EFFECTS OF RESISTANCES AND LEAKAGE INDUCTANCES ON CROSS REGULATION IN SRC

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ABSTRACT

This paper analyzes the effect of parasitic resistances on the cross regulation performance of multi-output SRC topologies. The steady state cross regulation characteristics have been derived for the example of a two-output SRC. Comparison is given between the characteristic curves of SRC with parasitic resistances and SRC with leakage inductances. The results show that the cross regulation due to parasitic resistances is much less than that due to the leakage inductances. Our theoretical results in parasitic resistance case are verified by computer simulation.

I. INTRODUCTION

Next to the leakage inductances, tha parasitic resistances have probably the greatest effect on the cross regulation performance of the multi-output converters. The winding resistances in the transformer and the inductor, the forward resistances of the rectifiers and ESR in the filter capacitor must be incorporated in the analysis of high power converters. In this paper, the resistances will be incorporated into the equivalent circuits of multi-output series resonant converters. The performance characteristic curves will be derived by using the state-plane techniques. This technique is very general and can be applied to other resonant converter topologies.

II. MULTI-OUTPUT SRC MODEL

In general, a multi-output SRC can be represented by the model which consists of a resonant tank driven by a multistate voltage source and terminated into multistate voltage sinks. A multistate voltage source is an equivalent source which can be synthesized by connecting a dc source with a network of switches. A multistate sink is an equivalent dependent source representing the output circuit consisting of rectifiers, filters and the resistive loads. A current sink is used to represent the output circuit with an inductive filter, whereas a voltage sink is used for a capacitive filter. The multistate sinks are connected across the secondary windings of a single multi-winding transformer. In our analysis, we use the low frequency model for the multi-winding transformer proposed previously [9].

One of the important considerations in designing a multi-output converter is the cross coupling between outputs. In our model, we define the converter gain of the i-th output, $i = 1, 2 ....$, as the output voltage in the i-th winding normalized to the source voltage. The normalized output load, Q_i, in the i-th output is defined as the load resistance in the output divided by the characteristic impedance Z_0, of the tank circuit. The converter gain in i-th output, M_i, can be expressed in the functional form as follows :

$$M_i = f_i(r, M_j, Q_j \; ; j = 1, 2,...) \tag{1}$$

where r is the control variable such as the normalized switching frequency f_{ns}, the pulse width or the phase difference in the transistor drive waveforms.

In most practical cases, only one of the outputs is regulated by feedback control and all other outputs are either unregulated or post-regulated. The regulated output in general delivers the bulk of the total rated power of the multi-output converter. The feedback control removes r as an independent variables in eq. (1) which can be rewritten as follows ;

$$M_i = f_i(M_j, Q_j \; ; j = 1, 2, ...) \tag{2}$$

The percentage change in M_i due to variations in Q_i is termed the self regulation, and the percentage change in M_i due to variation in Q_j and M_j for $j \neq i$ is called the cross regulation. Minimizing the cross regulation effect is an important design consideration.

We will illustrate our modelling and the method of analysis for a two-output lossy SRC for which the equivalent circuit is given in Fig. 1. All the quantities and the variables

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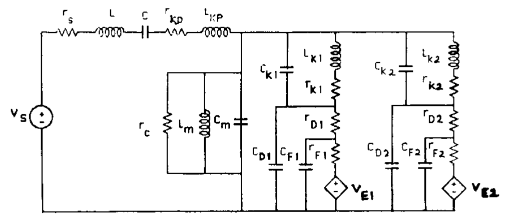

Fig. 1 Equivalent circuit of a Two-output SRC with all parasitics.

are referred to the primary side of the transformer. In the converter model of Fig. 1, we assume that the switches are driven by a square wave of frequency f_s with 50% duty ratio. The input source is modeled by a two-state voltage source V_S ($+V_g$, $-V_g$). The output circuit with the capacitive filter is modeled by a 3-state voltage sink V_{Ei}, as given below,

$$V_{Ei} = \frac{i_{oi}}{|i_{oi}|} V_{oi} \qquad i = 1,2 \qquad i_{oi} \neq 0$$

$$= open \quad (very\ high\ impedence) \qquad v_{si} < V_{oi} \tag{3}$$

where V_{oi} and i_{oi} are indicated in Fig. 1. v_{si} is the voltage across the secondary of the output transformer. The ON resistance of the switching transistor is represented by r_s. Although the parasitic elements in the transformer are actually distributed, they can be approximated by the lumped components. L_m and r_c which are respectively the magnetizing inductance and a hypothetical resistance depicting the eddy and hysteresis losses. The resistances r_{kp} and r_{kj} ; $j = 1,2,...$ are the winding resistances in the primary and the secondary windings. The resistors r_{Dj} ; $j = 1,2,...$ are the forward resistances of the diodes in the rectifier networks. The resistances r_{Fj} ; $j = 1,2,...$ are the ESRs of the capacitive output filters.

In the well designed transformer, L_m is much higher than the leakage inductances and hence it can be omitted from the equivalent circuit. In general, the core losses represented by r_c can also be ignored. The simplified equivalent circuit is shown in Fig. 2 . The resistance r in series with the resonant inductor L is the equivalent sum of r_s and r_{kp}. The resistance r_j in the j-th output circuit is the equivalent sum of r_{kj}, r_{Dj} and r_{Fj}.

The state-plane analysis of the combined effects of leakage inductances and parasitic resistances on the cross regulation is feasible, but even after much simplification the problem is still a difficult one. Hence we will analyze these two effects separately and present the comparison of the performance characteristics. The effect of leakage inductances in SRC has been analyzed previously in [10]. In the next section, we will present the analysis of a two output SRC with resistances included.

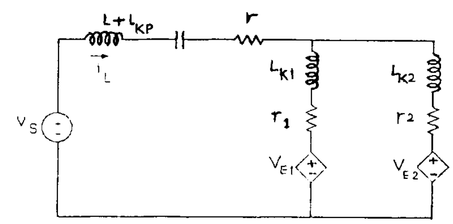

Fig. 2 Simplified equivalent circuit

III EXAMPLE OF A TWO-OUTPUT LOSSY SRC

A state-plane technique has been developed for the analysis of the lossy resonant converters in our laboratory. Using this technique, we can determine the cross regulation and control characteristics in SRC when the resistances are incorporated in the equivalent circuit. We will use the equivalent circuit of Fig. 2 minus the leakage inductances. For this example, we will also assume that the resistance r_s, which represents the ON resistance of the switches and r_{kp}, is quite small compared to r_{k1} and r_{k2} in output circuits.

3.1 Converter Operation in CCM

The converter operation in Continuous Conduction Mode over a half switching period can be described by the mode sequence, $m_1 \rightarrow m_2 \rightarrow m_3$ as shown in Fig. 3. In circuit mode m_1, The source V_S is in the V_g state. The two

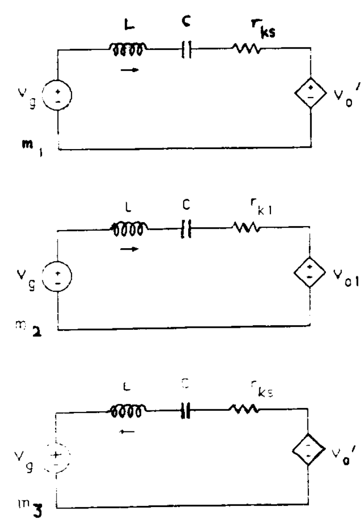

Fig. 3 Circuit modes over a half switching period

voltage sinks V_{o1} and V_{o2} and the resistances r_{k1} and r_{k2} of Fig. 2, are replaced by an equivalent voltage sink V_o' in series with an equivalent resistance r_{ks}, as given by

$$V_o' = V_{o1}\frac{r_{k1}}{r_{k1}+r_{k2}} + V_{o2}\frac{r_{k2}}{r_{k1}+r_{k2}} \tag{4}$$

where,

$$r_{ks} = \frac{r_{k1}r_{k2}}{r_{k1}+r_{k2}} \tag{5}$$

Circuit mode m_1 changes to circuit mode m_2 when output current i_{o2} becomes zero due to the following condition,

$$V_{o1} + r_{k1}i_{o1} - r_{k2}i_{o2} \le V_{o2} \tag{6}$$

Output current i_{o2} remains zero until circuit mode m_2 changes to circuit mode m_3 when i_L becomes zero. In circuit mode m_3, both output currents are negative and the circuit mode ends when source V_S is switched to $-V_g$ state.

3.2 State-plane Analysis

A 2nd order i-th circuit mode in the lossy SRC is transformed into an equivalent lossless circuit mode resonating at a new resonant frequency

$$\omega_{0i}' = \omega_0\sqrt{1-\alpha_i^2} \tag{7}$$

where,

$$\omega_0 = \sqrt{\frac{1}{LC}} \;;\; \alpha_i = \frac{r_i}{2Z_0} \;;\; Z_0 = \sqrt{\frac{L}{C}}$$

The r_i is the equivalent resistance in the i-th circuit mode.

The trajectory of the equivalent lossless circuit mode is a circular. The arc subtends a conduction angle given by

$$\text{Conduction angle} = \omega_{0i}'(t - t_0) \tag{8}$$

where t_0 is the instant at which the circuit mode starts.

The trajectory starts from the initial point of the lossy SRC on the state-plane. At the end point of the trajectory for the circuit mode under consideration, the corrections are made in the terminal values of the state variables to obtain the actual initial point for the following circuit mode.

The complete normalized state-plane trajectory, shown in Fig. 4, represents the steady state response of the converter over a half switching period. The normalized average current in output 2 can be calculated from the state-plane diagram as follows;

$$I_{no2} = \frac{M_2}{Q_2} = \frac{2}{T_s}\left[\int_{t_0}^{\tau}|i_{no2}(t)|\,dt + \int_{t_1}^{t_0+T_s/2}|i_{no2}(t)|\,dt\right] \tag{9}$$

where i_{no2} is the current in output 2, normalized to V_g/Z_0. The normalized average current from the source can be derived from

$$I_{ng} = \frac{2}{T_S}\left[\int_{t_0}^{\tau}|i_{nL}(t)|\,dt + \int_{\tau}^{t_1}|i_{nL}(t)|\,dt + \int_{t_1}^{t_0+T_s/2}|i_{nL}(t)|\,dt\right] \tag{10}$$

where i_{nL} is the current in resonant inductor, normalized to V_g/Z_0. These currents will be used to derive the cross regulation characteristics in section 3.2.

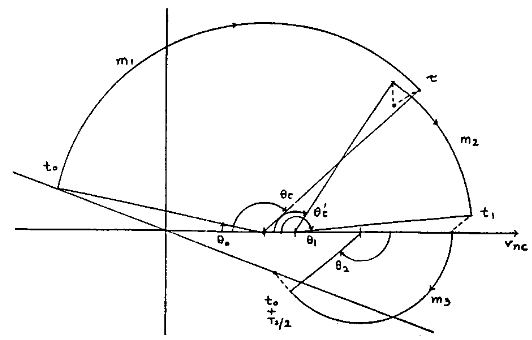

Fig. 4 State-plane Trajectory over a half switching period.

3.3 Performance Characteristics in CCM

By equating the power supplied by the source to the power delivered to the load and the resistive losses, we obtain

$$I_{ng} = \frac{M_1^2}{Q_1}\left[1 + 2\frac{\alpha_1}{Q_1}(A+1)\right] + \frac{M_2^2}{Q_2}\left[1 + 2\frac{\alpha_1}{Q_2}(\frac{A+1}{A})\right] \tag{11}$$

where,

$$\alpha_1 = \frac{r_{ks}}{2Z_0} \;;\; A = \frac{r_{k1}}{r_{k2}}$$

Using above equation and the results of the steady state analysis, the converter gain M_2 and the converter switching frequency normalized to the resonant frequency, f_{ns}, are determined as functions of the normalized load resistances Q_1, Q_2 and the converter gain M_1. The M_2 versus Q_1 curves are referred to as the cross regulation characteristics. The f_{ns} versus Q_1 curves are called the control characteristics.

3.3.1 Cross Regulation Characteristics

The percentage cross regulation of output 2 can be calculated from

$$X = \frac{\Delta M_{2x}}{M_2}\, x\, 100\% \tag{12}$$

where,

$$\Delta M_{2x} = \left[\frac{\partial M_2}{\partial Q_1}\Delta Q_1 + \frac{\partial M_2}{\partial M_1}\Delta M_1 \right]$$

The term $\frac{\partial M_2}{\partial Q_1}$ is the slope on the cross regulation characteristic curve. The term $\frac{\partial M_2}{\partial M_1}$ is the spread in the characteristic curves plotted with M_1 as parameter. Ideal cross regulation characteristic curves will have zero slope and zero spread.

The cross regulation characteristics curves in this paper are plotted as M_2/M_1 versus Q_1 with $A = r_{k1}/r_{k2}$, Q_2 and M_1 as parameters shown respectively in Fig. 5(a), 5(b) and 5(c).

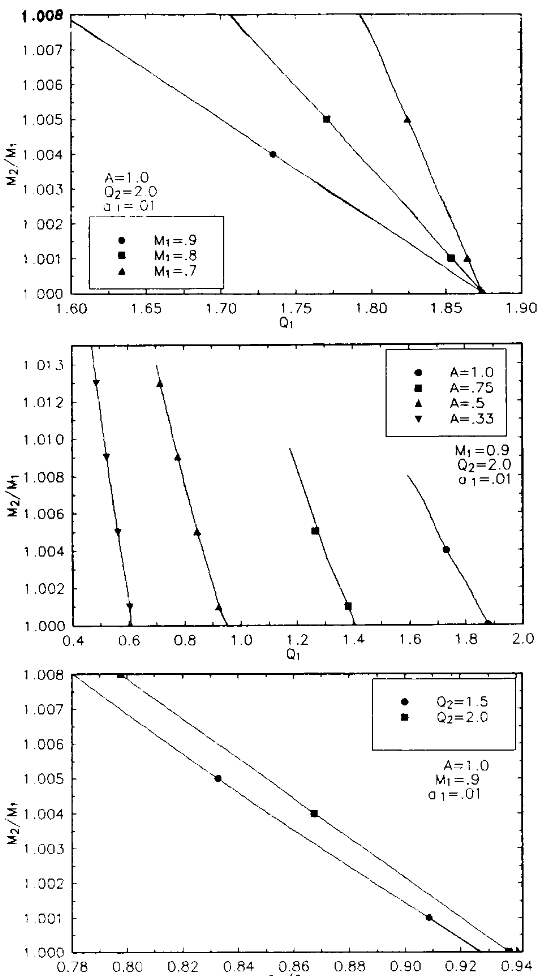

Fig. 5 (a) Cross regulation characteristics of lossy SRC in CCM with parameters (a) M_1 (b) $A = r_{k1}/r_{k2}$ (c)Q_2

The parameter of variation in the curves of Fig. 5(a) is the converter gain M_1 which is the ratio of output voltage V_{o1} to the input voltage V_g. The variation in M_1 may be either due to the different setting of voltage on output 1 or variation in the input line voltage. A different setting of voltage on output 1 may also be due to the different turn ratios of the output transformer. The curves for higher values of M_1 have smaller slopes and consequently the lower cross regulation. The higher values of M_1 means high transformer turn ratios required in the low output voltage supplies. The transformer with high turn ratios do not have high bandwidth. Therefore, the converter can not be operated effectively at high switching frequencies. However, at $M_1 = .9$, the change in converter gain M_2 due to a change in Q_1 from 1.9 to 1.6 is less than 1% which is much less than 60-70% obtained in the analysis of the effect of leakage inductances [10].

The parameter of variation in the curves of Fig. 5(b) is the ratio of parasitic resistances in the two output circuits. A small value of parameter A means that the resistance in output 2 is higher than that in output 1 which in practice means the rectifiers in output 2 have low current carrying capacity than in output 1. The slope of the curves and hence the amount of cross regulation in output 2 is higher for small values of A. Hence for achieving low cross regulation the unregulated outputs should use high current rectifiers.

The converter operation in CCM does not permit wide range of variation in Q_1. This range becomes smaller at small values of ratio A. The average Q_1 also shifts to lower values at small A.

Fig. 5(c) shows the cross regulation characteristic curves with the normalized load resistance, Q_2, in output 2 as a parameter. As Q_2 increases, which means a lower current in output 2, the ratio M_2/M_1 increases. The percentile cross regulation or the slope of the curves do not change appreciably. Hence the conclusion is that the cross regulation in output 2 is at its worst when it has a light load (low output current in output 2). It is not possible to vary Q_2 over a large range to sustain the converter operation in CCM.

3.3.2 Control Characteristics

The second output in the converter has a significant effect on the control characteristics of the main output (output 1) which is regulated by feedback. The control characteristics of the converter are plotted as the normalized switching frequency f_{ns} versus normalized load resistance Q_1. In Fig. 6(a), the curves are plotted with M_1 as the parameter. Lower switching frequencies are required for operation at small values of M_1, which became necessary to minimize the percentile cross regulation in output The control is easier at low switching frequencies.

Fig. 6(b) shows the control characteristic curves with the ratio of parasitic resistances in the output circuits ($A = r_{k1}/r_{k2}$) as the parameter. The switching frequencies varies because the average value of Q_1 varies with different values of A.

Fig. 6(c) shows the control characteristic curves with Q_2 as a parameter. The switching frequencies vary significantly for different values of Q_2. Higher switching frequencies are required when Q_2 is decreased (the current is increased on output 2).

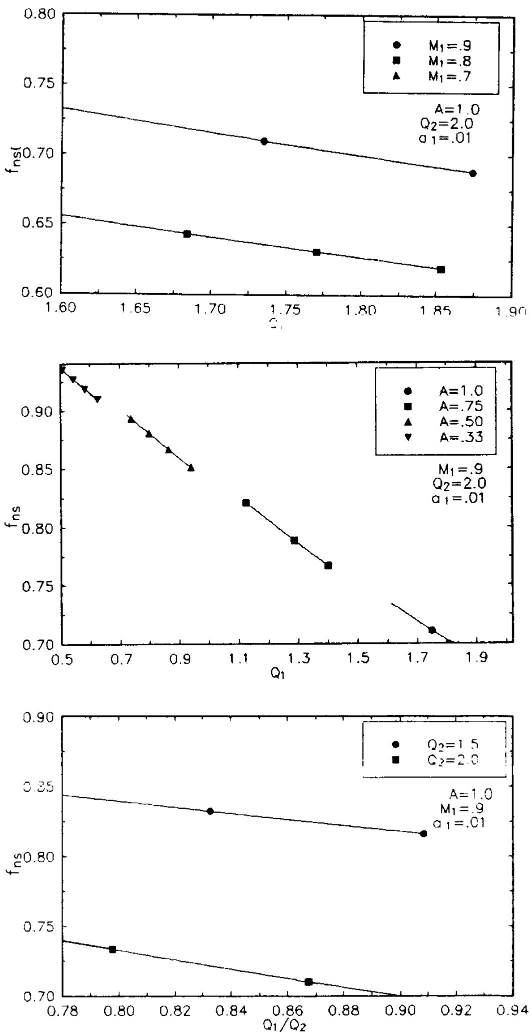

Fig. 6 Control characteristics of lossy SRC in CCM with parameters (a) M_1 (b) $A = r_{k1}/r_{k2}$ (c)Q_2

IV. THE EFFECTS OF RESISTANCES VERSUS LEAKAGE INDUCTANCES

The cross regulation characteristic curves for parasitic resistances in Fig. 5 (a) to (c) are compared with those for the leakage inductances [10] shown in Fig. 7 (a) to (c). We observe that the cross regulation (the slope of the curves) is higher in case of leakage inductances than in the case of parasitic resistances. Although the range of variation in Q_1 to sustain the operation in CCM is smaller in resistance case, the converter gain M_2 varies by less than 1%. This is far smaller than in leakage inductance case. Therefore, it may be concluded that in practice, the cross regulation is predominantly leakage inductances than the parasitic resistances.

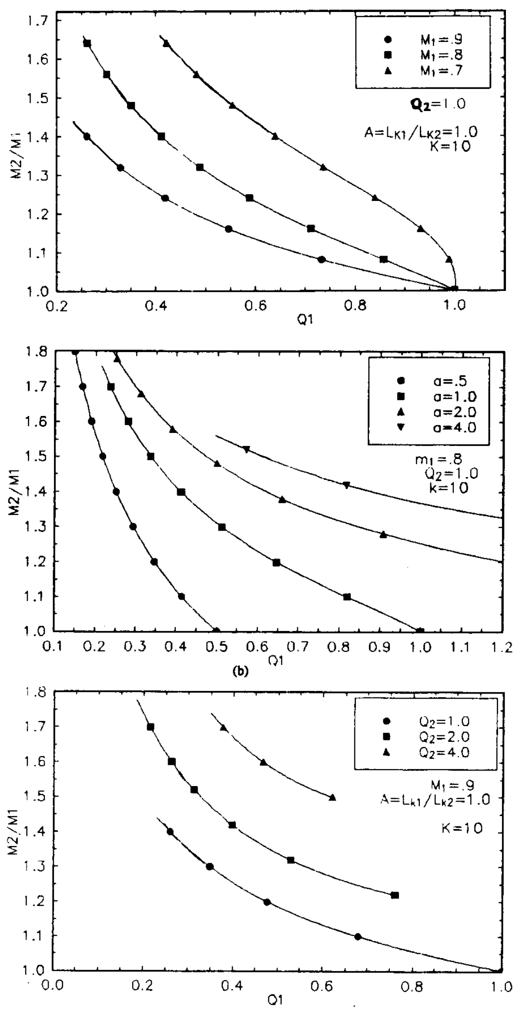

Fig. 7 (a) Cross regulation characteristics of lossless SRC in CCM with parameters (a) M_1 (b) $a = L_{k1}/L_{k2}$ (c)Q_2

V. SIMULATION RESULTS

An operating point was selected from the performance characteristic curves given in Fig. 5(a) and 6(a), which is defined by

$$Q_2 = 2.00 \ ; \ M_1 = .90 \ ; \ A = 1.00 \ ; \ \alpha_1 = .01 \ ;$$

$$Q_1 = 1.73 \ ; \ f_{ns} = .70$$

The component values for the simulation circuit were calculated using the above operating point and are given below

$$L = 6.70\ \mu H, \ C = 30\ nF, \ r_{k1} = .60\ ohms, \ r_{k2} = .60\ ohms,,$$

$$V_g = 75\ v, \ R_{L1} = 25.85\ ohms, \ R_{L2} = 29.89\ ohms, \ T_S = 4.0\ \mu S$$

The values of r_{k1} and r_{k2} may be the ON resistance of a full bridge rectifier circuit in 5-volt output using 10 amp rectifier diodes, referred to the primary side of the transformer.

Voltages on output 1 and output 2 were measured to be 63.8 and 64.0 volts respectively. We use full bridge rectifier circuits in the output circuits hence 1.6 volts were added to the above output voltages in order to find out the converter gains. The experimental and theoretical values are given below

$$\text{Simulation}: M_1 = .877 \ ; \ M_2 = .088 \ ; \ \frac{M_2}{M_1} = 1.0034$$

$$\text{Theoretical}: M_1 = .900 \ ; \ M_2 = .904 \ ; \ \frac{M_2}{M_1} = 1.004$$

Fig. 8 shows the simulation results of the dc voltages on two outputs and the current waveforms of i_{o2}, the current in output 2 and and i_L the current in the resonant inductor which is also the current supplied by the input voltage source. The current waveforms clearly shows that the converter is operating in CCM which was intended. The simulation results confirm the theoretical results completely.

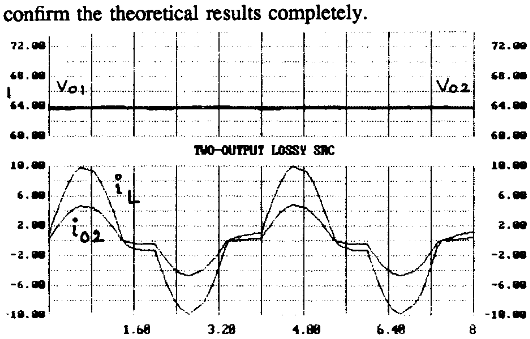

Fig. 8 Simulation of lossy SRC.

VI. CONCLUSIONS

From the performance characteristic curves of lossy SRC, it can be concluded that the effect of parasitic resistances on the cross regulation in the unregulated outputs is small. We may conclude that the leakage inductances in the transformers are the dominant sources of cross regulation found in practice.

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