

## ANALYSIS OF DC/DC CONVERTERS WITH RESONANT FILTERS

**Rene J Thibodeaux**

U. S. Air Force/Wright Laboratory  
 WL/POOC  
 1950 Fifth Street  
 Wright-Patterson AFB, OH 45433-7251  
 Tel: (937)255-6016/Fax: (937)255-3211

### ABSTRACT

Conventional DC/DC converter topologies contain a single semiconductor switch, a diode, and various arrangements of inductors and capacitors to filter the resulting waveforms into low ripple DC. The output or input voltage is limited to the rating of the semiconductor switch and diode. For high voltage performance, transformers are added to form forward, flyback, or push-pull topologies. For high input voltages, semiconductor switches are arranged in an inverter bridge topology with a transformer to operate at higher voltages. Generally, conventional DC/DC converters are limited to low voltage applications. The addition of a transformer to provide sufficient voltage gain adds significant weight to the converter. Series, parallel resonant circuits with dual bridge circuits can function like a DC/DC converter with a transformer. Resonant converter topologies concepts have existed for the past 30 years. This paper will present a simplified generalized view of resonant circuits as  $\pi$ -filters and T-filters which give insight into improving circuit performance and developing new resonant topologies for bidirectional power flow.

### INTRODUCTION

Switch-mode power converters generally have the disadvantage of producing losses when the converter attempts to abruptly turn off current while simultaneously attempting to standoff a voltage across the same switch. In reality, the current has a gradual decay with a rising voltage producing power that is not dissipated usefully in the load. Various types of resonant circuits give the power circuit the ability to turn off at a zero point in the resulting sinusoidal waveform, helping to reduce power losses. This reduction in losses permits high frequency operation of the switches, which in turn reduces the size of capacitors, inductors and transformers, making resonant converters relatively small even at high power levels.

Resonant circuits were applied to silicon controlled rectifiers (SCR) to make these early converter circuits efficient and smaller in size. Early SCR's would turn off relatively slow (20 $\mu$ s typically) so that any voltage present would produce power losses. SCR's could not tolerate large  $dV/dt$  and would actually turn on again if enough delay was given before full voltage appeared across the switch. Finally, early SCR's could not tolerate large  $dI/dt$  at turnon since the current carriers a limited to a small area, which looks like a high resistance and therefore high power dissipation. The power dissipation was usually high enough to cause hot spots that would burn the SCR out if  $dI/dt$  were too large. Resonant circuits produced zero current and voltage points to allow "soft" turnon and turnoff of the switch.

Traditionally, resonant converters are categorized into three types: series, parallel, and series-parallel. The series resonant converter contains a resonant circuit in series with the load and the parallel resonant converter contains a resonant circuit in parallel with the load. The series-parallel converter contains a resonant circuit parallel to the load with a series resonant circuit connected between the power source and parallel combination. The three conventional categories of resonant circuits can be viewed as subsets of  $\pi$ -filter and T-filter circuits. Using a simple impedance model gives a reasonable qualitative analysis of the influence of the different circuit elements on voltage and current. The simplified models gives a clearer picture if adding extra components will change the performance of the converter. Studies have shown that resonant  $\pi$ -filters and T-filters can act as voltage boost circuits or "resonant transformers." A high power resonant voltage step up converter for a MHD power conditioner was designed by Maxwell Laboratories in 1976 converting 30 KW, 2.5 KV source to 27 MW, 20 KV and a 3 MW, 200 KV.

Filter models easily apply to any type of resonant converter and with some stretch of the imagination, the analysis can apply to quasi-resonant circuits. A simplified analysis is basically

representing each inductive or capacitive element (or combination) as a simple impedance  $Z$ . The effects of initial voltage and current conditions are lost, however, important effects such as power quality, the degree of isolation of source and load, and bi-directional compatibility can be assessed.

### VOLTAGE-FED $\pi$ -FILTER TOPOLOGY

Fig. 1 shows a conceptual converter topology using a generalized  $\pi$ -filter fed by a constant voltage source with a Theven equivalent source impedance.

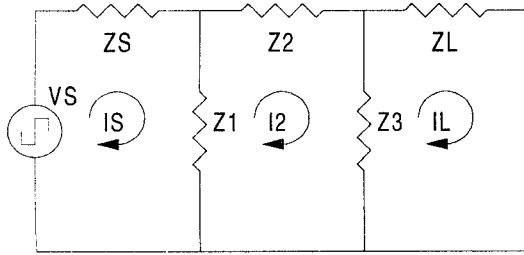


Fig. 1 - Voltage Fed  $\pi$ -filter converter

Switch-mode inverter generally have four basic states consisting of a positive on state, an off-state, a negative on state, and a final off-state. The dead-time state is usually required for to turn-off switches like thyristors and prevent shorting the power bus. In addition, the off-state helps to reduce  $dV/dt$  power losses.

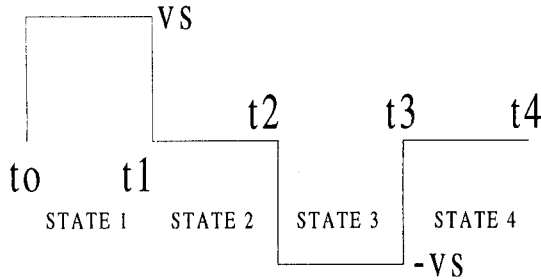


Fig. 2 - Switching States

The state equations are written in the frequency domain to produce simple algebraic relations. States 1 & 3 are basically equivalent circuits, where only polarity of the source changes. The simplified analysis ignores initial conditions which are require to determine the currents and voltages in States 2 & 4. Only a detailed analysis for specific circuit elements can fully account for the effects of initial voltage and current.

For the circuit shown in Fig. 1, the loop equations for State 1 ( $t_0 < t < t_1$ ),  $VS(t) = VS$ ,  $VS(s) = VS/s$ , (dropping the conventional frequency notation "s"),

$$\begin{aligned} \frac{VS}{s} &= ZS \cdot IS + Z1 \cdot (IS - I2) \\ 0 &= Z1 \cdot (I2 - IS) + Z2 \cdot I2 + Z3 \cdot (I2 - IL) \\ 0 &= Z3 \cdot (IL - I2) + ZL \cdot IL \end{aligned} \quad (1)$$

The load current  $IL$  becomes,

$$IL = \frac{VS}{s \cdot ZL} \cdot \frac{1}{\left[ \left( \frac{1}{ZL} + \frac{1}{Z3} \right) \cdot (Z2 + Z1) + 1 \right] \cdot \frac{ZS}{Z1} + \left( \frac{1}{Z3} + \frac{1}{ZL} \right) \cdot Z2 + 1} \quad (2)$$

We see that  $Z3$  and  $ZL$  work in parallel and that the source impedance  $ZS$  and the series impedance  $Z2$  have an influence on the performance of the circuit. A non-zero source impedance interacts with every impedance in the circuit. For minimal source impedance ( $ZS \cong 0$ ), the load current simplifies to,

$$IL = \frac{VS}{s \cdot ZL} \cdot \frac{1}{\left( \frac{1}{Z3} + \frac{1}{ZL} \right) \cdot Z2 + 1} \quad (3)$$

The load voltage ( $VL = ZL \cdot IL$ ) is given by,

$$VL = \frac{VS}{s} \cdot \frac{1}{\left[ \left( \frac{1}{ZL} + \frac{1}{Z3} \right) \cdot (Z2 + Z1) + 1 \right] \cdot \frac{ZS}{Z1} + \left( \frac{1}{Z3} + \frac{1}{ZL} \right) \cdot Z2 + 1} \quad (4)$$

and again for minimal source impedance,

$$VL = \frac{VS}{s} \cdot \frac{1}{\left( \frac{1}{Z3} + \frac{1}{ZL} \right) \cdot Z2 + 1} \quad (5)$$

The source current becomes,

$$IS = \frac{VS}{s \cdot Z1} \cdot \frac{\left( \frac{1}{ZL} + \frac{1}{Z3} \right) \cdot (Z2 + Z1) + 1}{\left[ \left( \frac{1}{ZL} + \frac{1}{Z3} \right) \cdot (Z2 + Z1) + 1 \right] \cdot \frac{ZS}{Z1} + \left( \frac{1}{Z3} + \frac{1}{ZL} \right) \cdot Z2 + 1} \quad (6)$$

The source current is a major component of power quality seen by the power system and all of the component effect power quality. The source current for minimal source impedance is,

$$IS = \frac{VS}{s \cdot Z1} \cdot \frac{\left( \frac{1}{ZL} + \frac{1}{Z3} \right) \cdot (Z2 + Z1) + 1}{\left( \frac{1}{Z3} + \frac{1}{ZL} \right) \cdot Z2 + 1} \quad (7)$$

The voltage gain from source to load is given by,

$$\frac{VL}{VS} = \frac{1}{\left[ \left( \frac{1}{ZL} + \frac{1}{Z3} \right) \cdot (Z2 + Z1) + 1 \right] \cdot \frac{ZS}{Z1} + \left( \frac{1}{Z3} + \frac{1}{ZL} \right) \cdot Z2 + 1} \quad (8)$$

We see the  $\pi$ -filter circuit is a natural buck circuit. To produce a voltage boost, the initial capacitor voltage must be higher than the source voltage or a transformer must be added.

The voltage gain for minimal source impedance,

$$\frac{V_L}{V_S} = \frac{1}{\left(\frac{1}{Z_3} + \frac{1}{Z_L}\right) \cdot Z_2 + 1} \quad (9)$$

When  $Z_S \ll Z_1$ , the influence of  $Z_S$  can be minimized. If  $Z_3$  is open ( $Z_3 = \infty$ ) then the circuit simplifies to a classic series converter which is really a narrow bandpass filter. For bi-directional power flow in a voltage-fed  $\pi$ -filter topology, given  $Z_2$  remains the same, the value of  $Z_3$  can be adjusted to produce a good impedance match with  $Z_L$ . For State 2 and State 4, the solutions are  $I_S(s) = 0$  and  $I_L(s) = 0$  when the initial conditions are ignored. For State 3 the solutions for current and voltage are the negatives of State 1.

### VOLTAGE-FED T-FILTER TOPOLOGY

Fig. 3 shows a conceptual converter using a generalized T-filter with a constant voltage source using a Thevenin equivalent impedance

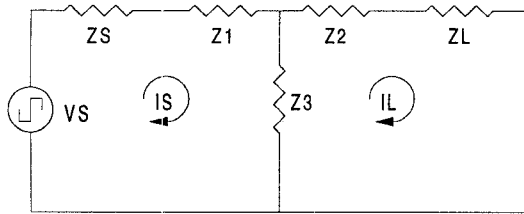


Fig. 3 - Voltage fed T-filter converter

The loop equations in State 1 where  $t_0 < t < t_1$ ,  $V_S(s) = V_S/s$ , are,

$$\begin{aligned} \frac{V_S}{s} &= Z_S \cdot I_S + Z_1 \cdot I_S + Z_3 \cdot (I_S - I_L) \\ 0 &= Z_3 \cdot (I_L - I_S) + Z_2 \cdot I_L + Z_L \cdot I_L \end{aligned} \quad (10)$$

The load current becomes,

$$I_L = \frac{V_S}{s \cdot Z_L \left(1 + \frac{Z_2 + Z_L}{Z_3}\right) \cdot \frac{Z_S + Z_1}{Z_L} + \frac{Z_2}{Z_L} + 1} \quad (11)$$

All circuit elements interact even if  $Z_S$  is minimized ( $Z_S = 0$ ),

$$I_L = \frac{V_S}{s \cdot Z_L \left(1 + \frac{Z_2 + Z_L}{Z_3}\right) \cdot \frac{Z_1}{Z_L} + \frac{Z_2}{Z_L} + 1} \quad (12)$$

The circuit will produce a more complex frequency response, which may be desired for some applications. Impedance matching would be difficult, making this topology less suited for bi-directional power flow.

The load voltage is again given by  $V_L = Z_L I_L$ ,

$$V_L = \frac{V_S}{s} \cdot \frac{1}{\left(1 + \frac{Z_2 + Z_L}{Z_3}\right) \cdot \frac{Z_1 + Z_S}{Z_L} + \frac{Z_2}{Z_L} + 1} \quad (13)$$

For the case of minimal source impedance,

$$V_L = \frac{V_S}{s} \cdot \frac{1}{\left(1 + \frac{Z_2 + Z_L}{Z_3}\right) \cdot \frac{Z_1}{Z_L} + \frac{Z_2}{Z_L} + 1} \quad (14)$$

The source current has a complex frequency response,

$$I_S = \frac{V_S}{s \cdot Z_L} \cdot \frac{1 + \frac{Z_L + Z_2}{Z_3}}{\left(1 + \frac{Z_2 + Z_L}{Z_3}\right) \cdot \frac{Z_1 + Z_S}{Z_L} + \frac{Z_2}{Z_L} + 1} \quad (15)$$

and even for minimal source impedance,

$$I_S = \frac{V_S}{s \cdot Z_L} \cdot \frac{1 + \frac{Z_L + Z_2}{Z_3}}{\left(1 + \frac{Z_2 + Z_L}{Z_3}\right) \cdot \frac{Z_1}{Z_L} + \frac{Z_2}{Z_L} + 1} \quad (16)$$

indicating more complications for power quality compared to the voltage fed  $\pi$ -filter converter. Finally the source to load voltage gain is,

$$\frac{V_L}{V_S} = \frac{1}{\left(1 + \frac{Z_2 + Z_L}{Z_3}\right) \cdot \frac{Z_1 + Z_S}{Z_L} + \frac{Z_2}{Z_L} + 1} \quad (17)$$

and for minimal source impedance,

$$\frac{V_L}{V_S} = \frac{1}{\left(1 + \frac{Z_2 + Z_L}{Z_3}\right) \cdot \frac{Z_1}{Z_L} + \frac{Z_2}{Z_L} + 1} \quad (18)$$

which implies a natural voltage step-down.

### CURRENT-FED PI-FILTER TOPOLOGY

Fig. 4 shows a conceptual DC/DC converter using a generalized resonant  $\pi$ -filter and a constant current source represented with a Norton equivalent impedance.

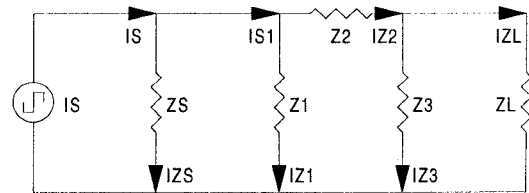


Fig 4 - Current-Fed  $\pi$ -filter converter

The nodal equations for the current fed  $\pi$ -filter converter shown in Fig 4 easily determine since there are only two node voltages, VS and VL. For State 1 ( $t_0 < t < t_1$ ),  $IS(s) = IS/s$ ,

$$\begin{aligned} \frac{IS}{s} \frac{VS}{Z_1} + \frac{VS - VL}{Z_2} + \frac{VS}{Z_3} \\ 0 = \frac{VL - VS}{Z_2} + \frac{VL}{Z_3} + \frac{VL}{Z_L} \end{aligned} \quad (19)$$

The solution for load voltage is given by,

$$VL = \frac{Z_L \cdot IS}{s} \frac{1}{\left[ \left( \frac{1}{Z_S} + \frac{1}{Z_1} \right) \cdot (Z_2 + Z_3) + 1 \right] \cdot \frac{Z_L}{Z_3} + \left( \frac{1}{Z_1} + \frac{1}{Z_S} \right) \cdot Z_2 + 1} \quad (20)$$

If the source impedance is maximized ( $Z_S \equiv \infty$ ), the load voltage becomes,

$$VL = \frac{Z_L \cdot IS}{s} \frac{1}{\left( \frac{Z_2 + Z_3}{Z_1} + 1 \right) \cdot \frac{Z_L}{Z_3} + \frac{Z_2}{Z_1} + 1} \quad (21)$$

The load current ( $IL = VL/Z_L$ ) is then,

$$IL = \frac{IS}{s} \frac{1}{\left[ \left( \frac{1}{Z_S} + \frac{1}{Z_1} \right) \cdot (Z_2 + Z_3) + 1 \right] \cdot \frac{Z_L}{Z_3} + \left( \frac{1}{Z_1} + \frac{1}{Z_S} \right) \cdot Z_2 + 1} \quad (22)$$

Even for a large source impedance, the load current is

$$IL = \frac{IS}{s} \frac{1}{\left( \frac{Z_2 + Z_3}{Z_1} + 1 \right) \cdot \frac{Z_L}{Z_3} + \frac{Z_2}{Z_1} + 1} \quad (23)$$

The load current has a complicated frequency response making impedance matching difficult. The source voltage becomes,

$$VS = \frac{Z_L \cdot IS}{s} \frac{\left( 1 + \frac{Z_2}{Z_3} \right) + \frac{Z_2}{Z_L}}{\left[ \left( \frac{1}{Z_1} + \frac{1}{Z_S} \right) \cdot (Z_2 + Z_3) + 1 \right] \cdot \frac{Z_L}{Z_3} + \left( \frac{1}{Z_1} + \frac{1}{Z_S} \right) \cdot Z_2 + 1} \quad (24)$$

so that power quality is more difficult to manage even for large source impedance,

$$VS = \frac{Z_L \cdot IS}{s} \frac{1 + \frac{Z_2}{Z_3} + \frac{Z_2}{Z_L}}{\left( \frac{Z_2 + Z_3}{Z_1} + 1 \right) \cdot \frac{Z_L}{Z_3} + \frac{Z_2}{Z_1} + 1} \quad (25)$$

The source to load current gain is,

$$\frac{IL}{IS} = \frac{1}{\left[ \left( \frac{1}{Z_S} + \frac{1}{Z_1} \right) \cdot (Z_2 + Z_3) + 1 \right] \cdot \frac{Z_L}{Z_3} + \left( \frac{1}{Z_1} + \frac{1}{Z_S} \right) \cdot Z_2 + 1} \quad (26)$$

for the large source impedance, the current gain becomes,

$$\frac{IL}{IS} = \frac{1}{\left( \frac{Z_2 + Z_3}{Z_1} + 1 \right) \cdot \frac{Z_L}{Z_3} + \frac{Z_2}{Z_1} + 1} \quad (27)$$

The current gain indicates the circuit has a natural tendency to reduce current.

## CURRENT-FED T-FILTER TOPOLOGY

Finally, the conceptual converter using a T-filter and a constant current source represented with a Norton equivalent impedance is shown in Fig. 5.

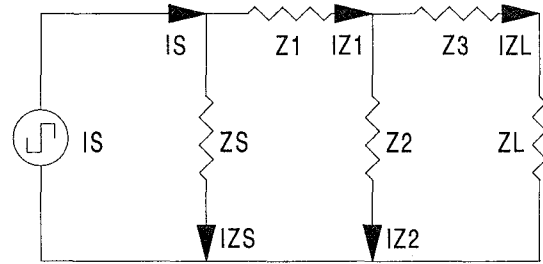


Fig. 5 - Current-Fed T-filter converter

The nodal equations for the current-fed T-filter converter for State 1 where  $t_0 < t < t_1$ ,  $IS(t) = IS$ ,  $IS(s) = IS/s$ ,

$$\begin{aligned} \frac{IS}{s} \frac{VS}{Z_S} + \frac{VS - V_2}{Z_1} \\ 0 = \frac{V_2 - VS}{Z_1} + \frac{V_2}{Z_2} + \frac{V_2 - VL}{Z_3} \\ 0 = \frac{VL - V_2}{Z_3} + \frac{VL}{Z_L} \end{aligned} \quad (28)$$

The load voltage becomes,

$$VL = \frac{Z_L \cdot IS}{s} \frac{1}{\left( \frac{Z_1 + Z_2}{Z_S} + 1 \right) \cdot \frac{Z_L + Z_3}{Z_2} + \frac{Z_1}{Z_S} + 1} \quad (29)$$

which indicates that  $Z_3$  and  $Z_L$  naturally work in series but that  $Z_1$  and  $Z_2$  work in series. If  $Z_S \gg Z_1 + Z_2$  (large source impedance), the load voltage reduces to,

$$VL = \frac{Z_L \cdot IS}{s} \frac{1}{\frac{Z_L + Z_3}{Z_2} + 1} \quad (30)$$

If  $Z_3 = 0$ , the current-fed T-filter topology simplifies to a classical parallel converter. In the bi-directional case where  $Z_2$  remains the same,  $Z_3$  can be chosen to match the converter impedance to the load impedance easily.

The load current is given by,

$$IL = \frac{IS}{s} \frac{1}{\left( \frac{Z_1 + Z_2}{Z_S} + 1 \right) \cdot \frac{Z_L + Z_3}{Z_2} + \frac{Z_1}{Z_S} + 1} \quad (31)$$

and for large source impedance, the load current is,

$$I_L = \frac{I_S}{s} \frac{1}{\frac{Z_L + Z_3}{Z_2} + 1} \quad (32)$$

The source voltage is,

$$V_S = \frac{Z_L I_S}{s} \frac{\left(\frac{Z_1}{Z_2} + 1\right) \frac{Z_L + Z_3}{Z_L} + \frac{Z_1}{Z_L}}{\left(\frac{Z_1 + Z_2}{Z_S} + 1\right) \frac{Z_L + Z_3}{Z_2} + \frac{Z_1}{Z_S} + 1} \quad (33)$$

For large source impedance, the source voltage is,

$$V_S = \frac{Z_L I_S}{s} \frac{\left(\frac{Z_1}{Z_2} + 1\right) \frac{Z_L + Z_3}{Z_L} + \frac{Z_1}{Z_L}}{\frac{Z_L + Z_3}{Z_2} + 1} \quad (34)$$

The source voltage quality is highly influenced by the converter.

The source to load current gain is given by,

$$\frac{I_L}{I_S} = \frac{1}{\left(\frac{Z_1 + Z_2}{Z_S} + 1\right) \frac{Z_L + Z_3}{Z_2} + \frac{Z_1}{Z_S} + 1} \quad (35)$$

The current gain equation implies that current is reduced at the output of the T-filter circuit so that large currents must flow through  $Z_2$ , a classic disadvantage of parallel converters. For large source impedance,

$$\frac{I_L}{I_S} = \frac{1}{\frac{Z_L + Z_3}{Z_2} + 1} \quad (36)$$

## CONCLUSION

Conventional series, parallel, and series-parallel circuits can be viewed as resonant  $\pi$ -filters and T-filters. Bi-directional resonant converters using full filter topologies provide the best impedance matching and isolation between the source and load. Filters naturally reduce the voltage or current seen by the load so that voltage or current boosting requires a transformer or adjusting the initial voltage of circuit capacitance through the turn-off time of the switches. The source and load can be reasonably isolated when the source side impedance of the voltage-fed  $\pi$ -filter is sufficiently greater than the Thevenin equivalent source impedance or the impedance of current-fed T-filter is sufficiently smaller than the Norton equivalent source impedance

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