Attia, John Okyere. "Two-Port Networks."
Electronics and Circuit Analysis using MATLAB.
Ed. John Okyere Attia
Boca Raton: CRC Press LLC, 1999

## CHAPTER SEVEN

## TWO-PORT NETWORKS

This chapter discusses the application of MATLAB for analysis of two-port networks. The describing equations for the various two-port network representations are given. The use of MATLAB for solving problems involving parallel, series and cascaded two-port networks is shown. Example problems involving both passive and active circuits will be solved using MATLAB.

### 7.1 TWO-PORT NETWORK REPRESENTATIONS

A general two-port network is shown in Figure 7.1.


Figure 7.1 General Two-Port Network
$I_{1}$ and $V_{1}$ are input current and voltage, respectively. Also, $I_{2}$ and $V_{2}$ are output current and voltage, respectively. It is assumed that the linear two-port circuit contains no independent sources of energy and that the circuit is initially at rest ( no stored energy). Furthermore, any controlled sources within the linear two-port circuit cannot depend on variables that are outside the circuit.

### 7.1.1 z-parameters

A two-port network can be described by z-parameters as

$$
\begin{align*}
& V_{1}=z_{11} I_{1}+z_{12} I_{2}  \tag{7.1}\\
& V_{2}=z_{21} I_{1}+z_{22} I_{2} \tag{7.2}
\end{align*}
$$

In matrix form, the above equation can be rewritten as

$$
\left[\begin{array}{l}
V_{1}  \tag{7.3}\\
V_{2}
\end{array}\right]=\left[\begin{array}{ll}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
I_{2}
\end{array}\right]
$$

The $z$-parameter can be found as follows

$$
\begin{align*}
& z_{11}=\left.\frac{V_{1}}{I_{1}}\right|_{I_{2}=0}  \tag{7.4}\\
& z_{12}=\left.\frac{V_{1}}{I_{2}}\right|_{I_{1}=0}  \tag{7.5}\\
& z_{21}=\left.\frac{V_{2}}{I_{1}}\right|_{I_{2}=0}  \tag{7.6}\\
& z_{22}=\left.\frac{V_{2}}{I_{2}}\right|_{I_{1}=0} \tag{7.7}
\end{align*}
$$

The z-parameters are also called open-circuit impedance parameters since they are obtained as a ratio of voltage and current and the parameters are obtained by open-circuiting port $2\left(I_{2}=0\right)$ or port1 $\left(I_{1}=0\right)$. The following example shows a technique for finding the z-parameters of a simple circuit.

## Example 7.1

For the T-network shown in Figure 7.2, find the z-parameters.


Figure 7.2 T-Network

## Solution

Using KVL

$$
\begin{align*}
& V_{1}=Z_{1} I_{1}+Z_{3}\left(I_{1}+I_{2}\right)=\left(Z_{1}+Z_{3}\right) I_{1}+Z_{3} I_{2}  \tag{7.8}\\
& V_{2}=Z_{2} I_{2}+Z_{3}\left(I_{1}+I_{2}\right)=\left(Z_{3}\right) I_{1}+\left(Z_{2}+Z_{3}\right) I_{2} \tag{7.9}
\end{align*}
$$

thus

$$
\left[\begin{array}{l}
V_{1}  \tag{7.10}\\
V_{2}
\end{array}\right]=\left[\begin{array}{cc}
Z_{1}+Z_{3} & Z_{3} \\
Z_{3} & Z_{2}+Z_{3}
\end{array}\right]\left[\begin{array}{c}
I_{1} \\
I_{2}
\end{array}\right]
$$

and the z-parameters are

$$
[Z]=\left[\begin{array}{cc}
Z_{1}+Z_{3} & Z_{3}  \tag{7.11}\\
Z_{3} & Z_{2}+Z_{3}
\end{array}\right]
$$

### 7.1.2 y-parameters

A two-port network can also be represented using y-parameters. The describing equations are

$$
\begin{align*}
& I_{1}=y_{11} V_{1}+y_{12} V_{2}  \tag{7.12}\\
& I_{2}=y_{21} V_{1}+y_{22} V_{2} \tag{7.13}
\end{align*}
$$

where

$$
\begin{aligned}
& V_{1} \text { and } V_{2} \text { are independent variables and } \\
& I_{1} \text { and } I_{2} \text { are dependent variables. }
\end{aligned}
$$

In matrix form, the above equations can be rewritten as

$$
\left[\begin{array}{l}
I_{1}  \tag{7.14}\\
I_{2}
\end{array}\right]=\left[\begin{array}{ll}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{array}\right]\left[\begin{array}{l}
V_{1} \\
V_{2}
\end{array}\right]
$$

The y-parameters can be found as follows:

$$
\begin{align*}
& y_{11}=\left.\frac{I_{1}}{V_{1}}\right|_{V_{2}=0}  \tag{7.15}\\
& y_{12}=\left.\frac{I_{1}}{V_{2}}\right|_{V_{1}=0}  \tag{7.16}\\
& y_{21}=\left.\frac{I_{2}}{V_{1}}\right|_{V_{2}=0}  \tag{7.17}\\
& y_{22}=\left.\frac{I_{2}}{V_{2}}\right|_{V_{1}=0} \tag{7.18}
\end{align*}
$$

The y-parameters are also called short-circuit admittance parameters. They are obtained as a ratio of current and voltage and the parameters are found by short-circuiting port $2\left(V_{2}=0\right)$ or port $1\left(V_{1}=0\right)$. The following two examples show how to obtain the y-parameters of simple circuits.

## Example 7.2

Find the y-parameters of the pi $(\pi)$ network shown in Figure 7.3.


Figure 7.3 Pi-Network

## Solution

Using KCL, we have

$$
\begin{equation*}
I_{1}=V_{1} Y_{a}+\left(V_{1}-V_{2}\right) Y_{b}=V_{1}\left(Y_{a}+Y_{b}\right)-V_{2} Y_{b} \tag{7.19}
\end{equation*}
$$

$$
\begin{equation*}
I_{2}=V_{2} Y_{c}+\left(V_{2}-V_{1}\right) Y_{b}=-V_{1} Y_{b}+V_{2}\left(Y_{b}+Y_{c}\right) \tag{7.20}
\end{equation*}
$$

Comparing Equations (7.19) and (7.20) to Equations (7.12) and (7.13), the yparameters are

$$
[Y]=\left[\begin{array}{cc}
Y_{a}+Y_{b} & -Y_{b}  \tag{7.21}\\
-Y_{b} & Y_{b}+Y_{c}
\end{array}\right]
$$

## Example 7.3

Figure 7.4 shows the simplified model of a field effect transistor. Find its yparameters.


Figure 7.4 Simplified Model of a Field Effect Transistor

Using KCL,

$$
\begin{align*}
& I_{1}=V_{1} s C_{1}+\left(V_{1}-V_{2}\right) s C_{3}=V_{1}\left(s C_{1}+s C_{3}\right)+V_{2}\left(-s C_{3}\right)  \tag{7.22}\\
& I_{2}=V_{2} Y_{2}+g_{m} V_{1}+\left(V_{2}-V_{1}\right) s C_{3}=V_{1}\left(g_{m}-s C_{3}\right)+V_{2}\left(Y_{2}+s C_{3}\right) \tag{7.23}
\end{align*}
$$

Comparing the above two equations to Equations (7.12) and (7.13), the yparameters are

$$
[Y]=\left[\begin{array}{cc}
s C_{1}+s C_{3} & -s C_{3}  \tag{7.24}\\
g_{m}-s C_{3} & Y_{2}+s C_{3}
\end{array}\right]
$$

### 7.1.3 h-parameters

A two-port network can be represented using the h-parameters. The describing equations for the h-parameters are

$$
\begin{align*}
& V_{1}=h_{11} I_{1}+h_{12} V_{2}  \tag{7.25}\\
& I_{2}=h_{21} I_{1}+h_{22} V_{2} \tag{7.26}
\end{align*}
$$

where

$$
\begin{aligned}
& I_{1} \text { and } V_{2} \text { are independent variables and } \\
& V_{1} \text { and } I_{2} \text { are dependent variables. }
\end{aligned}
$$

In matrix form, the above two equations become

$$
\left[\begin{array}{l}
V_{1}  \tag{7.27}\\
I_{2}
\end{array}\right]=\left[\begin{array}{ll}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
V_{2}
\end{array}\right]
$$

The h-parameters can be found as follows:

$$
\begin{align*}
& h_{11}=\left.\frac{V_{1}}{I_{1}}\right|_{V_{2}=0}  \tag{7.28}\\
& h_{12}=\left.\frac{V_{1}}{V_{2}}\right|_{I_{1}=0}  \tag{7.29}\\
& h_{21}=\left.\frac{I_{2}}{I_{1}}\right|_{V_{2}=0}  \tag{7.30}\\
& h_{22}=\left.\frac{I_{2}}{V_{2}}\right|_{I_{1}=0} \tag{7.31}
\end{align*}
$$

The h-parameters are also called hybrid parameters since they contain both open-circuit parameters $\left(I_{1}=0\right)$ and short-circuit parameters $\left(V_{2}=0\right)$. The h-parameters of a bipolar junction transistor are determined in the following example.

## Example 7.4

A simplified equivalent circuit of a bipolar junction transistor is shown in Figure 7.5, find its h-parameters.


Figure 7.5 Simplified Equivalent Circuit of a Bipolar Junction Transistor

## Solution

Using KCL for port 1,

$$
\begin{equation*}
V_{1}=I_{1} Z_{1} \tag{7.32}
\end{equation*}
$$

Using KCL at port 2, we get

$$
\begin{equation*}
I_{2}=\beta I_{1}+Y_{2} V_{2} \tag{7.33}
\end{equation*}
$$

Comparing the above two equations to Equations (7.25) and (7.26) we get the h-parameters.

$$
[h]=\left[\begin{array}{cc}
Z_{1} & 0  \tag{7.34}\\
\beta & Y_{2}
\end{array}\right]
$$

### 7.1.4 Transmission parameters

A two-port network can be described by transmission parameters. The describing equations are

$$
\begin{align*}
& V_{1}=a_{11} V_{2}-a_{12} I_{2}  \tag{7.35}\\
& I_{1}=a_{21} V_{2}-a_{22} I_{2} \tag{7.36}
\end{align*}
$$

where

$$
\begin{aligned}
& V_{2} \text { and } I_{2} \text { are independent variables and } \\
& V_{1} \text { and } I_{1} \text { are dependent variables. }
\end{aligned}
$$

In matrix form, the above two equations can be rewritten as

$$
\left[\begin{array}{l}
V_{1}  \tag{7.37}\\
I_{1}
\end{array}\right]=\left[\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right]\left[\begin{array}{c}
V_{2} \\
-I_{2}
\end{array}\right]
$$

The transmission parameters can be found as

$$
\begin{align*}
& a_{11}=\left.\frac{V_{1}}{V_{2}}\right|_{I_{2}=0}  \tag{7.38}\\
& a_{12}=-\left.\frac{V_{1}}{I_{2}}\right|_{V_{2}=0}  \tag{7.39}\\
& a_{21}=\left.\frac{I_{1}}{V_{2}}\right|_{I_{2}=0}  \tag{7.40}\\
& a_{22}=-\left.\frac{I_{1}}{I_{2}}\right|_{V_{2}=0} \tag{7.41}
\end{align*}
$$

The transmission parameters express the primary (sending end) variables $V_{1}$ and $I_{1}$ in terms of the secondary (receiving end) variables $V_{2}$ and $-I_{2}$. The negative of $I_{2}$ is used to allow the current to enter the load at the receiving end. Examples 7.5 and 7.6 show some techniques for obtaining the transmission parameters of impedance and admittance networks.

## Example 7.5

Find the transmission parameters of Figure 7.6.


Figure 7.6 Simple Impedance Network

## Solution

By inspection,

$$
\begin{equation*}
I_{1}=-I_{2} \tag{7.42}
\end{equation*}
$$

Using KVL,

$$
\begin{equation*}
V_{1}=V_{2}+Z_{1} I_{1} \tag{7.43}
\end{equation*}
$$

Since $I_{1}=-I_{2}$, Equation (7.43) becomes

$$
\begin{equation*}
V_{1}=V_{2}-Z_{1} I_{2} \tag{7.44}
\end{equation*}
$$

Comparing Equations (7.42) and (7.44) to Equations (7.35) and (7.36), we have

$$
\begin{array}{ll}
a_{11}=1 & a_{12}=Z_{1} \\
a_{21}=0 & a_{22}=1 \tag{7.45}
\end{array}
$$

## Example 7.6

Find the transmission parameters for the network shown in Figure 7.7.


Figure 7.7 Simple Admittance Network

## Solution

By inspection,

$$
\begin{equation*}
V_{1}=V_{2} \tag{7.46}
\end{equation*}
$$

Using KCL, we have

$$
\begin{equation*}
I_{1}=V_{2} Y_{2}-I_{2} \tag{7.47}
\end{equation*}
$$

Comparing Equations (7.46) and 7.47) to equations (7.35) and (7.36) we have

$$
\begin{array}{ll}
a_{11}=1 & a_{12}=0 \\
a_{21}=Y_{2} & a_{22}=1 \tag{7.48}
\end{array}
$$

Using the describing equations, the equivalent circuits of the various two-port network representations can be drawn. These are shown in Figure 7.8.

(a)


Figure 7.8 Equivalent Circuit of Two-port Networks (a) zparameters, (b) y-parameters and (c ) h-parameters

### 7.2 INTERCONNECTION OF TWO-PORT NETWORKS

Two-port networks can be connected in series, parallel or cascade. Figure 7.9 shows the various two-port interconnections.

(a) Series-connected Two-port Network

(b) Parallel-connected Two-port Network

(c ) Cascade Connection of Two-port Network
Figure 7.9 Interconnection of Two-port Networks (a) Series (b) Parallel (c ) Cascade

It can be shown that if two-port networks with z-parameters $[Z]_{1},[Z]_{2},[Z]_{3}, \ldots,[Z]_{n}$ are connected in series, then the equivalent twoport z-parameters are given as

$$
\begin{equation*}
[Z]_{e q}=[Z]_{1}+[Z]_{2}+[Z]_{3}+\ldots+[Z]_{n} \tag{7.49}
\end{equation*}
$$

If two-port networks with y-parameters $[Y]_{1},[Y]_{2},[Y]_{3}, \ldots,[Y]_{n}$ are connected in parallel, then the equivalent two-port y-parameters are given as

$$
\begin{equation*}
[Y]_{e q}=[Y]_{1}+[Y]_{2}+[Y]_{3}+\ldots+[Y]_{n} \tag{7.50}
\end{equation*}
$$

When several two-port networks are connected in cascade, and the individual networks have transmission parameters $[A]_{1},[A]_{2},[A]_{3}, \ldots,[A]_{n}$, then the equivalent two-port parameter will have a transmission parameter given as

$$
\begin{equation*}
[A]_{e q}=[A]_{1} *[A]_{2} *[A]_{3} * \ldots *[A]_{n} \tag{7.51}
\end{equation*}
$$

The following three examples illustrate the use of MATLAB for determining the equivalent parameters of interconnected two-port networks.

## Example 7.7

Find the equivalent y-parameters for the bridge T-network shown in Figure 7.10 .


Figure 7.10 Bridge-T Network

## Solution

The bridge-T network can be redrawn as


Figure 7.11 An Alternative Representation of Bridge-T Network

From Example 7.1, the z-parameters of network N2 are

$$
[Z]=\left[\begin{array}{cc}
Z_{1}+Z_{3} & Z_{3} \\
Z_{3} & Z_{2}+Z_{3}
\end{array}\right]
$$

We can convert the z-parameters to y-parameters [refs. 4 and 6] and we get

$$
\begin{align*}
& y_{11}=\frac{Z_{2}+Z_{3}}{Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}} \\
& y_{12}=\frac{-Z_{3}}{Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}} \\
& y_{21}=\frac{-Z_{3}}{Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}}  \tag{7.52}\\
& y_{22}=-\frac{Z_{1}+Z_{3}}{Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}}
\end{align*}
$$

From Example 7.5, the transmission parameters of network N1 are

$$
\begin{array}{ll}
a_{11}=1 & a_{12}=Z_{4} \\
a_{21}=0 & a_{22}=1
\end{array}
$$

We convert the transmission parameters to y-parameters[ refs. 4 and 6] and we get

$$
\begin{align*}
& y_{11}=\frac{1}{Z_{4}} \\
& y_{12}=-\frac{1}{Z_{4}} \\
& y_{21}=-\frac{1}{Z_{4}}  \tag{7.53}\\
& y_{22}=\frac{1}{Z_{4}}
\end{align*}
$$

Using Equation (7.50), the equivalent $y$-parameters of the bridge-T network are

$$
\begin{align*}
& y_{11 e q}=\frac{1}{Z_{4}}+\frac{Z_{2}+Z_{3}}{Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}} \\
& y_{12 e q}=-\frac{1}{Z_{4}}-\frac{Z_{3}}{Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}} \\
& y_{21 e q}=-\frac{1}{Z_{4}}-\frac{Z_{3}}{Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}}  \tag{7.54}\\
& y_{22 e q}=\frac{1}{Z_{4}}+\frac{Z_{1}+Z_{3}}{Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}}
\end{align*}
$$

## Example 7.8

Find the transmission parameters of Figure 7.12.


Figure 7.12 Simple Cascaded Network

## Solution

Figure 7.12 can be redrawn as


Figure 7.13 Cascade of Two Networks N1 and N2

From Example 7.5, the transmission parameters of network N1 are

$$
\begin{array}{ll}
a_{11}=1 & a_{12}=Z_{1} \\
a_{21}=0 & a_{22}=1
\end{array}
$$

From Example 7.6, the transmission parameters of network N2 are

$$
\begin{array}{ll}
a_{11}=1 & a_{12}=0 \\
a_{21}=Y_{2} & a_{22}=1
\end{array}
$$

From Equation (7.51), the transmission parameters of Figure 7.13 are

$$
\left[\begin{array}{ll}
a_{11} & a_{12}  \tag{7.55}\\
a_{21} & a_{22}
\end{array}\right]_{e q}=\left[\begin{array}{cc}
1 & Z_{1} \\
0 & 1
\end{array}\right]\left[\begin{array}{cc}
1 & 0 \\
Y_{2} & 1
\end{array}\right]=\left[\begin{array}{cc}
1+Z_{1} Y_{2} & Z_{1} \\
Y_{2} & 1
\end{array}\right]
$$

## Example 7.9

Find the transmission parameters for the cascaded system shown in Figure 7.14. The resistance values are in Ohms.


Figure 7.14 Cascaded Resistive Network

## Solution

Figure 7.14 can be considered as four networks, N1, N2, N3, and N4 connected in cascade. From Example 7.8, the transmission parameters of Figure 7.12 are

$$
\begin{aligned}
& {[a]_{N 1}=\left[\begin{array}{ll}
3 & 2 \\
1 & 1
\end{array}\right]} \\
& {[a]_{N 2}=\left[\begin{array}{cc}
3 & 4 \\
0.5 & 1
\end{array}\right]} \\
& {[a]_{N 3}=\left[\begin{array}{cc}
3 & 8 \\
0.25 & 1
\end{array}\right]} \\
& {[a]_{N 4}=\left[\begin{array}{cc}
3 & 16 \\
0.125 & 1
\end{array}\right]}
\end{aligned}
$$

The transmission parameters of Figure 7.14 can be obtained using the following MATLAB program.

## MATLAB Script

diary ex7_9.dat
\% Transmission parameters of cascaded network
al $=\left[\begin{array}{llll}3 & 2 & 1 & 1\end{array}\right]$;
$\mathrm{a} 2=\left[\begin{array}{llll}3 & 4 & 0.5 & 1\end{array}\right]$;
a3 $=[38 ; 0.251] ;$
a4 $=[316 ; 0.1251] ;$
\% equivalent transmission parameters
$a=a 1 *(a 2 *(a 3 * a 4))$
diary
The value of matrix a is

$$
a=\begin{array}{rr}
\mathrm{a}= \\
112.2500 & 630.0000 \\
39.3750 & 221.0000
\end{array}
$$

### 7.3 TERMINATED TWO-PORT NETWORKS

In normal applications, two-port networks are usually terminated. A terminated two-port network is shown in Figure 7.4.


Figure 7.15 Terminated Two-Port Network
In the Figure 7.15, $V_{g}$ and $Z_{g}$ are the source generator voltage and impedance, respectively. $Z_{L}$ is the load impedance. If we use z-parameter representation for the two-port network, the voltage transfer function can be shown to be

$$
\begin{equation*}
\frac{V_{2}}{V_{g}}=\frac{z_{21} Z_{L}}{\left(z_{11}+Z_{g}\right)\left(z_{22}+Z_{L}\right)-z_{12} z_{21}} \tag{7.56}
\end{equation*}
$$

and the input impedance,

$$
\begin{equation*}
Z_{i n}=z_{11}-\frac{z_{12} z_{21}}{z_{22}+Z_{L}} \tag{7.57}
\end{equation*}
$$

and the current transfer function,

$$
\begin{equation*}
\frac{I_{2}}{I_{1}}=-\frac{z_{21}}{z_{22}+Z_{L}} \tag{7.58}
\end{equation*}
$$

A terminated two-port network, represented using the y-parameters, is shown in Figure 7.16.


Figure 7.16 A Terminated Two-Port Network with y-parameters Representation

It can be shown that the input admittance, $Y_{i n}$, is

$$
\begin{equation*}
Y_{i n}=y_{11}-\frac{y_{12} y_{21}}{y_{22}+Y_{L}} \tag{7.59}
\end{equation*}
$$

and the current transfer function is given as

$$
\begin{equation*}
\frac{I_{2}}{I_{g}}=\frac{y_{21} Y_{L}}{\left(y_{11}+Y_{g}\right)\left(y_{22}+Y_{L}\right)-y_{12} y_{21}} \tag{7.60}
\end{equation*}
$$

and the voltage transfer function

$$
\begin{equation*}
\frac{V_{2}}{V_{g}}=-\frac{y_{21}}{y_{22}+Y_{L}} \tag{7.61}
\end{equation*}
$$

A doubly terminated two-port network, represented by transmission parameters, is shown in Figure 7.17.


Figure 7.17 A Terminated Two-Port Network with Transmission Parameters Representation

The voltage transfer function and the input impedance of the transmission parameters can be obtained as follows. From the transmission parameters, we have

$$
\begin{align*}
& V_{1}=a_{11} V_{2}-a_{12} I_{2}  \tag{7.62}\\
& I_{1}=a_{21} V_{2}-a_{22} I_{2} \tag{7.63}
\end{align*}
$$

From Figure 7.6,

$$
\begin{equation*}
V_{2}=-I_{2} Z_{L} \tag{7.64}
\end{equation*}
$$

Substituting Equation (7.64) into Equations (7.62) and (7.63), we get the input impedance,

$$
\begin{equation*}
Z_{i n}=\frac{a_{11} Z_{L}+a_{12}}{a_{21} Z_{L}+a_{22}} \tag{7.65}
\end{equation*}
$$

From Figure 7.17, we have

$$
\begin{equation*}
V_{1}=V_{g}-I_{1} Z_{g} \tag{7.66}
\end{equation*}
$$

Substituting Equations (7.64) and (7.66) into Equations (7.62) and (7.63), we have

$$
\begin{align*}
& V_{g}-I_{1} Z_{g}=V_{2}\left[a_{11}+\frac{a_{12}}{Z_{L}}\right]  \tag{7.67}\\
& I_{1}=V_{2}\left[a_{21}+\frac{a_{22}}{Z_{L}}\right] \tag{7.68}
\end{align*}
$$

Substituting Equation (7.68) into Equation (7.67), we get

$$
\begin{equation*}
V_{g}-V_{2} Z_{g}\left[a_{21}+\frac{a_{22}}{Z_{L}}\right]=V_{2}\left[a_{11}+\frac{a_{12}}{Z_{L}}\right] \tag{7.69}
\end{equation*}
$$

Simplifying Equation (7.69), we get the voltage transfer function

$$
\begin{equation*}
\frac{V_{2}}{V_{g}}=\frac{Z_{L}}{\left(a_{11}+a_{21} Z_{g}\right) Z_{L}+a_{12}+a_{22} Z_{g}} \tag{7.70}
\end{equation*}
$$

The following examples illustrate the use of MATLAB for solving terminated two-port network problems.

## Example 7.10

Assuming that the operational amplifier of Figure 7.18 is ideal,
(a) Find the z-parameters of Figure 7.18.
(b) If the network is connected by a voltage source with source resistance of $50 \Omega$ and a load resistance of $1 \mathrm{~K} \Omega$, find the voltage gain.
(c) Use MATLAB to plot the magnitude response.


Figure 7.18 An Active Lowpass Filter

## Solution

Using KVL,

$$
\begin{align*}
& V_{1}=R_{1} I_{1}+\frac{I_{1}}{s C}  \tag{7.71}\\
& V_{2}=R_{4} I_{2}+R_{3} I_{3}+R_{2} I_{3} \tag{7.72}
\end{align*}
$$

From the concept of virtual circuit discussed in Chapter 11,

$$
\begin{equation*}
R_{2} I_{3}=\frac{I_{1}}{s C} \tag{7.73}
\end{equation*}
$$

Substituting Equation (7.73) into Equation (7.72), we get

$$
\begin{equation*}
V_{2}=\frac{\left(R_{2}+R_{3}\right) I_{1}}{s C R_{2}}+R_{4} I_{2} \tag{7.74}
\end{equation*}
$$

Comparing Equations (7.71) and (7.74) to Equations (7.1) and (7.2), we have

$$
\begin{align*}
& z_{11}=R_{1}+\frac{1}{s C} \\
& z_{12}=0 \\
& z_{21}=\left(1+\frac{R_{3}}{R_{2}}\right)\left(\frac{1}{s C}\right)  \tag{7.75}\\
& z_{22}=R_{4}
\end{align*}
$$

From Equation (7.56), we get the voltage gain for a terminated two-port network. It is repeated here.

$$
\frac{V_{2}}{V_{g}}=\frac{z_{21} Z_{L}}{\left(z_{11}+Z_{g}\right)\left(z_{22}+Z_{L}\right)-z_{12} z_{21}}
$$

Substituting Equation (7.75) into Equation (7.56), we have

$$
\begin{equation*}
\frac{V_{2}}{V_{g}}=\frac{\left(1+\frac{R_{3}}{R_{2}}\right) Z_{L}}{\left(R_{4}+Z_{L}\right)\left[1+s C\left(R_{1}+Z_{g}\right)\right]} \tag{7.76}
\end{equation*}
$$

For $Z_{g}=50 \Omega, Z_{L}=1 K \Omega, R_{3}=10 \mathrm{~K} \Omega, R_{2}=1 \mathrm{~K} \Omega, R_{4}=2 \mathrm{~K} \Omega$ and $C=0.1 \mu F$, Equation (7.76) becomes

$$
\begin{equation*}
\frac{V_{2}}{V_{g}}=\frac{2}{\left[1+1.05 * 10^{-4} s\right]} \tag{7.77}
\end{equation*}
$$

The MATLAB script is

```
%
num = [2];
den=[1.05e-4 1];
w = logspace(1,5);
h = freqs(num,den,w);
f= w/(2*pi);
mag =20*\operatorname{log}10(abs(h)); % magnitude in dB
semilogx(f,mag)
title('Lowpass Filter Response')
xlabel('Frequency, Hz')
```

ylabel('Gain in dB')
The frequency response is shown in Figure 7.19.


Figure 7.19 Magnitude Response of an Active Lowpass Filter

## SELECTED BIBLIOGRAPHY

1. MathWorks, Inc., MATLAB, High-Performance Numeric Computation Software, 1995.
2. Biran, A. and Breiner, M., MATLAB for Engineers, AddisonWesley, 1995.
3. Etter, D.M., Engineering Problem Solving with MATLAB, $2^{\text {nd }}$ Edition, Prentice Hall, 1997.
4. Nilsson, J.W., Electric Circuits, $3^{\text {rd }}$ Edition, Addison-Wesley Publishing Company, 1990.
5. Meader, D.A., Laplace Circuit Analysis and Active Filters, Prentice Hall, 1991.
6. Johnson, D. E. Johnson, J.R., and Hilburn, J.L. Electric Circuit Analysis, $3^{\text {rd }}$ Edition, Prentice Hall, 1997.
7. Vlach, J.O., Network Theory and CAD, IEEE Trans. on Education, Vol. 36, No. 1, Feb. 1993, pp. 23-27.

## EXERCISES

7.1 (a) Find the transmission parameters of the circuit shown in Figure P7.1a. The resistance values are in ohms.


Figure P7.1a Resistive T-Network
(b) From the result of part (a), use MATLAB to find the transmission parameters of Figure P7.2b. The resistance values are in ohms.


Figure P7.1b Cascaded Resistive Network
7.2 Find the y-parameters of the circuit shown in Figure P7.2 The resistance values are in ohms.


Figure P7.2 A Resistive Network
7.3 (a) Show that for the symmetrical lattice structure shown in Figure P7.3,

$$
\begin{aligned}
& z_{11}=z_{22}=0.5\left(Z_{c}+Z_{d}\right) \\
& z_{12}=z_{21}=0.5\left(Z_{c}-Z_{d}\right)
\end{aligned}
$$

(b) If $Z_{c}=10 \Omega, Z_{d}=4 \Omega$, find the equivalent y parameters.


Figure P7.3 Symmetrical Lattice Structure
7.4 (a) Find the equivalent z-parameters of Figure P7.4.
(b) If the network is terminated by a load of 20 ohms and connected to a source of $V_{S}$ with a source resistance of 4 ohms , use MATLAB to plot the frequency response of the circuit.


Figure P7.4 Circuit for Problem 7.4
7.5 For Figure P7.5
(a) Find the transmission parameters of the RC ladder network.
(b) Obtain the expression for $\frac{V_{2}}{V_{1}}$.
(c) Use MATLAB to plot the phase characteristics of $\frac{V_{2}}{V_{1}}$.


Figure P7.5 RC Ladder Network
7.6 For the circuit shown in Figure P7.6,
(a) Find the y-parameters.
(b) Find the expression for the input admittance.
(c) Use MATLAB to plot the input admittance as a function of frequency.


Figure P7.6 Circuit for Problem 7.6
7.7 For the op amp circuit shown in Figure P7.7, find the y-parameters.


Figure P7.7 Op Amp Circuit

