Attia, John Okyere. "Transient Analysis."
Electronics and Circuit Analysis using MATLAB.
Ed. John Okyere Attia
Boca Raton: CRC Press LLC, 1999

CHAPTER FIVE

TRANSIENT ANALYSIS

5.1 RC NETWORK

Considering the RC Network shown in Figure 5.1, we can use KCL to write Equation (5.1).

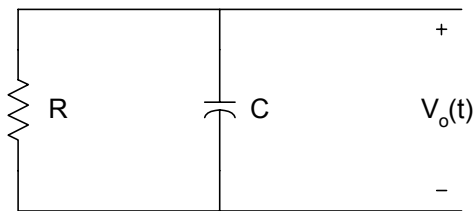**Figure 5.1** Source-free RC Network

$$C\frac{dv_o(t)}{dt} + \frac{v_o(t)}{R} = 0 \tag{5.1}$$

i.e.,

$$\frac{dv_o(t)}{dt} + \frac{v_o(t)}{CR} = 0$$

If V_m is the initial voltage across the capacitor, then the solution to Equation (5.1) is

$$v_0(t) = V_m e^{-\left(\frac{t}{CR}\right)} \tag{5.2}$$

where
$\qquad$ CR is the time constant

Equation (5.2) represents the voltage across a discharging capacitor. To obtain the voltage across a charging capacitor, let us consider Figure 5.2.
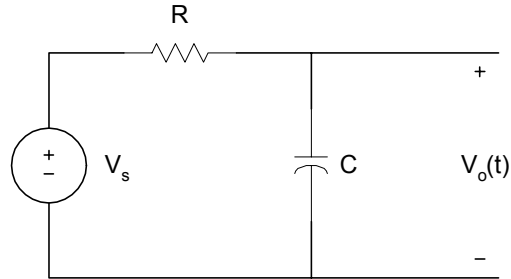
Figure 5.2 Charging of a Capacitor

Using KCL, we get

$$C\frac{dv_o(t)}{dt} + \frac{v_o(t) - V_s}{R} = 0 \tag{5.3}$$

If the capacitor is initially uncharged, that is $v_0(t) = 0$ at $t = 0$, the solution to Equation (5.3) is given as

$$v_0(t) = V_S\left(1 - e^{-\left(\frac{t}{CR}\right)}\right) \tag{5.4}$$

Examples 5.1 and 5.2 illustrate the use of MATLAB for solving problems related to RC Network.

Example 5.1

Assume that for Figure 5.2 $C = 10$ μF, use MATLAB to plot the voltage across the capacitor if R is equal to (a) 1.0 kΩ, (b) 10 kΩ and (c) 0.1 kΩ.

Solution

MATLAB Script

```
% Charging of an RC circuit
%
c = 10e-6;
r1 = 1e3;
```

```
tau1 = c*r1;
t = 0:0.002:0.05;
v1 = 10*(1-exp(-t/tau1));
r2 = 10e3;
tau2 = c*r2;
v2 = 10*(1-exp(-t/tau2));
r3 = .1e3;
tau3 = c*r3;
v3 = 10*(1-exp(-t/tau3));
plot(t,v1,'+',t,v2,'o', t,v3,'*')
axis([0 0.06 0 12])
title('Charging of a capacitor with three time constants')
xlabel('Time, s')
ylabel('Voltage across capacitor')
text(0.03, 5.0, '+ for R = 1 Kilohms')
text(0.03, 6.0, 'o for R = 10 Kilohms')
text(0.03, 7.0, '* for R = 0.1 Kilohms')
```
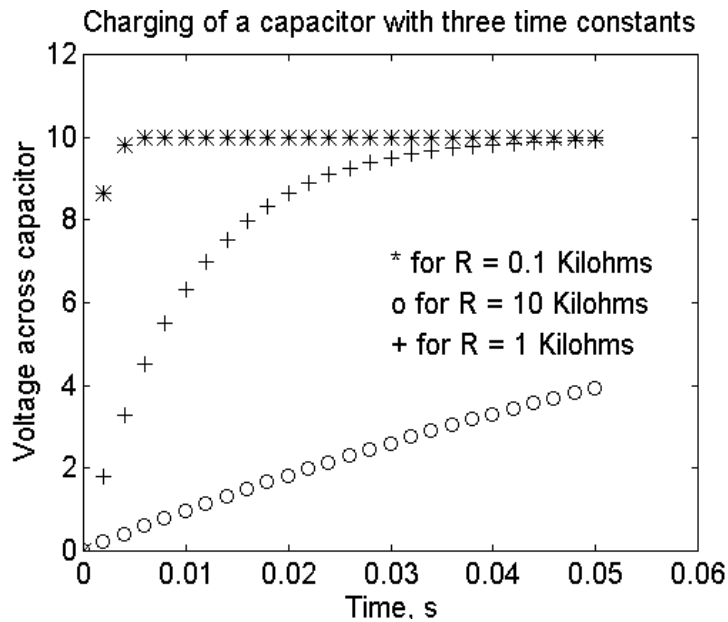
Figure 5.3 shows the charging curves.

Figure 5.3 Charging of Capacitor

From Figure 5.3, it can be seen that as the time constant is small, it takes a short time for the capacitor to charge up.

Example 5.2

For Figure 5.2, the input voltage is a rectangular pulse with an amplitude of 5V and a width of 0.5s. If $C = 10$ μF, plot the output voltage, $v_0(t)$, for resistance R equal to (a) 1000 Ω, and (b) 10,000 Ω. The plots should start from zero seconds and end at 1.5 seconds.

Solution

MATLAB Script

```
%  The problem will be solved using a function program rceval
function [v, t] = rceval(r, c)
% rceval is a function program for calculating
%       the output voltage given the values of
%       resistance and capacitance.
% usage [v, t] = rceval(r, c)
%       r is the resistance value(ohms)
%       c is the capacitance value(Farads)
%       v is the output voltage
%       t is the time corresponding to voltage v
tau  = r*c;
for i=1:50
    t(i) = i/100;
    v(i) = 5*(1-exp(-t(i)/tau));
end
vmax = v(50);

for i = 51:100
    t(i) = i/100;
    v(i) = vmax*exp(-t(i-50)/tau);
end
end

% The problem will be solved using function program
% rceval
% The output is obtained for the various resistances
c = 10.0e-6;
r1 = 2500;
```

```
[v1,t1] = rceval(r1,c);
r2 = 10000;
[v2,t2] = rceval(r2,c);

% plot the voltages
plot(t1,v1,'*w', t2,v2,'+w')
axis([0 1 0 6])
title('Response of an RC circuit to pulse input')
xlabel('Time, s')
ylabel('Voltage, V')
text(0.55,5.5,'* is for 2500 Ohms')
text(0.55,5.0, '+ is for 10000 Ohms')
```

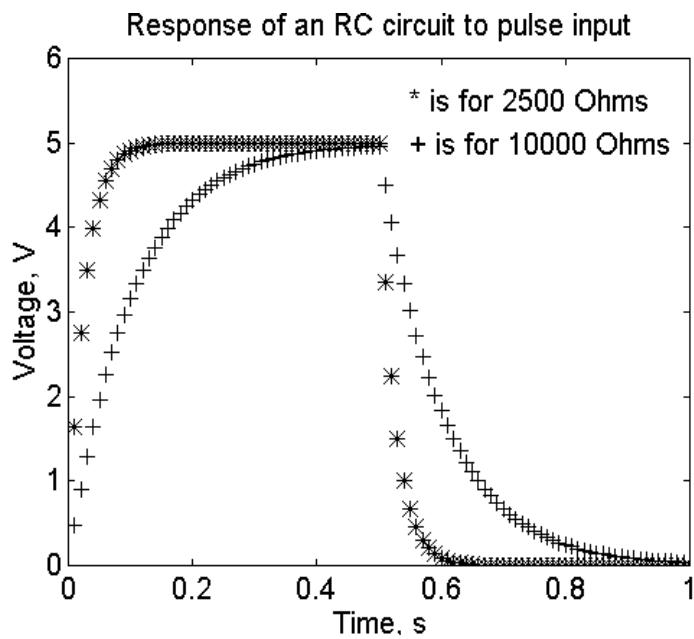Figure 5.4 shows the charging and discharging curves.

Figure 5.4 Charging and Discharging of a Capacitor with Different
Time Constants

5.2 RL NETWORK
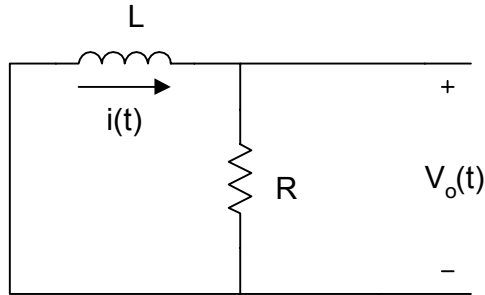
Consider the RL circuit shown in Figure 5.5.

Figure 5.5 Source-free RL Circuit

Using the KVL, we get

$$L\frac{di(t)}{dt} + Ri(t) = 0 \tag{5.5}$$

If the initial current flowing through the inductor is I_m, then the solution to Equation (5.5) is

$$i(t) = I_m e^{-\left(\frac{t}{\tau}\right)} \tag{5.6}$$

where

$$\tau = \frac{L}{R} \tag{5.7}$$

Equation (5.6) represents the current response of a source-free RL circuit with initial current I_m, and it represents the natural response of an RL circuit.

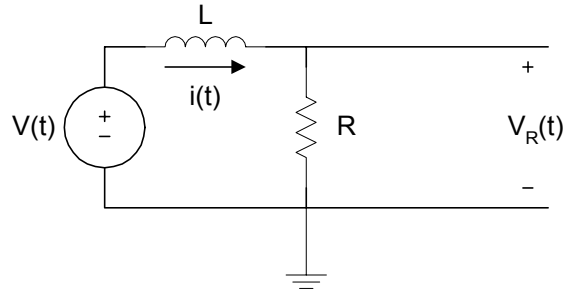Figure 5.6 is an RL circuit with source voltage $v(t) = V_S$.

Figure 5.6 RL Circuit with a Voltage Source

Using KVL, we get

$$L\frac{di(t)}{dt} + Ri(t) = V_S \tag{5.8}$$

If the initial current flowing through the series circuit is zero, the solution of Equation (5.8) is

$$i(t) = \frac{V_S}{R}\left(1 - e^{-\left(\frac{Rt}{L}\right)}\right) \tag{5.9}$$

The voltage across the resistor is

$$v_R(t) = Ri(t)$$
$$= V_S\left(1 - e^{-\left(\frac{Rt}{L}\right)}\right) \tag{5.10}$$

The voltage across the inductor is

$$v_L(t) = V_S - v_R(t)$$
$$= V_S e^{-\left(\frac{Rt}{L}\right)} \tag{5.11}$$

The following example illustrates the use of MATLAB for solving RL circuit problems.

Example 5.3

For the sequential circuit shown in Figure 5.7, the current flowing through the inductor is zero. At $t = 0$, the switch moved from position a to b, where it remained for 1 s. After the 1 s delay, the switch moved from position b to position c, where it remained indefinitely. Sketch the current flowing through the inductor versus time.
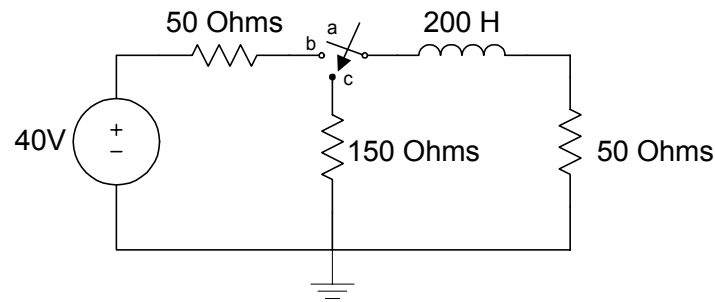
Figure 5.7 RL Circuit for Example 5.3

Solution

For $0 < t < 1$ s, we can use Equation (5.9) to find the current

$$i(t) = 0.4\left(1 - e^{-\left(\frac{t}{\tau_1}\right)}\right) \qquad (5.12)$$

where

$$\tau_1 = \frac{L}{R} = \frac{200}{100} = 2 \text{ s}$$

At $t = 1$ s

$$i(t) = 0.4\left(1 - e^{-0.5}\right) \qquad (5.13)$$
$$= I_{max}$$

For $t > 1$ s, we can use Equation (5.6) to obtain the current

$$i(t) = I_{max} e^{-\left(\frac{t-0.5}{\tau_2}\right)} \qquad (5.14)$$

where

$$\tau_2 = \frac{L}{R_{eq2}} = \frac{200}{200} = 1 \text{ s}$$

The MATLAB program for plotting $i(t)$ is shown below.

MATLAB Script

```
% Solution to Example 5.3
% tau1 is time constant when switch is at b
% tau2 is the time constant when the switch is in position c
%

tau1 = 200/100;

for k=1:20
  t(k) = k/20;
  i(k) = 0.4*(1-exp(-t(k)/tau1));
end

imax = i(20);
tau2 = 200/200;
for k = 21:120
  t(k) = k/20;
  i(k) = imax*exp(-t(k-20)/tau2);
end

% plot the current
plot(t,i,'o')
axis([0 6 0 0.18])
title('Current of an RL circuit')
xlabel('Time, s')
ylabel('Current, A')
```

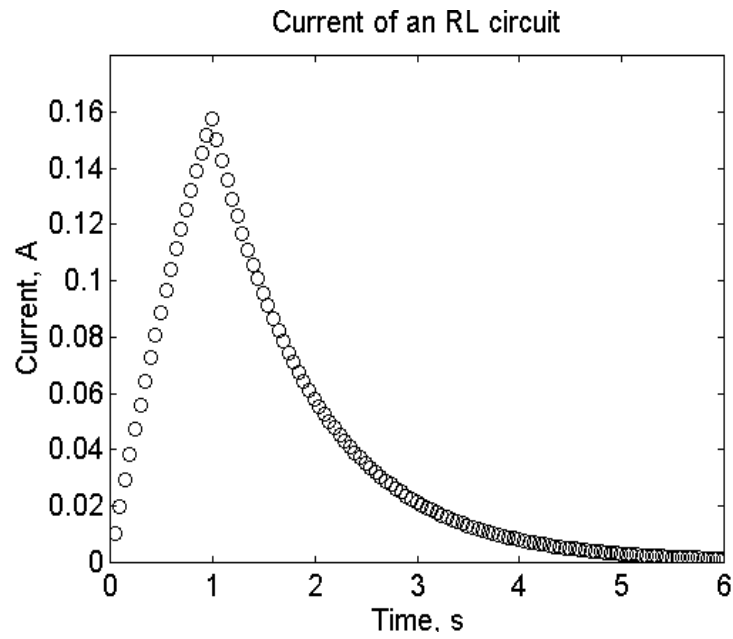Figure 5.8 shows the current $i(t)$.

Figure 5.8 Current Flowing through Inductor

5.3   RLC CIRCUIT

For the series RLC circuit shown in Figure 5.9, we can use KVL to obtain
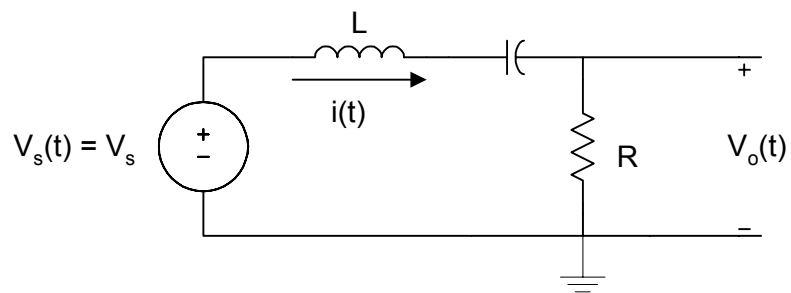the Equation (5.15).

Figure 5.9   Series RLC Circuit

$$v_S(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau + Ri(t) \qquad (5.15)$$

Differentiating the above expression, we get

$$\frac{dv_S(t)}{dt} = L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C}$$

i.e.,

$$\frac{1}{L}\frac{dv_S(t)}{dt} = \frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{i(t)}{LC} \qquad (5.16)$$

The homogeneous solution can be found by making $v_S(t)$ = constant, thus

$$0 = \frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{i(t)}{LC} \qquad (5.17)$$

The characteristic equation is

$$0 = \lambda^2 + a\lambda + b \qquad (5.18)$$

where

$$a = R/L \quad \text{and}$$

$$b = 1/LC$$

The roots of the characteristic equation can be determined. If we assume that the roots are

$$\lambda = \alpha, \beta$$

then, the solution to the homogeneous solution is

$$i_h(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} \qquad (5.19)$$

where

A_1 and A_2 are constants.

If $v_S(t)$ is a constant, then the forced solution will also be a constant and be given as

$$i_f(t) = A_3 \qquad\qquad (5.20)$$

The total solution is given as

$$i(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} + A_3 \qquad\qquad (5.21)$$

where

A_1, A_2, and A_3 are obtained from initial conditions.

Example 5.4 illustrates the use of MATLAB for finding the roots of characteristic equations. The MATLAB function **roots,** described in Section 6.3.1, is used to obtain the roots of characteristic equations.

Example 5.4

For the series RLC circuit shown in , If L = 10 H, R = 400 Ohms and C = 100µF, $v_S(t) = 0$, $i(0) = 4$ A and $\dfrac{di(0)}{dt} = 15$ A/s, find $i(t)$.

Solution

Since $v_S(t) = 0$, we use Equation (5.17) to get

$$0 = \frac{d^2 i(t)}{dt^2} + \frac{400}{10}\frac{di(t)}{dt} + 1000 i(t)$$

The characteristic equation is

$$0 = \lambda^2 + 40\lambda + 1000$$

The MATLAB function **roots** is used to obtain the roots of the characteristics equation.

MATLAB Script

```
p = [1 40 1000];
lambda = roots(p)

lambda =
        -20.0000 +24.4949i
        -20.0000 -24.4949i
```

Using the roots obtained from MATLAB, $i(t)$ is given as

$$i(t) = e^{-20t}(A_1 \cos(24.4949t) + A_2 \sin(24.4949t)$$

$$i(0) = e^{-0}(A_1 + A_2(0)) \qquad \Rightarrow A_1 = 4$$

$$\frac{di(t)}{dt} = -20e^{-20t}\left[A_1 \cos(24.4949t) + A_2 \sin(24.4949t)\right] +$$

$$e^{-20t}\left[-24.4949 A_1 \sin(24.4949t) + 24.4949 A_2 \cos(24.4949t)\right]$$

$$\frac{di(0)}{dt} = 24.4949 A_2 - 20 A_1 = 15$$

Since $A_1 = 4$, $A_2 = 3.8784$

$$i(t) = e^{-20t}\left[4 \cos(24.4949t) + 3.8784 \sin(24.4949t)\right]$$

Perhaps the simplest way to obtain voltages and currents in an RLC circuit is to use Laplace transform. Table 5.1 shows Laplace transform pairs that are useful for solving RLC circuit problems.

From the RLC circuit, we write differential equations by using network analysis tools. The differential equations are converted into algebraic equations using the Laplace transform. The unknown current or voltage is then solved in the s-domain. By using an inverse Laplace transform, the solution can be expressed in the time domain. We will illustrate this method using Example 5.5

Table 5.1
Laplace Transform Pairs

	f (t)	f(s)	
1	1	$\dfrac{1}{s}$	s>0
2	t	$\dfrac{1}{s^2}$	s>0
3	t^n	$\dfrac{n!}{s^{n+1}}$	s>0
4	e^{-at}	$\dfrac{1}{s+a}$	s>a
5	te^{-at}	$\dfrac{1}{(s+a)^2}$	s>a
6	$\sin(wt)$	$\dfrac{w}{s^2+w^2}$	s>0
7	$\cos(wt)$	$\dfrac{s}{s^2+w^2}$	s>0
8	$e^{at}\sin(wt)$	$\dfrac{w}{(s+a)^2+w^2}$	
9	$e^{at}\cos(wt)$	$\dfrac{s+a}{(s+a)^2+w^2}$	
10	$\dfrac{df}{dt}$	$sF(s)-f(0^+)$	
11	$\int_0^t f(\tau)d\tau$	$\dfrac{F(s)}{s}$	
12	$f(t-t_1)$	$e^{-t_1 s}F(s)$	

Example 5.5

The switch in Figure 5.10 has been opened for a long time. If the switch opens at t = 0, find the voltage $v(t)$. Assume that $R = 10\ \Omega$, $L = 1/32$ H, $C = 50\mu F$ and $I_S = 2\ A$.
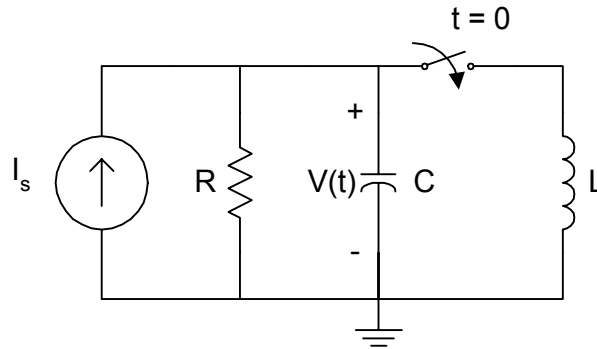

Figure 5.10   Circuit for Example 5.5

At t < 0, the voltage across the capacitor is

$$v_C(0) = (2)(10) = 20 \quad V$$

In addition, the current flowing through the inductor

$$i_L(0) = 0$$

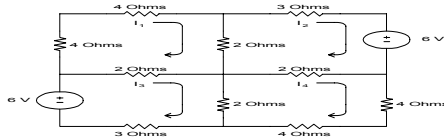At   t > 0, the switch closes and all the four elements of Figure 5.10 remain in parallel. Using KCL, we get

$$I_S = \frac{v(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int_0^t v(\tau)d\tau + i_L(0)$$

Taking the Laplace transform of the above expression, we get

$$\frac{I_S}{s} = \frac{V(s)}{R} + C[sV(s) - V_C(0)] + \frac{V(s)}{sL} + \frac{i_L(0)}{s}$$

Simplifying the above expression, we get

For I_S = 2A, $C = 50\mu$ F, $R = 10\Omega$, $L = 1/32$ H, $V(s)$ becomes

$$V(s) = \frac{40000 + 20s}{s^2 + 2000s + 64*10^4}$$

$$V(s) = \frac{40000 + 20s}{(s+1600)(s+400)} = \frac{A}{(s+1600)} + \frac{B}{(s+400)}$$

$$A = \underset{s \to -1600}{Lim} \quad V(s)(s+1600) \quad = \text{-6.67}$$

$$B = \underset{s \to -400}{Lim} \quad V(s)(s+400) \quad = \text{26.67}$$

$$v(t) = -6.67e^{-1600t} + 26.67e^{-400t}$$

The plot of $v(t)$ is shown in Figure 5.13.

5.4   STATE VARIABLE APPROACH

Another method of finding the transient response of an RLC circuit is the state variable technique.   The later method   (i) can be used to analyze and synthesize control systems, (ii) can be applied to time-varying and nonlinear systems, (iii) is suitable for digital and computer solution and (iv) can be used to develop the general system characteristics.

A state of a system is a minimal set of variables chosen such that if their values are known at the time t, and all inputs are known for times greater than t_1, one can calculate the output of the system for times greater than t_1. In general, if we designate x as the state variable, u as the input, and y as the output of a system, we can express the input u and output y as

$$\overset{\bullet}{x}(t) = Ax(t) + Bu(t) \tag{5.22}$$

$$y(t) \quad = Cx(t) + Du(t) \tag{5.23}$$

where

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ . \\ . \\ u_n(t) \end{bmatrix} \qquad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ x_n(t) \end{bmatrix} \qquad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ . \\ . \\ y_n(t) \end{bmatrix}$$

and *A, B, C*, and *D* are matrices determined by constants of a system.

For example, consider a single-input and a single-output system described by the differential equation

$$\frac{d^4 y(t)}{dt^4} + 3\frac{d^3 y(t)}{dt^3} + 4\frac{d^2 y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 2y(t) \quad = \quad 6u(t)$$

$$\tag{5.24}$$

We define the components of the state vector as

$$x_1(t) = y(t)$$

$$x_2(t) = \frac{dy(t)}{dt} = \frac{dx_1(t)}{dt} = \overset{\bullet}{x}_1(t)$$

$$x_3(t) = \frac{d^2 y(t)}{dt^2} = \frac{dx_2(t)}{dt} = \overset{\bullet}{x}_2(t)$$

$$x_4(t) = \frac{d^3 y(t)}{dt^3} = \frac{dx_3(t)}{dt} = \overset{\bullet}{x}_3(t)$$

$$x_5(t) = \frac{d^4 y(t)}{dt^4} = \frac{dx_4(t)}{dt} = \dot{x}_4(t) \tag{5.25}$$

Using Equations (5.24) and (5.25), we get

$$\dot{x}_4(t) = 6u(t) - 3x_4(t) - 4x_3(t) - 8x_2(t) - 2x_1(t) \tag{5.26}$$

From the Equations (5.25) and (5.26), we get

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -8 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6 \end{bmatrix} u(t) \tag{5.27}$$

or $\qquad \dot{x}(t) = Ax(t) + Bu(t) \tag{5.28}$

where

$$\dot{x} = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix}; \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -8 & -4 & -3 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6 \end{bmatrix} \tag{5.29}$$

Since

$$y(t) = x_1(t)$$

we can express the output $y(t)$ in terms of the state $x(t)$ and input $u(t)$ as

$$y(t) = Cx(t) + Du(t) \tag{5.30}$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 \end{bmatrix} \qquad (5.31)$$

In RLC circuits, if the voltage across a capacitor and the current flowing in an inductor are known at some initial time t, then the capacitor voltage and inductor current will allow the description of system behavior for all subsequent times. This suggests the following guidelines for the selection of acceptable state variables for RLC circuits:

1.   Currents associated with inductors are state variables.

2.   Voltages associated with capacitors are state variables.

3.   Currents or voltages associated with resistors do not specify independent state variables.

4.   When closed loops of capacitors or junctions of inductors exist in a circuit, the state variables chosen according to rules 1 and 2 are not independent.
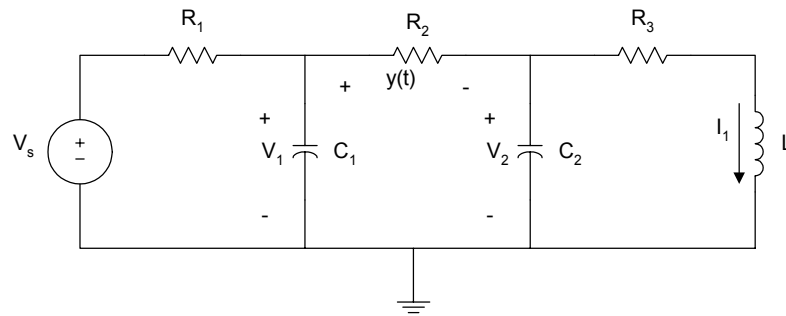
Consider the circuit shown in Figure 5.11.

Figure 5.11   Circuit for State Analysis

Using the above guidelines, we select the state variables to be V_1, V_2, and i_1.

Using nodal analysis, we have

$$C_1 \frac{dv_1(t)}{dt} + \frac{V_1 - V_s}{R_1} + \frac{V_1 - V_2}{R_2} = 0 \qquad (5.32)$$

$$C_2 \frac{dv_2(t)}{dt} + \frac{V_2 - V_1}{R_2} + i_1 = 0 \qquad (5.33)$$

Using loop analysis

$$V_2 = i_1 R_3 + L \frac{di_1(t)}{dt} \qquad (5.34)$$

The output $y(t)$ is given as

$$y(t) = v_1(t) - v_2(t) \qquad (5.35)$$

Simplifying Equations (5.32) to (5.34), we get

$$\frac{dv_1(t)}{dt} = -\left(\frac{1}{C_1 R_1} + \frac{1}{C_1 R_2}\right)V_1 + \frac{V_2}{C_1 R_2} + \frac{V_s}{C_1 R_1} \qquad (5.36)$$

$$\frac{dv_2(t)}{dt} = \frac{V_1}{C_2 R_2} - \frac{V_2}{C_2 R_2} - \frac{i_1}{C_2} \qquad (5.37)$$

$$\frac{di_1(t)}{dt} = \frac{V_2}{L} - \frac{R_3}{L} i_1 \qquad (5.38)$$

Expressing the equations in matrix form, we get

$$\begin{bmatrix} \dot{V_1} \\ \dot{V_2} \\ \dot{i_1} \end{bmatrix} = \begin{bmatrix} -\left(\dfrac{1}{C_1 R_1} + \dfrac{1}{C_1 R_2}\right) & \dfrac{1}{C_1 R_2} & 0 \\ \dfrac{1}{C_2 R_2} & -\dfrac{1}{C_2 R_2} & -\dfrac{1}{C_2} \\ 0 & \dfrac{1}{L} & -\dfrac{R_3}{L} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i_1 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{C_1 R_1} \\ 0 \\ 0 \end{bmatrix} V_s$$

$$(5.39)$$

and the output is

$$y = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i_1 \end{bmatrix} \qquad (5.40)$$

MATLAB functions for solving ordinary differential equations are ODE functions. These are described in the following section.

5.4.1   MATLAB Ode Functions

MATLAB has two functions, **ode23 and ode45**, for computing numerical solutions to ordinary differential equations. The **ode23** function integrates a system of ordinary differential equations using second- and third-order Runge-Kutta formulas; the **ode45** function uses fourth- and fifth-order Runge-Kutta integration equations.

The general forms of the ode functions are

[t,x] = ode23 (xprime, tstart, tfinal, xo, tol,trace)

or

[t,x] = ode45 (xprime, tstart, tfinal, xo, tol, trace)

where

xprime   is the name (in quotation marks) of the MATLAB function or m-file that contains the differential equations to be integrated. The function will compute the state derivative vector $\dot{x}(t)$ given the current time t, and state vector $x(t)$. The function must have 2 input arguments, scalar t (time) and column vector x (state) and the function returns the output argument $xdot$, $(\dot{x})$, a column vector of state derivatives

$$\dot{x}(t_1) = \frac{dx(t_1)}{dt}$$

Let us illustrate the use of MATLAB ode functions with the following two examples.

Example 5.6

For Figure 5.2, V_S = 10V, R = 10,000 Ω, C = 10µF. Find the output voltage $v_0(t)$, between the interval 0 to 20 ms, assuming $v_0(0) = 0$ and by (a) using a numerical solution to the differential equation; and (b) analytical solution.

Solution

From Equation (5.3), we have

$$C\frac{dv_o(t)}{dt} + \frac{v_o(t) - V_s}{R} = 0$$

thus

$$\frac{dv_o(t)}{dt} = \frac{V_s}{CR} - \frac{v_o(t)}{CR} = 100 - 10v_0(t)$$

From Equation(5.4), the analytical solution is

$$v_0(t) = 10\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)$$

MATLAB Script

```
% Solution for first order differential equation
% the function diff1(t,y) is created to evaluate
% the differential equation
% Its m-file is diff1.m
%
% Transient analysis of RC circuit using ode
% function and analytical solution
% numerical solution using ode

t0 = 0;
tf = 20e-3;
xo = 0;  % initial conditions
[t, vo] = ode23('diff1',t0,tf,xo);

% the analytical solution given by Equation(5.4) is
vo_analy = 10*(1-exp(-10*t));

% plot two solutions
subplot(121)
plot(t,vo,'b')
title('State Variable Approach')
xlabel('Time, s'),ylabel('Capacitor Voltage, V'),grid
subplot(122)
plot(t,vo_analy,'b')
title('Analytical Approach')
xlabel('Time, s'),ylabel('Capacitor Voltage, V'),grid

%
function dy = diff1(t,y)
dy = 100 - 10*y;
end
```

Figure 5.12 shows the plot obtained using Equation (5.4) and that obtained from the MATLAB ode23 function. From the two plots, we can see that the two results are identical.

Figure 5.12   Output Voltage $v_0(t)$ Obtained from (a) State Variable Approach and (b) Analytical Method

Example 5.7

For Figure 5.10, if $R = 10\Omega$, $L = 1/32$ H, $C = 50\mu$F, use a numerical solution of the differential equation to solve $v(t)$. Compare the numerical solution to the analytical solution obtained from Example 5.5.

Solution

From Example 5.5, $v_C(0) = 20$V, $i_L(0) = 0$, and

$$L\frac{di_L(t)}{dt} = v_C(t)$$

$$C\frac{dv_C(t)}{dt} + i_L + \frac{v_C(t)}{R} - I_S = 0$$

Simplifying, we get

$$\frac{di_L(t)}{dt} = \frac{v_C(t)}{L}$$

$$\frac{dv_C(t)}{dt} = \frac{I_S}{C} - \frac{i_L(t)}{C} - \frac{v_C(t)}{RC}$$

Assuming that

$$x_1(t) = i_L(t)$$
$$x_2(t) = v_C(t)$$

We get

$$\dot{x}_1(t) = \frac{1}{L}x_2(t)$$

$$\dot{x}_2(t) = \frac{I_S}{C} - \frac{1}{C}x_1(t) - \frac{1}{RC}x_2(t)$$

We create function m-file containing the above differential equations.

MATLAB Script

```
%  Solution of second-order differential equation
%  The function diff2(x,y) is created to evaluate the diff. equation
%  the name of the m-file is diff2.m
%  the function is defined as:
%
function xdot = diff2(t,x)
is = 2;
c = 50e-6;  L = 1/32;  r = 10;
k1 = 1/c ;  %  1/C
k2 =  1/L ;  %  1/L
k3 = 1/(r*c);   %  1/RC

xdot(1) = k2*x(2);
xdot(2) = k1*is - k1*x(1) - k3*x(2);
end
```

To simulate the differential equation defined in diff2 in the interval $0 \leq t \leq 30$ ms, we note that

$$x_1(0) = i_L(0) = 0 \text{ V}$$

$$x_2(0) = v_C(0) = 20$$

Using the MATLAB ode23 function, we get

```
% solution of second-order differential equation
% the function diff2(x,y) is created to evaluate
% the differential equation
% the name of m-file is diff2.m
%
% Transient analysis of RLC circuit using ode function
% numerical solution

t0 = 0;
tf = 30e-3;
x0 = [0 20]; % Initial conditions
[t,x] = ode23('diff2',t0,tf,x0);

% Second column of matrix x represent capacitor voltage
subplot(211), plot(t,x(:,2))
xlabel('Time, s'), ylabel('Capacitor voltage, V')
text(0.01, 7, 'State Variable Approach')

% Transient analysis of RLC circuit from Example 5.5
t2 =0:1e-3:30e-3;
vt = -6.667*exp(-1600*t2) + 26.667*exp(-400*t2);
subplot(212), plot(t2,vt)
xlabel('Time, s'), ylabel('Capacitor voltage, V')
text(0.01, 4.5, 'Results from Example 5.5')
```
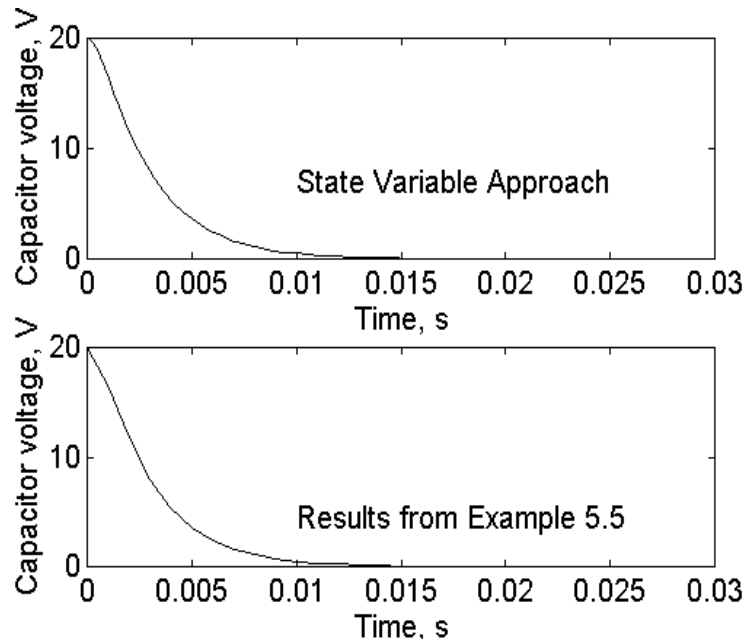
The plot is shown in Figure 5.13.

Figure 5.13 Capacitor Voltage $v_0(t)$ Obtained from Both State Variable Approach and Laplace Transform

The results from the state variable approach and those obtained from Example 5.5 are identical.

Example 5.8

For Figure 5.11, if $v_S(t) = 5u(t)$ where $u(t)$ is the unit step function and $R_1 = R_2 = R_3 = 10 \ K\Omega$, $C_1 = C_2 = 5 \ \mu F$, and $L = 10$ H, find and plot the voltage $v_0(t)$ within the intervals of 0 to 5 s.

Solution

Using the element values and Equations (5.36) to (5.38), we have

$$\frac{dv_1(t)}{dt} = -40v_1(t) + 20v_2(t) + 20V_s$$

$$\frac{dv_2(t)}{dt} = 20v_1(t) - 20v_2(t) - i_1(t)$$

$$\frac{di_1(t)}{dt} = 0.1v_2(t) - 1000i_1(t)$$

We create an m-file containing the above differential equations.

MATLAB Script

```
%
% solution of a set of first order differential equations
% the function diff3(t,v) is created to evaluate
% the differential equation
% the name of the m-file is diff3.m
%

function vdot = diff3(t,v)

vdot(1) = -40*v(1) + 20*v(2) + 20*5;
vdot(2) = 20*v(1) - 20*v(2) - v(3);
vdot(3) = 0.1*v(2) -1000*v(3);
end
```

To obtain the output voltage in the interval of $0 \leq t \leq 5$ s, we note that the output voltage

$$v_0(t) = v_1(t) - v_2(t)$$

Note that at t < 0, the step signal is zero so

$$v_0(0) = v_2(0) = i_1(0) = 0$$

Using ode45 we get

```
% solution of a set of first-order differential equations
% the function diff3(t,v) is created to evaluate
% the differential equation
% the name of the m-file is diff3.m
%
% Transient analysis of RLC circuit using state
```

```
% variable approach

t0 = 0;
tf = 2;
x0 = [0 0 0]; % initial conditions

[t,x] = ode23('diff3', t0, tf, x0);

tt = length(t);

for i = 1:tt
   vo(i) = x(i,1) - x(i,2);
end

plot(t, vo)
title('Transient analysis of RLC')
xlabel('Time, s'), ylabel('Output voltage')
```

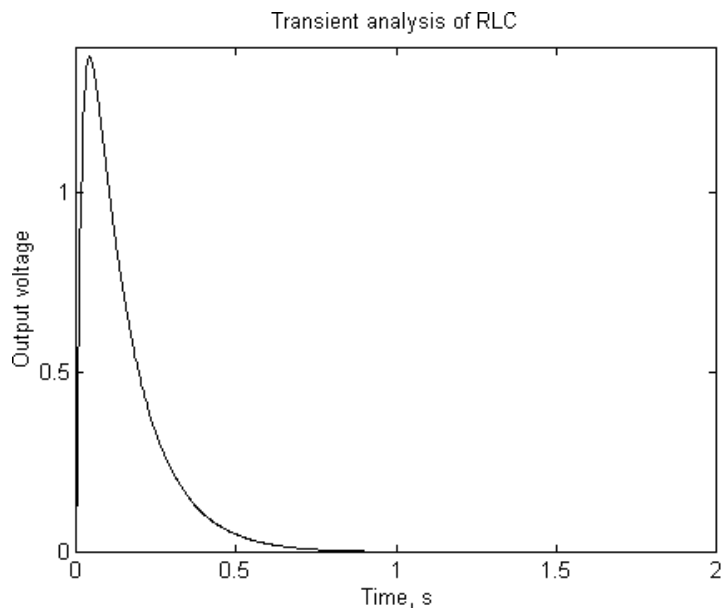The plot of the output voltage is shown in .

Figure 5.14 Output Voltage

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1.   MathWorks, Inc., *MATLAB, High-Performance Numeric Computation Software*, 1995.

2.   Biran, A. and Breiner, M. *MATLAB for Engineers*, Addison-Wesley, 1995.

3.   Etter, D.M., *Engineering Problem Solving with MATLAB*, 2nd Edition, Prentice Hall, 1997.

4.   Nilsson, J.W., *Electric Circuits*, 3rd Edition, Addison-Wesley Publishing Company, 1990.

5.   Vlach, J.O., Network Theory and CAD, *IEEE Trans. on Education*, Vol. 36, No. 1, Feb. 1993, pp. 23 - 27.

6.   Meader, D. A., *Laplace Circuit Analysis and Active Filters*, Prentice Hall, New Jersey, 1991.

EXERCISES

5.1 If the switch is opened at $t = 0$, find $v_0(t)$. Plot $v_0(t)$ between the time interval $0 \leq t \leq 5$ s.
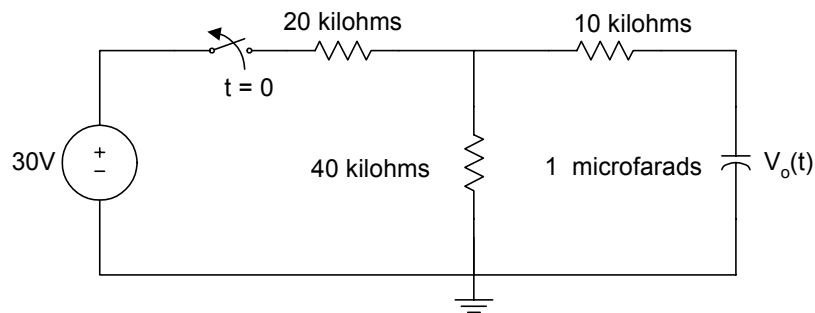

Figure P5.1 Figure for Exercise 5.1

5.2   The switch is close at t = 0; find $i(t)$ between the intervals 0 to 10 ms. The resistance values are in ohms.
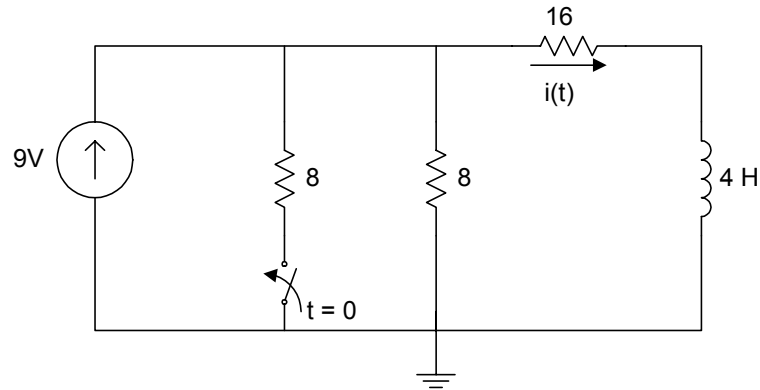

Figure P5.2 Figure for Exercise 5.2

5.3   For the series RLC circuit, the switch is closed at t = 0. The initial energy in the storage elements is zero. Use MATLAB to find $v_0(t)$.
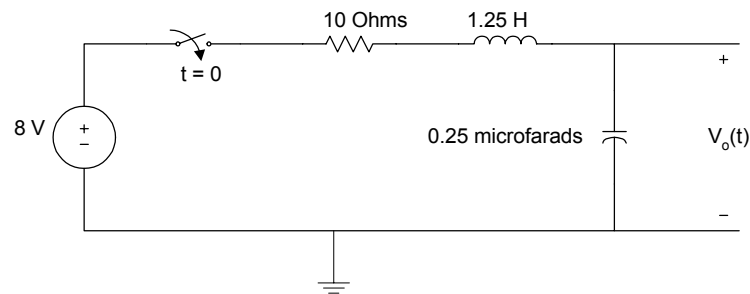

Figure P5.3 Circuit for Exercise 5.3

5.4   Use MATLAB to solve the following differential equation

$$\frac{d^3 y(t)}{dt^3} + 7\frac{d^2 y(t)}{dt^2} + 14\frac{dy(t)}{dt} + 12y(t) = 10$$

with initial conditions

$$y(0) = 1, \qquad \frac{dy(0)}{dt} = 2, \qquad \frac{d^2 y(0)}{dt^2} = 5$$

Plot y(t) within the intervals of 0 and 10 s.

5.5 For Figure P5.5, if $V_S = 5u(t)$, determine the voltages V₁(t), V₂(t), V₃(t) and V₄(t) between the intervals of 0 to 20 s. Assume that the initial voltage across each capacitor is zero.
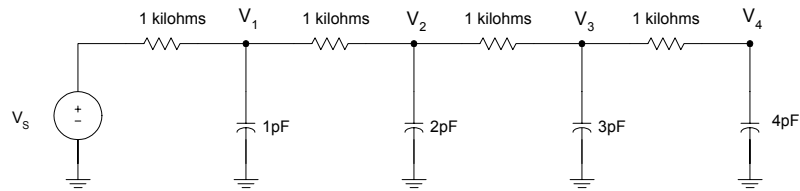

Figure P5.5 RC Network

5.6 For the differential equation

$$\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 3\sin(t) + 7\cos(t)$$

with initial conditions $y(0) = 4$ and $\frac{dy(0)}{dt} = -1$

(a) Determine $y(t)$ using Laplace transforms.

(b) Use MATLAB to determine $y(t)$.

(c) Sketch $y(t)$ obtained in parts (a) and (b).

(d) Compare the results obtained in part c.