphi > \cos\varphi_1$$

? 并电容、串电容？

串电容改变负载的运行状态

2)从无功功率补偿来看



$Q < Q_1$ $\varphi < \varphi_1$ $\cos\varphi > \cos\varphi_1$

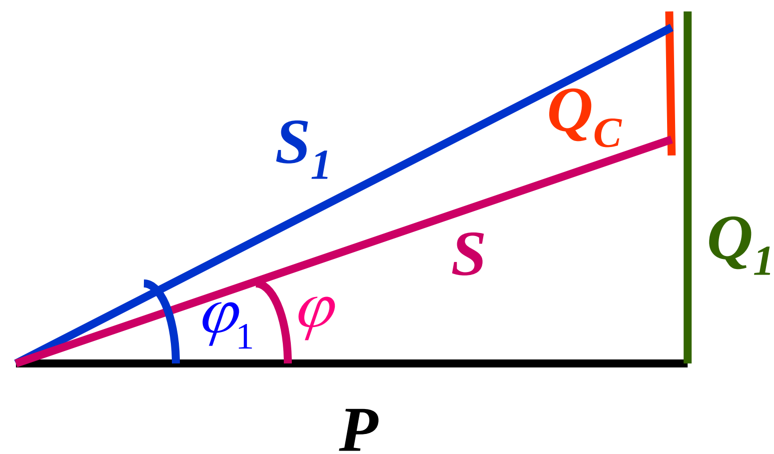
有功功率不变

$$Q_1 = UI_1 \sin\varphi_1$$

$$Q_C = -UI_C$$

$$Q = Q_1 + Q_C$$

$$= UI_1 \sin\varphi_1 - UI_C$$



功率因数提高多少？视技术经济两种指标综合比较

技术： 功率因数低 损耗大 系统稳定

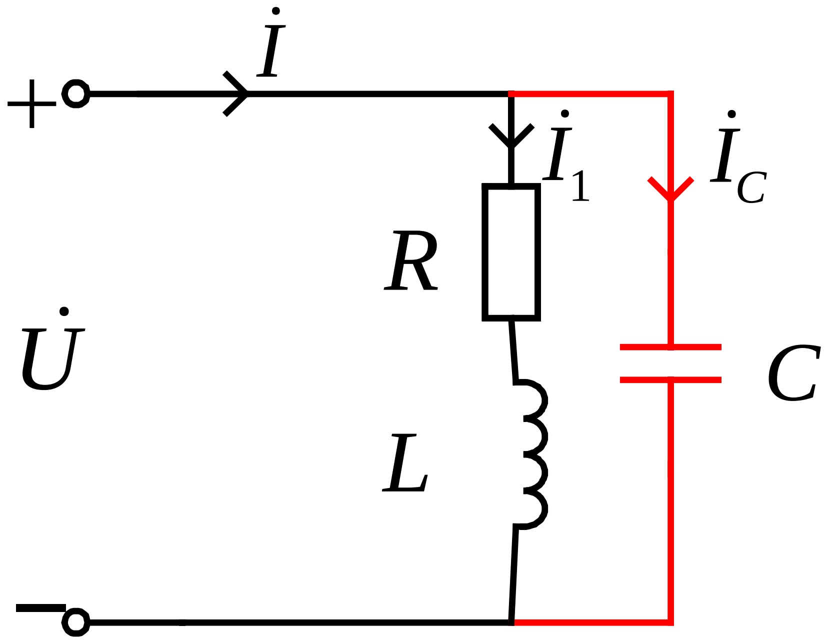
功率因数高 损耗小 系统不稳定

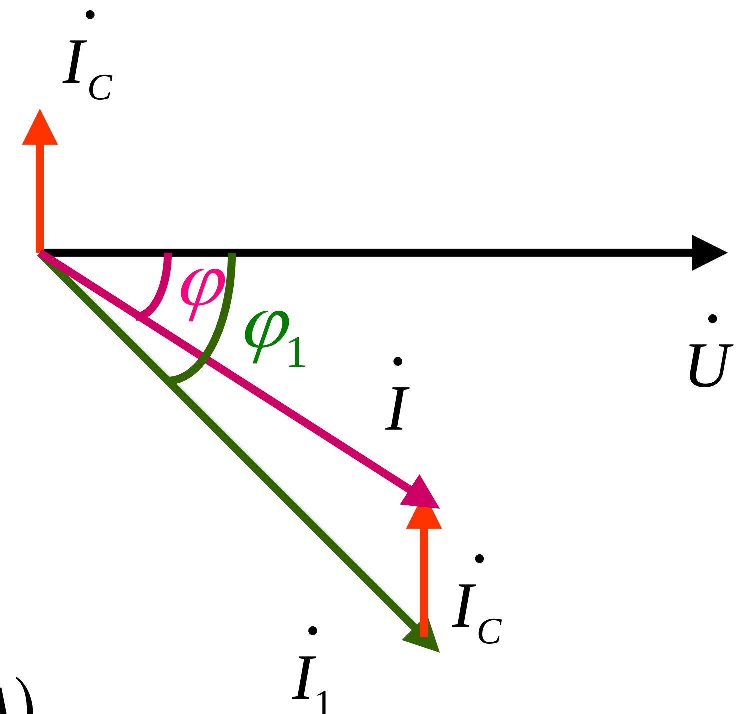
经济： 提高功率因数 并联C 要投资

投资、损耗 熟大熟小

例1. 有一负载接在电压10KV、ω=314rad/s的输电线上，有功功率P=1000KV，功率因数为0.8(感性).欲将功率因数提高至0.95，问应并联多少微法的电容？

- 解1：





$$I_1 = \frac{P}{U\cos\varphi_1} = \frac{1000\times 10^3}{10\times 10^3\times 0.8} = 125(A)$$

$$I = \frac{P}{U\cos\varphi} = \frac{1000\times 10^3}{10\times 10^3\times 0.95} = 105(A)$$

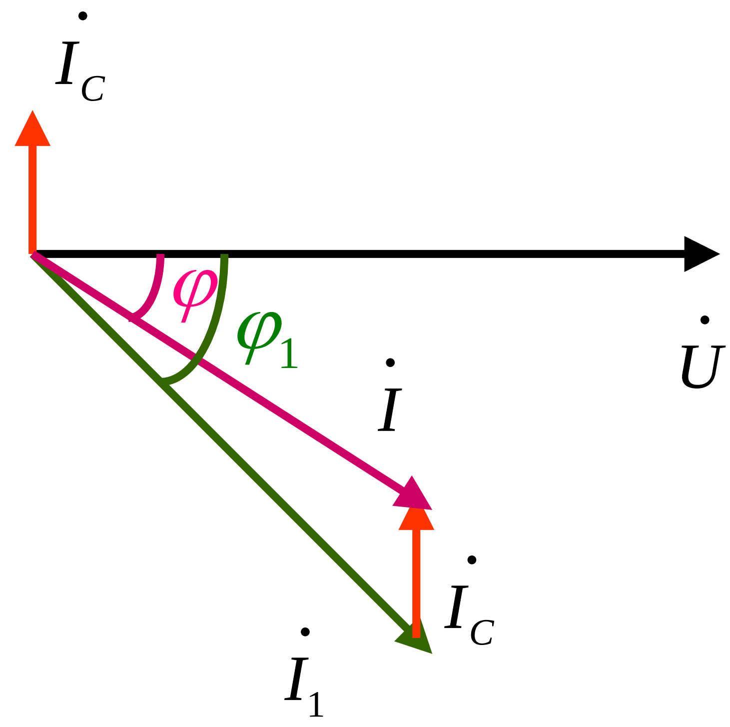
$$I_{1X} = I_1 \sin\varphi_1 = 125\times 0.6 = 75(A)$$

$$I_X = I \sin\varphi = 105\times 0.31 = 32.8(A)$$

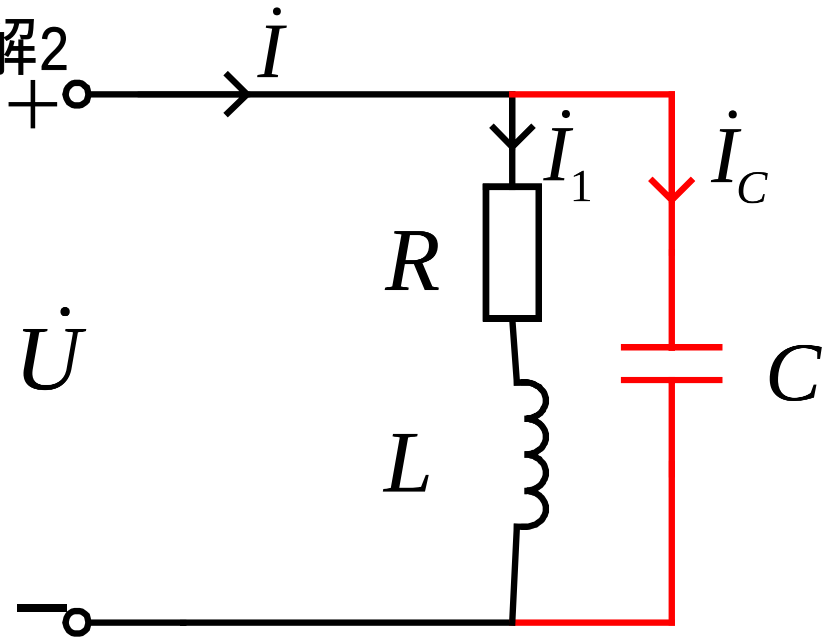
$$I_C = I_{1X} - I_X = 42.2(A)$$

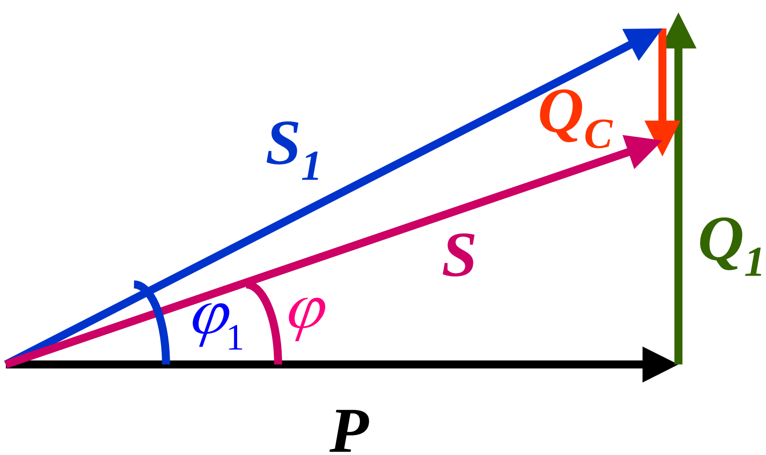
$$I_C = U_C B_C = U\cdot\omega C$$

$$C = \frac{I_C}{U\omega} = \frac{42.2}{10\times 10^3\times 314} 13.4(\mu F)$$



解2





$$Q_1 = Ptg\varphi_1 = 1000\times10^3\times0.75$$
$$= 750(KVAR)$$

$$Q = Ptg\varphi = 1000\times10^3\times0.328$$
$$= 328(KVAR)$$

$$Q_1 + Q_C = Q$$
$$Q_C = Q - Q_1 = -422(KVAR)$$

$$Q_C = -U_C I = -U^2\cdot\omega C$$

$$C = \frac{Q_C}{U^2\cdot\omega} = \frac{422\times10^3}{\left(10\times10^3\right)\times314}$$

$$=13.4\left(\mu F\right)$$