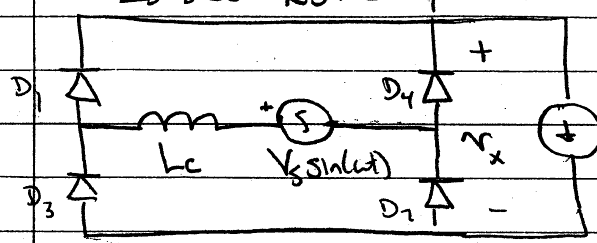
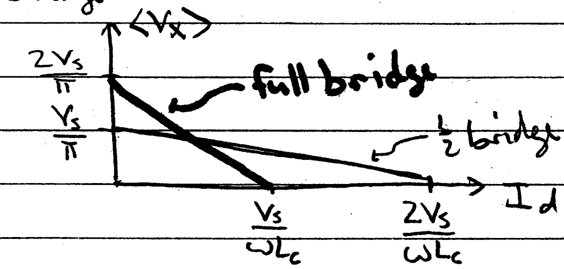
Power Electronics Notes D. Perreault

Full-bridge rectifier has similar problem (similar analysis)
⇒ see KSV Chap 4 for full bridge





★★ POWER FACTOR + MEASURES OF DISTORTION
→ Read KS+V Ch. 3)

look at the AC side

Definitions + Identities

2 functions x, y orthogonal over [a, b] if

$$\int_a^b x(t)\,y(t)\,dt = 0$$

Now $\int_0^{2\pi} \sin(m\omega t)\sin(n\omega t + \phi)\,d\omega t = 0$ if $n \neq m$

⇒ sinusoids of different frequencies are orthogonal

$$\int_0^{2\pi} \sin(\omega t)\cos(\omega t)\,d(\omega t) = 0$$

⇒ sin. + cosine are orthogonal

In general $\boxed{\frac{1}{2\pi}\int_0^{2\pi} \sin(\omega t)\sin(\omega t + \phi) = \frac{1}{2}\cos\phi}$

These definitions will be useful for calculating power, etc.

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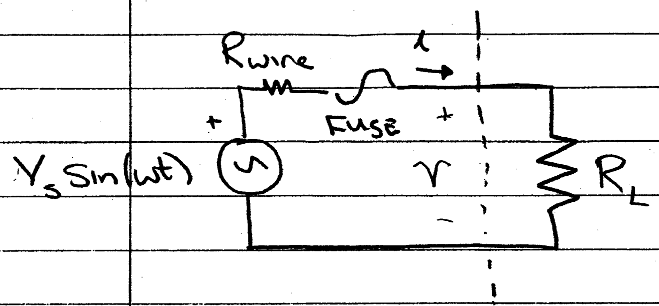
Background

Suppose we plug a resistor into the wall



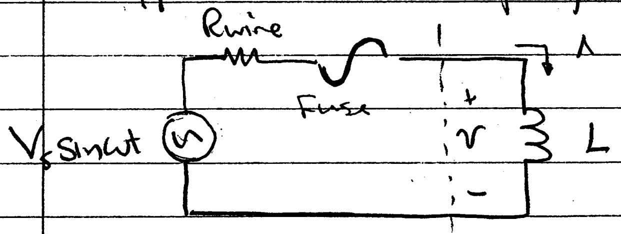
$$P = \langle v i \rangle = V_{rms} I_{rms}$$
$$= I_{rms}^2 R$$

The fuse is rated for a specified Rms current

→ above that, it will blow so that dissipation in R_{wire} does not start a fire

→ 115 $V_{AC,Rms}$, 15 A_{Rms} fuse we get ~ 1.7 KW max from wall (neglecting R_{wire})

Suppose instead we plug an inductor into the wall



neglecting R_{wire},

$$i = -\frac{V_s}{\omega L}\cos(\omega t)$$

$$\langle P \rangle = \frac{1}{2\pi}\int v(t)\, i(t)\, d(\omega t) = -\frac{V_s^2}{2\pi \omega L}\int \sin(\omega t)\cos(\omega t)\, d\omega t = 0!$$

(of course!)

mathematically, it is because v, i are orthogonal

While we draw no real power, we __still__ draw current

$$I_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\, d(\omega t)} = \frac{1}{\sqrt{2}}\frac{V_s}{\omega L}$$

@ 115V, 60 Hz, $L \le 20\text{mH} \rightarrow I_{Rms} \geq 15A$

So we still will blow the fuse (to protect the wall wiring) even though we do not draw any real power at the output!

(→ some power dissipated in R_{wire})

⇒ in this case we are not utilizing the source well!

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★ To provide a measure of the utilization of the source we define **Power Factor**

$$\boxed{P.F. \triangleq \frac{\langle P \rangle}{V_{rms} I_{rms}}}$$

Real Power P

apparent power S

For a resistor $\langle P \rangle = V_{rms} I_{rms}$ → P.F. = 1 best utilization

for an inductor $\langle P \rangle = 0$ → P.F. = 0 worst utilization

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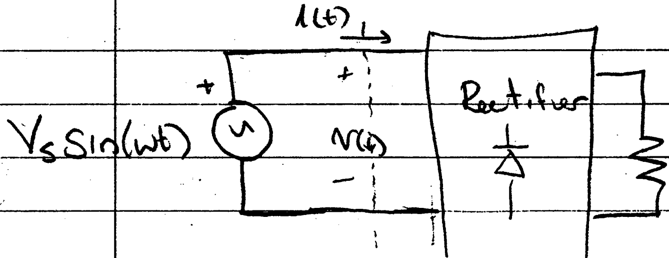
Consider a rectifier drawing some current waveform



Note: $I_{RMS} =$

$$\sqrt{\tfrac{1}{2} I_1^2 + \tfrac{1}{2} I_2^2 + \cdots \tfrac{1}{2} I_n^2 + \cdots}$$

Express $i(t)$ as a fourier series

sum of weighted shifted sinusoids

$$i(t) = \sum_{n=0}^{\infty} I_n \sin(n\omega t + \phi_n)$$

$$\langle P \rangle = \frac{1}{2\pi} \int_{2\pi} v(t)\, i(t)\, d(\omega t)$$

$$= \frac{1}{2\pi} \int_{2\pi} V_s \sin(\omega t) \sum_n I_n \sin(n\omega t + \phi_n)$$

$$= \sum_{n=0}^{\infty} \frac{1}{2\pi} \int_{2\pi} V_s I_n \sin(\omega t) \sin(n\omega t + \phi_n)$$

by orthogonality all terms except fundamental drop out!

$$\langle P \rangle = \frac{1}{2\pi} \int_{2\pi} V_s I_1 \sin(\omega t) \sin(\omega t + \phi_1)$$

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$$\langle P \rangle = \frac{1}{2\pi}\int_{2\pi} V_s I_1 \sin(\omega t)\sin(\omega t + \phi_1)$$

$$\boxed{\langle P \rangle = \frac{V_s I_1}{2}\cos\phi_1 = V_{s,rms} I_{1,rms}\cos(\phi_1)}$$

So the only current that contributes to real power is the fundamental component in phase with the voltage

$$P.F. = \frac{V_{rms} I_{1,rms}}{V_{rms} I_{rms}}\cos\phi_1 = \frac{I_{1,rms}}{I_{rms}}\cos(\phi_1)$$

★ we can break down into two factors

$$P.F. = K_d \cdot K_\theta$$

distortion factor (K_d) displacement factor (K_θ)

$$P.F. = \underbrace{\left(\frac{I_{1,rms}}{I_{RMS}}\right)}_{K_d} \cdot \underbrace{\cos(\phi_1)}_{K_\theta}$$

K_d distortion factor (≤ 1) tells us how much the utilization of the source is reduced because of harmonic currents that don't contribute to power

K_θ displacement factor (≤ 1) tells us how much utilization is reduced due to phase shift between the voltage + fundamental current.

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★ Consider another measure of distortion: Total Harmonic Distortion (THD)

$$\boxed{THD \triangleq \sqrt{\frac{\sum_{n\neq 1} i_n^2}{i_1^2}}}$$

This measures the rms of the harmonics, normalized to the rms of the fundamental (sq. root of the power ratio)

Distortion factor + THD are related!

$$THD = \sqrt{\frac{\sum_{n\neq 1} i_n^2}{i_1^2}} = \sqrt{\frac{i_{rms}^2 - i_{1,rms}^2}{i_{1,rms}^2}}$$

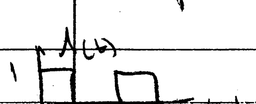
$$THD^2 = \frac{i_{rms}^2}{i_{1,rms}^2} - 1$$

$$\frac{i_{rms}^2}{i_{1,rms}^2} = 1 + THD^2$$

$$\boxed{K_d = \frac{i_{1,rms}}{i_{rms}} = \sqrt{\frac{1}{1+(THD)^2}}}$$

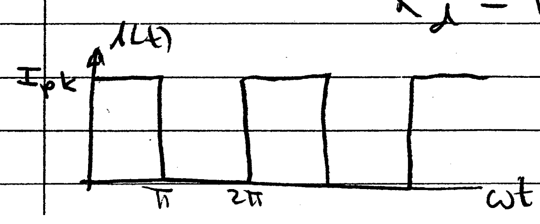
Example: $V = V_r \sin(\omega t)$

$i(t) = $ square wave $\begin{cases} i_n = \frac{4}{\pi n} & n \text{ odd} \\ i_0 = i_{ave} = \frac{1}{2} i_{pk} \end{cases}$



Can find $\rightarrow THD = 121\%$

$$K_d = P.F. = \frac{4/\pi\sqrt{2}}{\sqrt{2}} = \frac{2}{\pi} \simeq .63$$



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★ (Passive) Power Factor Compensation — KSV chap 3.4.1

Lets focus on the displacement factor component of power factor. For simplicity, lets assume a linear load (e.g. R-L) so that voltages + currents are sinusoidal.

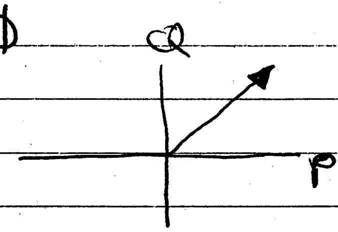
For sinusoidal v, i $\qquad P.F = \dfrac{\langle p \rangle}{V_{rms} I_{rms}} = \cos\phi$

ϕ is the <u>power factor angle</u> $\begin{cases} \text{leading } \phi < 0 & \text{capacitive} \\ \text{lagging } \phi > 0 & \text{inductive} \end{cases}$

Real Power $P = V_{rms} I_{rms} \cos\phi$

★ Define reactive power as

$$Q \triangleq V_{rms} I_{rms} \sin\phi$$



In vector form $\vec{S} = P + jQ$

In phasor form $\vec{V}, \vec{I} \rightarrow \vec{S} = \langle V I^* \rangle$

		Units
apparent power	$S = \lVert \vec{S} \rVert = V_{rms} I_{rms}$	VA
average power	$Re\{S\} = P = V_{rms} I_{rms} \cos\phi$	W
reactive power	$Im\{S\} = Q = V_{rms} I_{rms} \sin\phi$	VAR

We can use these results to help adjust the displacement factor of a system! (make $Q_{net} \rightarrow 0$)

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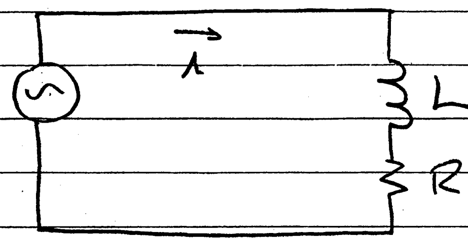
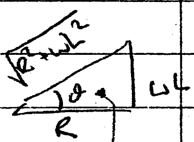
Suppose we have an R-L load (e.g. an induction machine)

$V_s \cos \omega t$



$$i(t) = \frac{V_s}{\sqrt{\omega^2 L^2 + R^2}} \sin\left(\omega t - \text{ATAN}\left(\frac{\omega L}{R}\right)\right)$$

voltage-current phase $\phi = \text{ATAN}\left(\frac{\omega L}{R}\right)$



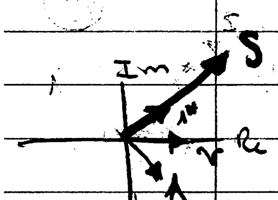
flip for neg.

$$\text{P.F.} = \cos\left(\text{ATAN}\left(\frac{\omega L}{R}\right)\right) = \frac{R}{\sqrt{R^2 + \omega L^2}} < 1$$

Voltage-Current phase because $S \triangleq V i^*$

we can add some additional reactive load to balance out + give net unity power factor.

$$S = V_{rms} I_{rms} = \frac{1}{2} \frac{V_s^2}{\sqrt{\omega^2 L^2 + R^2}}$$



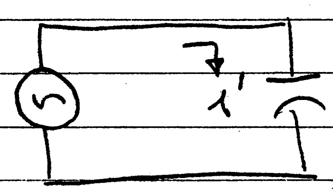
$$P = S \cos\phi = V_{rms} I_{rms} \cos\phi = \frac{V_s^2 R}{2(\omega^2 L^2 + R^2)}$$

$$jQ = jS \sin\phi = jV_{rms} I_{rms} \sin\phi = j\frac{\omega L V_s^2}{2(\omega^2 L^2 + R^2)}$$

so we have real + Reactive power

Suppose we add a Capacitor in parallel

$V_s \sin(\omega t)$



$$Z_c = \frac{1}{j\omega C} = \frac{1}{\omega C} e^{-j\pi/2}$$

$$\frac{1}{Z_c} = \omega C e^{j\pi/2}$$

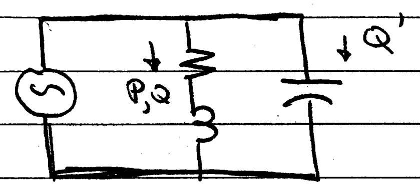
$V_{phase} - I_{phase} = -90°$

$$i' = V_s \omega C \sin(\omega t + \pi/2)$$

$$S' = V_{rms} I_{RMS} = \tfrac{1}{2} V_s^2 \omega C \qquad P' = 0$$

$$Q' = -j \tfrac{1}{2} V_s^2 \omega C$$

So by placing the Capacitor in parallel



$$S = P + jQ + jQ'$$

make jQ, jQ' cancel

$$Q + Q' = j\frac{\omega L V_s^2}{2(\omega^2 L^2 + R^2)} - j\frac{1}{2}V_s^2 \omega C = 0$$

$$\boxed{C = \frac{L}{\omega^2 L^2 + R}}$$

$\omega = 377$ RAD/sec (60 Hz)
$R = 1\Omega$
$L = 2.7$ mH

$\Rightarrow C = 1.32$ mF

If we know our load, we can add reactive elements to compensate so that no displacement factor reduction of Line utilization occurs.

Real, Reactive Power Defs. are useful to help us do this.

This does <u>not</u> help with distortion factor.