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2

Voltage and Current Sources

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2.1 Step, Impulse, Ramp, Sinusoidal, Exponential, and DC Signals

Richard C. Dorf and Zhen Wan

The important signals for circuits include the step, impulse, ramp, sinusoid, and dc signals. These signals are widely used and are described here in the time domain. All of these signals have a Laplace transform.

Step Function

The **unit-step** function $u(t)$ is defined mathematically by

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Here *unit step* means that the amplitude of $u(t)$ is equal to 1 for $t \geq 0$. Note that we are following the convention that $u(0) = 1$. From a strict mathematical standpoint, $u(t)$ is not defined at $t = 0$. Nevertheless, we usually take $u(0) = 1$. If A is an arbitrary nonzero number, $Au(t)$ is the step function with amplitude A for $t \geq 0$. The unit step function is plotted in Fig. 2.1.

The Impulse

The **unit impulse** $\delta(t)$, also called the *delta function* or the *Dirac distribution*, is defined by

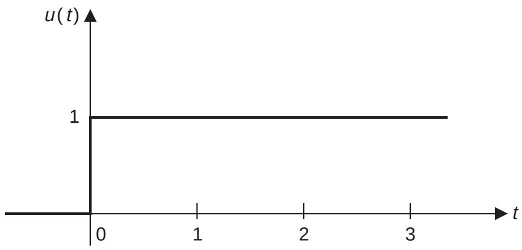


FIGURE 2.1 Unit-step function.

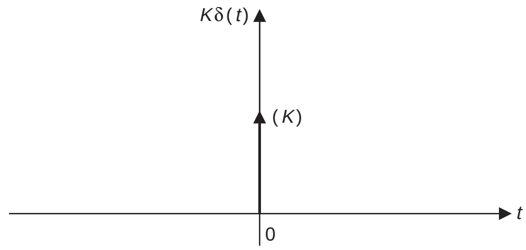


FIGURE 2.2 Graphical representation of the impulse $K\delta(t)$

$$\delta(t) = 0, \qquad t \neq 0$$

$$\int_{-\varepsilon}^{\varepsilon} \delta(\lambda)\, d\lambda = 1, \qquad \text{for any real number } \varepsilon > 0$$

The first condition states that $\delta(t)$ is zero for all nonzero values of t, while the second condition states that the area under the impulse is 1, so $\delta(t)$ has unit area. It is important to point out that the value $\delta(0)$ of $\delta(t)$ at $t = 0$ is not defined; in particular, $\delta(0)$ is not equal to infinity. For any real number K, $K\delta(t)$ is the impulse with area K. It is defined by

$$K\delta(t) = 0, \qquad t \neq 0$$

$$\int_{-\varepsilon}^{\varepsilon} K\delta(\lambda)\, d\lambda = K, \qquad \text{for any real number } \varepsilon > 0$$

The graphical representation of $K\delta(t)$ is shown in Fig. 2.2. The notation K in the figure refers to the area of the impulse $K\delta(t)$.

The unit-step function $u(t)$ is equal to the integral of the unit impulse $\delta(t)$; more precisely, we have

$$u(t) = \int_{-\infty}^{t} \delta(\lambda)\, d\lambda, \qquad \text{all } t \text{ except } t = 0$$

Conversely, the first derivative of $u(t)$, with respect to t, is equal to $\delta(t)$, except at $t = 0$, where the derivative of $u(t)$ is not defined.

Ramp Function

The *unit-ramp function* $r(t)$ is defined mathematically by

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Note that for $t \geq 0$, the slope of $r(t)$ is 1. Thus, $r(t)$ has *unit slope*, which is the reason $r(t)$ is called the unit-ramp function. If K is an arbitrary nonzero scalar (real number), the ramp function $Kr(t)$ has slope K for $t \geq 0$. The unit-ramp function is plotted in Fig. 2.3.

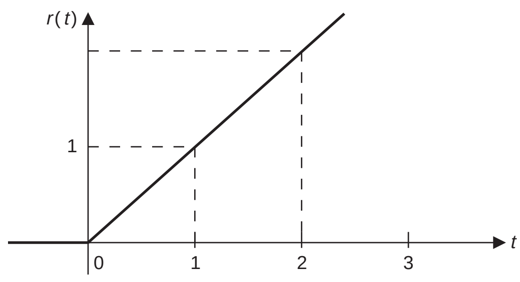


FIGURE 2.3 Unit-ramp function

The unit-ramp function $r(t)$ is equal to the integral of the unit-step function $u(t)$; that is,

$$r(t) = \int_{-\infty}^{t} u(\lambda)\, d\lambda$$

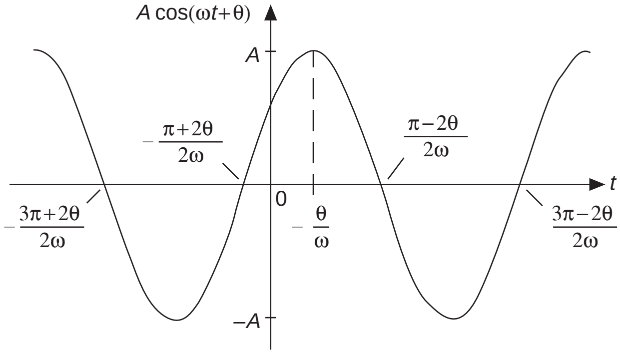


FIGURE 2.4 The sinusoid $A\cos(\omega t + \theta)$ with $-\pi/2 < \theta < 0$.

Conversely, the first derivative of $r(t)$ with respect to t is equal to $u(t)$, except at $t = 0$, where the derivative of $r(t)$ is not defined.

Sinusoidal Function

The sinusoid is a continuous-time signal: $A\cos(\omega t + \theta)$.

Here A is the amplitude, ω is the frequency in radians per second (rad/s), and θ is the phase in radians. The frequency f in cycles per second, or hertz (Hz), is $f = \omega/2\pi$. The sinusoid is a periodic signal with period $2\pi/\omega$. The sinusoid is plotted in Fig. 2.4.

Decaying Exponential

In general, an exponentially decaying quantity (Fig. 2.5) can be expressed as

$$a = A\, e^{-t/\tau}$$

where a = instantaneous value
A = amplitude or maximum value
e = base of natural logarithms = 2.718 …
τ = time constant in seconds
t = time in seconds

The current of a discharging capacitor can be approximated by a decaying exponential function of time.

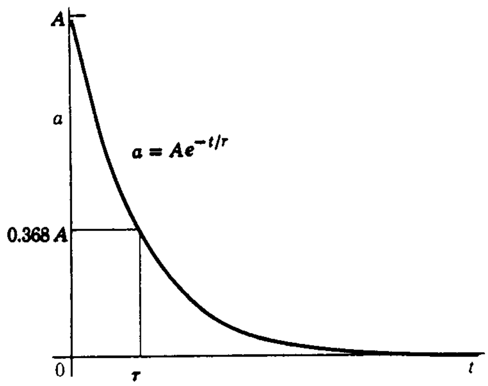


FIGURE 2.5 The decaying exponential.

Time Constant

Since the exponential factor only *approaches* zero as t increases without limit, such functions theoretically last forever. In the same sense, all radioactive disintegrations last forever. In the case of an exponentially decaying current, it is convenient to use the value of time that makes the exponent –1. When $t = \tau$ = the *time constant,* the value of the exponential factor is

$$e^{-t/\tau} = e^{-1} = \frac{1}{e} = \frac{1}{2.718} = 0.368$$

In other words, after a time equal to the time constant, the exponential factor is reduced to approximately 37% of its initial value.

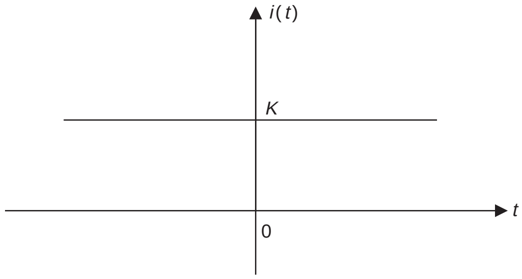


FIGURE 2.6 The dc signal with amplitude K.

DC Signal

The direct current signal (dc signal) can be defined mathematically by

$$i(t) = K \qquad -\infty < t < +\infty$$

Here, K is any nonzero number. The dc signal remains a constant value of K for any $-\infty < t < \infty$. The dc signal is plotted in Fig. 2.6.

Defining Terms

Ramp: A continually growing signal such that its value is zero for $t \leq 0$ and proportional to time t for $t > 0$.
Sinusoid: A periodic signal $x(t) = A\cos(\omega t + \theta)$ where $\omega = 2\pi f$ with frequency in hertz.
Unit impulse: A very short pulse such that its value is zero for $t \neq 0$ and the integral of the pulse is 1.
Unit step: Function of time that is zero for $t < t_0$ and unity for $t > t_0$. At $t = t_0$ the magnitude changes from zero to one. The unit step is dimensionless.

Related Topic

11.1 Introduction

References

R.C. Dorf, *Introduction to Electric Circuits,* 3rd ed., New York: Wiley, 1996.
R.E. Ziemer, *Signals and Systems,* 2nd ed., New York: Macmillan, 1989.

Further Information

IEEE Transactions on Circuits and Systems
IEEE Transactions on Education

2.2 Ideal and Practical Sources

Clayton R. Paul

A *mathematical model* of an electric circuit contains *ideal models* of physical circuit elements. Some of these ideal circuit elements (e.g., the resistor, capacitor, inductor, and transformer) were discussed previously. Here we will define and examine both *ideal* and *practical voltage and current sources.* The terminal characteristics of these models will be compared to those of actual sources.

All-Plastic Battery

Researchers at the U.S. Air Force's Rome Laboratory and Johns Hopkins University have developed an all-plastic battery using polymers instead of conventional electrode materials. All-plastic power cells could be a safer, more flexible substitute for use in electronic devices and other commercial applications. In addition, all-polymer cells reduce toxic waste disposal, negate environmental concerns, and can meet EPA and FAA requirements.

Applications include powering GPS receivers, communication transceivers, remote sensors, backup power systems, cellular phones, pagers, computing products and other portable equipment. Potential larger applications include remote monitoring stations, highway communication signs and electric vehicles.

The Johns Hopkins scientists are among the first to create a potentially practical battery in which both of the electrodes and the electrolyte are made of polymers. Fluoro-substituted thiophenes polymers have been developed with potential differences of up to 2.9 volts, and with potential specific energy densities of 30 to 75 watt hours/kg.

All plastic batteries can be recharged hundreds of times and operate under extreme hot and cold temperature conditions without serious performance degradation. The finished cell can be as thin as a business card and malleable, allowing battery manufacturers to cut a cell to a specific space or make the battery the actual case of the device to be powered. (Reprinted with permission from *NASA Tech Briefs,* 20(10), 26, 1996.)

Ideal Sources

The *ideal independent voltage source* shown in Fig. 2.7 constrains the terminal voltage across the element to a prescribed function of time, $v_S(t)$, as $v(t) = v_S(t)$. The polarity of the source is denoted by ± signs within the circle which denotes this as an ideal *independent* source. Controlled or *dependent* ideal voltage sources will be discussed in Section 2.3. The current through the element will be determined by the circuit that is attached to the terminals of this source.

The *ideal independent current source* in Fig. 2.8 constrains the terminal current through the element to a prescribed function of time, $i_S(t)$, as $i(t) = i_S(t)$. The polarity of the source is denoted by an arrow within the

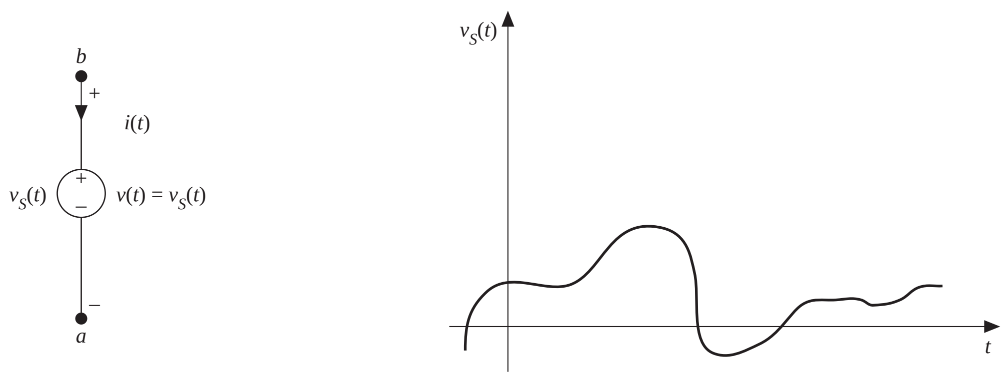


FIGURE 2.7 Ideal independent voltage source.

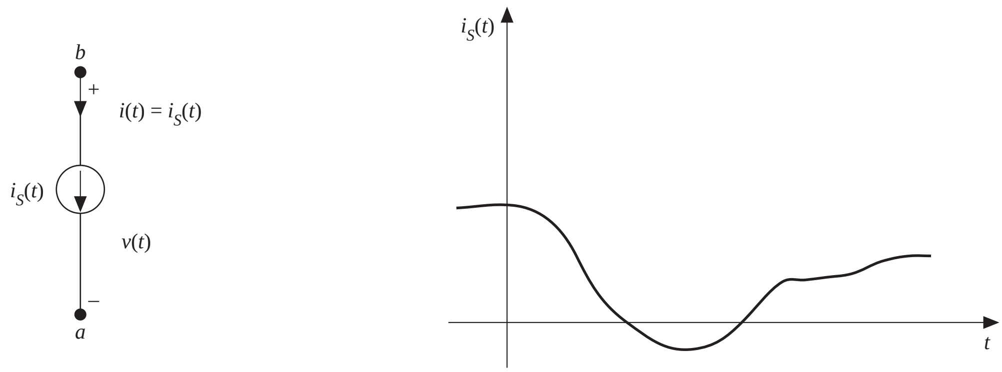


FIGURE 2.8 Ideal independent current source.

circle which also denotes this as an ideal *independent* source. The voltage across the element will be determined by the circuit that is attached to the terminals of this source.

Numerous functional forms are useful in describing the source variation with time. These were discussed in Section 2.1—the step, impulse, ramp, sinusoidal, and dc functions. For example, an ideal independent dc voltage source is described by $v_S(t) = V_S$, where V_S is a constant. An ideal independent sinusoidal current source is described by $i_S(t) = I_S \sin(\omega t + \phi)$ or $i_S(t) = I_S \cos(\omega t + \phi)$, where I_S is a constant, $\omega = 2\pi f$ with f the *frequency* in hertz and ϕ is a phase angle. Ideal sources may be used to model actual sources such as temperature transducers, phonograph cartridges, and electric power generators. Thus usually the time form of the output cannot generally be described with a simple, basic function such as dc, sinusoidal, ramp, step, or impulse waveforms. We often, however, represent the more complicated waveforms as a linear combination of more basic functions.

Practical Sources

The preceding ideal independent sources constrain the terminal voltage or current to a *known* function of time *independent of the circuit that may be placed across its terminals.* Practical sources, such as batteries, have their terminal voltage (current) dependent upon the terminal current (voltage) caused by the circuit attached to the source terminals. A simple example of this is an automobile storage battery. The battery's terminal voltage is approximately 12 V when no load is connected across its terminals. When the battery is applied across the terminals of the starter by activating the ignition switch, a large current is drawn from its terminals. During starting, its terminal voltage drops as illustrated in Fig. 2.9(a). How shall we construct a *circuit model* using the ideal elements discussed thus far to model this nonideal behavior? A model is shown in Fig. 2.9(b) and consists of the *series* connection of an ideal resistor, R_S, and an ideal independent voltage source, $V_S = 12$ V. To determine the terminal voltage–current relation, we sum Kirchhoff's voltage law around the loop to give

$$v = V_S - R_S i \tag{2.1}$$

This equation is plotted in Fig. 2.9(b) and approximates that of the actual battery. The equation gives a straight line with slope $-R_S$ that intersects the v axis ($i = 0$) at $v = V_S$. The resistance R_S is said to be the *internal resistance* of this nonideal source model. It is a fictitious resistance but the model nevertheless gives an equivalent *terminal behavior.*

Although we have derived an approximate model of an actual source, another equivalent form may be obtained. This alternative form is shown in Fig. 2.9(c) and consists of the *parallel* combination of an ideal independent current source, $I_S = V_S/R_S$, and the same resistance, R_S, used in the previous model. Although it may seem strange to model an automobile battery using a current source, the model is completely equivalent to the series voltage source–resistor model of Fig. 2.9(b) *at the output terminals a–b.* This is shown by writing Kirchhoff's current law at the upper node to give

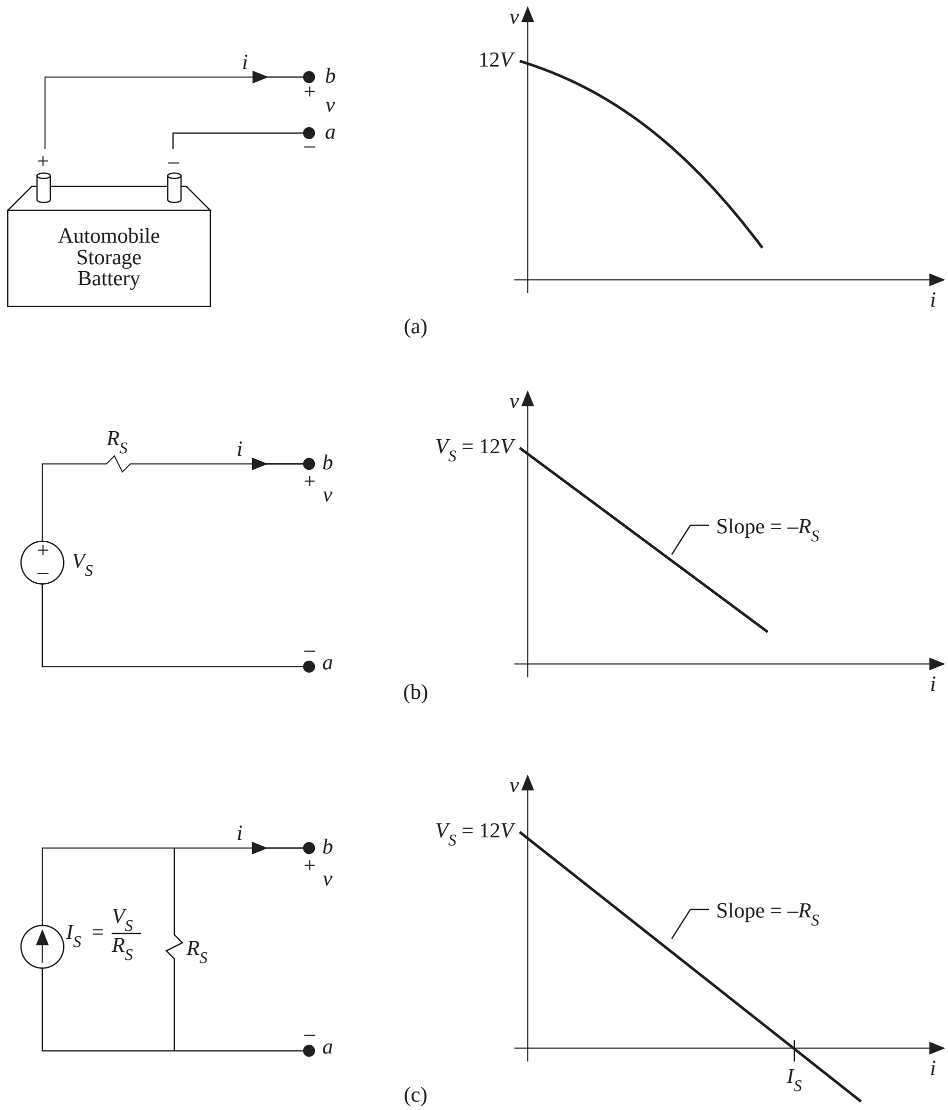


FIGURE 2.9 Practical sources. (a) Terminal v-i characteristic; (b) approximation by a voltage source; (c) approximation by a current source.

$$i = I_S - \frac{1}{R_S} v \tag{2.2}$$

Rewriting this equation gives

$$v = R_S I_S - R_S i \tag{2.3}$$

Comparing Eq. (2.3) to Eq. (2.1) shows that

$$V_S = R_S I_S \tag{2.4}$$

Therefore, we can convert from one form (voltage source in series with a resistor) to another form (current source in parallel with a resistor) very simply.

An ideal voltage source is represented by the model of Fig. 2.9(b) with $R_S = 0$. An actual battery therefore provides a close approximation of an ideal voltage source since the source resistance R_S is usually quite small. An ideal current source is represented by the model of Fig. 2.9(c) with $R_S = \infty$. This is very closely represented by the bipolar junction transistor (BJT).

Related Topic

1.1 Resistors

Defining Term

Ideal source: An ideal model of an actual source that assumes that the parameters of the source, such as its magnitude, are independent of other circuit variables.

Reference

C.R. Paul, *Analysis of Linear Circuits,* New York: McGraw-Hill, 1989.

2.3 Controlled Sources

J. R. Cogdell

When the analysis of electronic (nonreciprocal) circuits became important in circuit theory, controlled sources were added to the family of circuit elements. Table 2.1 shows the four types of controlled sources. In this section, we will address the questions: What are controlled sources? Why are controlled sources important? How do controlled sources affect methods of circuit analysis?

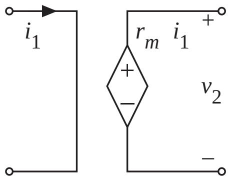
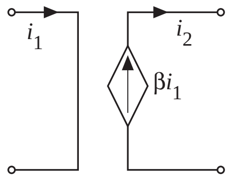
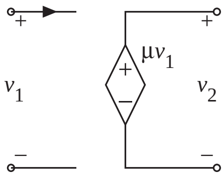
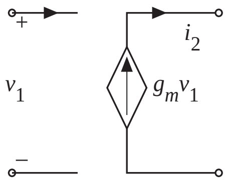
What Are Controlled Sources?

By *source* we mean a voltage or current source in the usual sense. By *controlled* we mean that the strength of such a source is controlled by some circuit variable(s) elsewhere in the circuit. Figure 2.10 illustrates a simple circuit containing an (independent) current source, i_s, two resistors, and a controlled voltage source, whose magnitude is controlled by the current i_1. Thus, i_1 determines two voltages in the circuit, the voltage across R_1 via Ohm's law and the controlled voltage source via some unspecified effect.

A controlled source may be controlled by more than one circuit variable, but we will discuss those having a single controlling variable since multiple controlling variables require no new ideas. Similarly, we will deal only with resistive elements, since inductors and capacitors introduce no new concepts. The controlled voltage or current source may depend on the controlling variable in a linear or nonlinear manner. When the relationship is nonlinear, however, the equations are frequently linearized to examine the effects of small variations about some dc values. When we linearize, we will use the customary notation of small letters to represent general and time-variable voltages and currents and large letters to represent constants such as the dc value or the peak value of a sinusoid. On subscripts, large letters represent the total voltage or current and small letters represent the **small-signal** component. Thus, the equation $i_B = I_B + I_b \cos \omega t$ means that the total base current is the sum of a constant and a small-signal component, which is sinusoidal with an amplitude of I_b.

To introduce the context and use of controlled sources we will consider a circuit model for the bipolar junction transistor (BJT). In Fig. 2.11 we show the standard symbol for an *npn* BJT with base (B), emitter (E), and collector (C) identified, and voltage and current variables defined. We have shown the common emitter configuration, with the emitter terminal shared to make input and output terminals. The base current, i_B, ideally depends upon the base-emitter voltage, v_{BE}, by the relationship

TABLE 2.1 Names, Circuit Symbols, and Definitions for the Four Possible Types of Controlled Sources

Name	Circuit Symbol	Definition and Units
Current-controlled voltage source (CCVS)		$v_2 = r_m i_1$ r_m = transresistance units, ohms
Current-controlled current source (CCCS)		$i_2 = \beta i_1$ β, current gain, dimensionless
Voltage-controlled voltage source (VCVS)		$v_2 = \mu v_1$ μ, voltage gain, dimensionless
Voltage-controlled current source (VCCS)		$i_2 = g_m v_1$ g_m, transconductance units, Siemans (mhos)

$$i_B = I_0\left\{\exp\left[\frac{v_{BE}}{V_T}\right] - 1\right\} \tag{2.5}$$

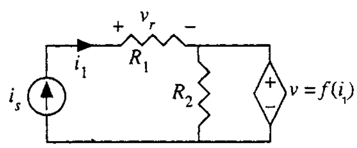


FIGURE 2.10 A simple circuit containing a controlled source.

where I_0 and V_T are constants. We note that the base current depends on the base-emitter voltage only, but in a nonlinear manner. We can represent this current by a voltage-controlled current source, but the more common representation would be that of a nonlinear conductance, $G_{BE}(v_{BE})$, where

$$G_{BE}(v_{BE}) = \frac{i_B}{v_{BE}}$$

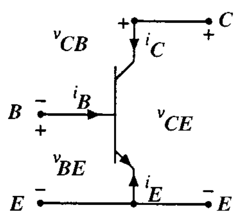


FIGURE 2.11 An *npn* BJT in the common emitter configuration.

Let us model the effects of small changes in the base current. If the changes are small, the nonlinear nature of the conductance can be ignored and the circuit model becomes a linear conductance (or resistor). Mathematically this conductance arises from a first-order expansion of the nonlinear function. Thus, if $v_{BE} = V_{BE} + v_{be}$, where v_{BE} is the total base-emitter voltage, V_{BE} is a (large) constant voltage and v_{be} is a (small) variation in the base-emitter voltage, then the first two terms in a Taylor series expansion are

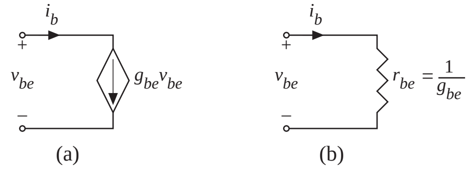


FIGURE 2.12 Equivalent circuits for the base circuit: (a) uses a controlled source and (b) uses a resistor.

$$i_B = I_0\left\{\exp\left[\frac{V_{BE} + v_{be}}{V_T}\right] - 1\right\} \cong I_0\left\{\exp\left[\frac{V_{BE}}{V_T}\right] - 1\right\} + \frac{I_0}{V_T}\exp\left[\frac{V_{BE}}{V_T}\right]v_{be} \tag{2.6}$$

We note that the base current is approximated by the sum of a constant term and a term that is first order in the small variation in base-emitter voltage, v_{be}. The multiplier of this small voltage is the linearized conductance, g_{be}. If we were interested only in small changes in currents and voltages, only this conductance would be required in the model. Thus, the input (base-emitter) circuit can be represented for the small-signal base variables, i_b and v_{be}, by either equivalent circuit in Fig. 2.12.

The voltage-controlled current source, $g_{be}v_{be}$, can be replaced by a simple resistor because the small-signal voltage and current associate with the same branch. The process of **linearization** is important to the modeling of the collector-emitter characteristic, to which we now turn.

The collector current, i_C, can be represented by one of the Eber and Moll equations

$$i_C = \beta I_0\left\{\exp\left[\frac{v_{BE}}{V_T}\right] - 1\right\} - I_0'\left\{\exp\left[\frac{v_{BC}}{V_T}\right] - 1\right\} \tag{2.7}$$

where β and I_0' are constants. If we restrict our model to the amplifying region of the transistor, the second term is negligible and we may express the collector current as

$$i_C = \beta I_0\left\{\exp\left[\frac{v_{BE}}{V_T}\right] - 1\right\} = \beta i_B \tag{2.8}$$

Thus, for the ideal transistor, the collector-emitter circuit may be modeled by a current-controlled current source, which may be combined with the results expressed in Eq. (2.5) to give the model shown in Fig. 2.13.

Using the technique of small-signal analysis, we may derive either of the small-signal equivalent circuits shown in Fig. 2.14.

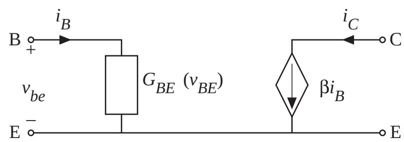


FIGURE 2.13 Equivalent circuit for BJT.

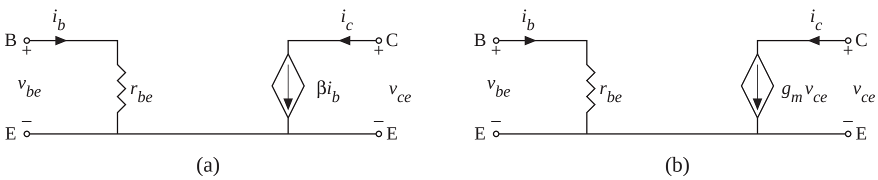


FIGURE 2.14 Two BJT small-signal equivalent circuits ($g_m = \beta/r_{be}$): (a) uses a CCCS and (b) uses a VCCS.

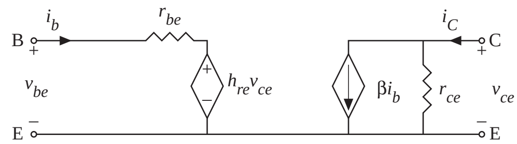


FIGURE 2.15 Full hybrid parameter model for small-signal BJT.

The small-signal characteristics of the *npn* transistor in its amplifying region is better represented by the equivalent circuit shown in Fig. 2.15. Note we have introduced a voltage-controlled voltage source to model the influence of the (output) collector-emitter voltage on the (input) base-emitter voltage, and we have placed a resistor, r_{ce}, in parallel with the collector current source to model the influence of the collector-emitter voltage on the collector current.

The four parameters in Fig. 2.15 (r_{be}, h_{re}, β, and r_{ce}) are the hybrid parameters describing the transistor properties, although our notation differs from that commonly used. The parameters in the small-signal equivalent circuit depend on the operating point of the device, which is set by the time-average voltages and currents (V_{BE}, I_C, etc.) applied to the device. All of the parameters are readily measured for a given transistor and operating point, and manufacturers commonly specify ranges for the various parameters for a type of transistor.

What Is the Significance of Controlled Sources?

Commonplace wisdom in engineering education and practice is that information and techniques that are presented visually are more useful than abstract, mathematical forms. Equivalent circuits are universally used in describing electrical engineering systems and devices because circuits portray interactions in a universal, pictorial language. This is true generally, and it is doubly necessary when circuit variables interact through the mysterious coupling modeled by controlled sources. This is the primary significance of controlled sources: that they represent unusual couplings of circuit variables in the universal, visual language of circuits.

A second significance is illustrated by our equivalent circuit of the *npn* bipolar transistor, namely, the characterization of a class of similar devices. For example, the parameter β in Eq. (2.8) gives important information about a single transistor, and similarly for the range of β for a type of transistor. In this connection, controlled sources lead to a vocabulary for discussing some property of a class of systems or devices, in this case the current gain of an *npn* BJT.

How Does the Presence of Controlled Sources Affect Circuit Analysis?

The presence of nonreciprocal elements, which are modeled by controlled sources, affects the analysis of the circuit. Simple circuits may be analyzed through the direct application of Kirchhoff's laws to branch circuit variables. Controlled sources enter this process similar to the constitutive relations defining *R*, *L*, and *C*, i.e., in defining relationships between branch circuit variables. Thus, controlled sources add no complexity to this basic technique.

The presence of controlled sources negates the advantages of the method that uses series and parallel combinations of resistors for voltage and current dividers. The problem is that the couplings between circuit variables that are expressed by controlled sources make all the familiar formulas unreliable.

When superposition is used, the controlled sources are left on in all cases as independent sources are turned on and off, thus reflecting the kinship of controlled sources to the circuit elements. In principle, little complexity is added; in practice, the repeated solutions required by superposition entail much additional work when controlled sources are involved.

The classical methods of nodal and loop (mesh) analysis incorporate controlled sources without great difficulty. For purposes of determining the number of independent variables required, that is, in establishing the topology of the circuit, the controlled sources are treated as ordinary voltage or current sources. The equations are then written according to the usual procedures. Before the equations are solved, however, the controlling variables must be expressed in terms of the unknowns of the problem. For example, let us say we are performing a nodal analysis on a circuit containing a current-controlled current source. For purposes of counting independent nodes, the controlled current source is treated as an open circuit. After equations are written for the unknown node voltages, the current source will introduce into at least one equation its controlling current, which is not one of the nodal variables. The additional step required by the controlled source is that of expressing the controlling current in terms of the nodal variables.

The parameters introduced into the circuit equations by the controlled sources end up on the left side of the equations with the resistors rather than on the right side with the independent sources. Furthermore, the symmetries that normally exist among the coefficients are disturbed by the presence of controlled sources.

The methods of Thévenin and Norton equivalent circuits continue to be very powerful with controlled sources in the circuits, but some complications arise. The controlled sources must be left on for calculation of the Thévenin (open-circuit) voltage or Norton (short-circuit) current and also for the calculation of the output impedance of the circuit. This usually eliminates the method of combining elements in series or parallel to determine the output impedance of the circuit, and one must either determine the output impedance from the ratio of the Thévenin voltage to the Norton current or else excite the circuit with an external source and calculate the response.

Defining Terms

Controlled source (dependent source): A voltage or current source whose intensity is controlled by a circuit voltage or current elsewhere in the circuit.

Linearization: Approximating nonlinear relationships by linear relationships derived from the first-order terms in a power series expansion of the nonlinear relationships. Normally the linearized equations are useful for a limited range of the voltage and current variables.

Small-signal: Small-signal variables are those first-order variables used in a linearized circuit. A small-signal equivalent circuit is a linearized circuit picturing the relationships between the small-signal voltages and currents in a linearized circuit.

Related Topics

2.2 Ideal and Practical Sources • 22.3 Electrical Equivalent Circuit Models and Device Simulators for Semiconductor Devices

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