# EFFECTS OF OUTPUT FILTER ON CROSS REGULATION IN SRC

Jai P. Agrawal and C. Q. Lee

Department of Electrical Engineering and Computer Science
University of Illinois at Chicago, P.O.Box 4348,
Chicago, IL 60680, Tel : (312)996-2664 .

## ABSTRACT

This paper presents the analysis of the cross regulation performance of multi-output SRC topologies using multi-winding transformers . The steady state cross regulation characteristics are derived using the state-plane techniques, illustrated by the examples of two-output SRC with capacitive and inductive filters. The characteristic curves show that over some range of output currents, the cross regulation performance of SRC with inductive filter is much improved over the SRC with capacitive filter and less dependent on the tolerances in the leakage inductances and the level ofloading on the unregulated outputs. The converter control characteristics of SRC with inductive filter are relatively unaffected by the addition of the second output. The theoretical results are verified by simulation.

## I. INTRODUCTION

Most instrument power supplies have more than one outputs. However , often only one of these outputs is regulated by feedback control, while the others are either unregulated or post regulated. The percentage change in one of the output voltages due to the load variations in other outputs, is known as the cross regulation problem. This problem may severely limit the controllibility range of each output of the converter.

Most multi-output converters use transformers with multiple secondary windings to drive their load resistances. The steady state cross regulation in these converters depend on the leakage inductances, the core and the copper losses in the transformer and unequal loading on the outputs.

In this paper, we present our analysis of cross regulation in multi-output series resonant converters and, as examples, give a comparision between the steady state cross regulation characteristics of two-output SRC with capacitive and inductive filters. The theoretical results are verified by computer simulation.

We use state-plane method of analysis from which we can easily derive the cross regulation characteristics. It also provides good physical insight to the effect of multi-output power conversion on the converter operation and the self regulation characteristics.

## II. MULTI-OUTPUT SRC MODEL

In general, a multi-output SRC can be represented by the model which consists of a resonant tank driven by a multistate voltage or current source and terminated into multistate voltage or current sink. A multistate source is an equivalent source which can be synthesized by connecting a dc source with a network of switches. A multistate sink is an equivalent dependent source representing the output circuit consisting of rectifiers, filters and the resistive loads. A current sink is used to represent the output circuit with an inductive filter, and a voltage sink is used for a capacitive filter. The sinks are connected across the secondary windings of a single multi-winding transformer. We use the low frequency model for the multi-winding transformer proposed previously [1]. The leakage inductance in the transformer primary can be included in the resonant inductor L of the tank circuit.

One of the important consideration in designing a multi-output converter is the cross coupling between outputs. In our model, we define the converter gain of i-th output, $i = 1, 2 ....$ as the output voltage in the i-th winding normalized to the source voltage. The i-th normalized output load, $Q_i$, is the load resistance in the i-th output normalized to the characteristic impedence,$Z_0$, of the tank circuit. The converter gain in i-th output, $M_i$, can be expressed in the functional form as follows :

$$M_i = f_i(r, M_j, Q_j \ ; \ j = 1, 2, ...) \qquad (1)$$

where r is the control variable such as the normalized switching frequency $f_{ns}$, the pulse width or the phase difference in the switched waveforms.

In practice, only one of the outputs is regulated by feedback control and all other outputs are either unregulated or post-regulated. The regulated output delivers the bulk of the total rated power of the multi-output converter. The feedback control removes r as the independent variables from which eq. (1) can be modified as follows ;

$$M_i = f_i(M_j, Q_j \ ; \ j = 1, 2, ...) \qquad (2)$$

A change in $M_i$ due to variations in $Q_i$ is termed the self regulation, and the change in $M_i$ due to $Q_j$ and $M_j$ for $j \neq i$ is called the cross regulation. Minimizing the cross regulation effect is an important design consideration because it may degrade the converter performance.

In this paper, we will analyze the half bridge two-output SRC topologies using capacitive and inductive filters. In the converter models for SRC, we assume that the switches are driven by a square wave of frequency $f_s$ with 50% duty ratio. The input source is modelled by a two-state voltage source $V_S$ $(+V_g, -V_g)$. Each output circuit consists of the rectifier network, the output filter and the load resistance.

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## III. STEADY STATE PERFORMANCE OF SRC WITH CAPACITIVE FILTER

In this section, we consider the steady state performance of SRC with two capacitive filter outputs, operating in the Continuous Conduction Mode (CCM). Consider the circuit of a two-output SRC with capacitive filter, shown in Fig. 1 and its equivalent circuit in Fig. 2. It has been assumed that the output $V_{o1}$ is feedback regulated and its average output current $I_{o1}$ is higher than the average output current $I_{o2}$ of the other output.

The output circuit with the capacitive filter is modeled by a 3-state voltage sink $V_{Ei}$, as given below,

$$V_{Ei} = \frac{i_{oi}}{|i_{oi}|} V_{oi} \qquad i = 1,2 \qquad i_{oi} \neq 0$$

$$= open \quad (very\ high\ impedence) \qquad v_{si} < V_{oi} \tag{3}$$

where $V_{oi}$ and $i_{oi}$ are indicated in Fig. 2. $v_{si}$ is the voltage across the secondary of the output transformer.

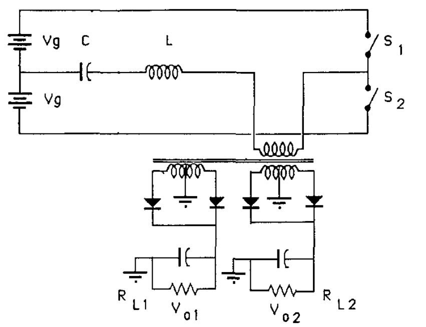


Fig. 1 Two-output SRC with capacitive filter (SRC-CF)

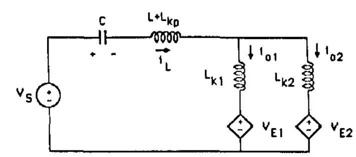


Fig. 2 Equivalent circuit of SRC-CF

### 3.1 Converter Operation

The converter operation in steady state is symmetrical in the two halves of a switching period which correspond to the two states of voltage source $V_S$ ( $+V_g$ and $-V_g$) in a switching period. Each half can be described by a mode sequence, $m_1 \rightarrow m_2 \rightarrow m_3$ as shown in Fig. 3.

In circuit mode $m_1$, the voltage sinks $V_{o1}$ and $V_{o2}$ and the leakage inductances $L_{k1}$ and $L_{k2}$ of Fig. 2 are replaced by an equivalent voltage sink $V_o'$ in series with an equivalent inductance $L_{ks}$, as given by,

$$V_o' = V_{o1}\frac{L_{k1}}{L_{k1}+L_{k2}} + V_{o2}\frac{L_{k2}}{L_{k1}+L_{k2}} \tag{4}$$

where,

$$L_{ks} = \frac{L_{k1}L_{k2}}{L_{k1}+L_{k2}} \tag{5}$$

Circuit mode $m_1$ changes to circuit mode $m_2$ when output current $i_{o2}$ becomes zero due to the following condition,

$$V_{o1} + L_{k1}\frac{di_{o1}}{dt} - L_{k2}\frac{di_{o2}}{dt} \leq V_{o2} \tag{6}$$

Output current $i_{o2}$ remains zero until circuit mode $m_2$ changes to circuit mode $m_3$ when $i_L$ becomes zero. In circuit mode $m_3$, both output currents are negative and the circuit mode ends when source $V_S$ is switched to $-V_g$ state. The normalized state-plane trajectory, shown in Fig. 4, represents the steady state response of the converter. The current waveforms for $i_L$, $i_{o1}$ and $i_{o2}$ can be deduced from the state-plane diagram and are shown in Fig. 5.

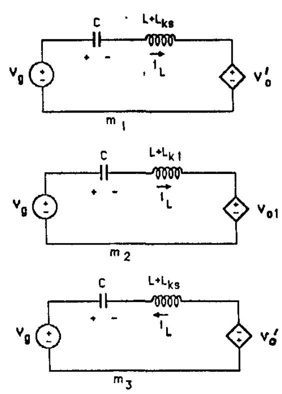


Fig. 3 Circuit modes over a half switching period

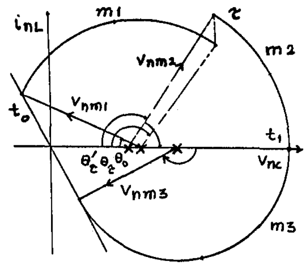


Fig. 4 State-plane Trajectory

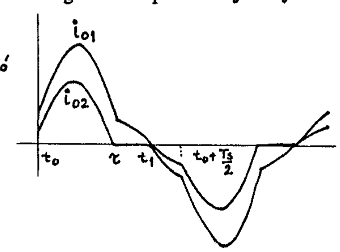


Fig. 5 Current waveforms

### 3.2 Steady State Analysis

In the following steady state analysis, the voltages and currents are normalized by $V_g$ and $V_g/Z_{0i}$, respectively, where $Z_{0i}$ is the characteristic impedence of circuit mode $m_i$ under consideration. We define normalized loads $Q_1$ and $Q_2$ for load resistances $R_{L1}$ and $R_{L2}$, normalized to $Z_{01}$ which is the characteristic impedence of circuit mode $m_1$.

The normalized average current in output 2 is calculated from the state-plane diagram as follows;

$$I_{no2} = \frac{M_2}{Q_2} = \frac{2}{T_s}\left[\int_{t_0}^{\tau} |i_{no2}(t)|\,dt + \int_{t_1}^{t_0+T_s/2} |i_{no2}(t)|\,dt\right] \tag{7}$$

The normalized average current from the source can be derived from

$$I_{ng} = \frac{2}{T_S}\left[\int_{t_0}^{\tau} |i_{nL}(t)|\,dt + \int_{\tau}^{t_1} |i_{nL}(t)|\,dt + \int_{t_1}^{t_0+T_s/2} |i_{nL}(t)|\,dt\right] \tag{8}$$

These currents will be used to derive the cross regulation characteristics in section 3.3.

### 3.3 Performance Characteristics

Based on the results from the steady-state analysis and the power conservation principles, the performance characteristics can be determined. The performance characteristics can be categorized into the cross regulation characteristics and the control characteristics.

### 3.3.1 Cross Regulation Characteristics

By equating the power supplied by the source to the power delivered to the load, we obtain

$$I_{ng} = \frac{M_1^2}{Q_1} + \frac{M_2^2}{Q_2} \tag{9}$$

Using above equation and the results of the steady state analysis, the converter gain $M_2$ is determined as a function of normalized load resistances $Q_1$, $Q_2$ and converter gain $M_1$. The cross regulation characteristics curves are plotted as $M_2/M_1$ versus $Q_1$ with $M_1$, $L_{k1}/L_{k2}$ and $Q_2$ as parameters. The percentage cross regulation of output 2 can be calculated from

$$X = \frac{\Delta M_{2x}}{M_2} x 100\% \tag{10}$$

where,

$$\Delta M_{2x} = \left[ \frac{\partial M_2}{\partial Q_1} \Delta Q_1 + \frac{\partial M_2}{\partial M_1} \Delta M_1 \right]$$

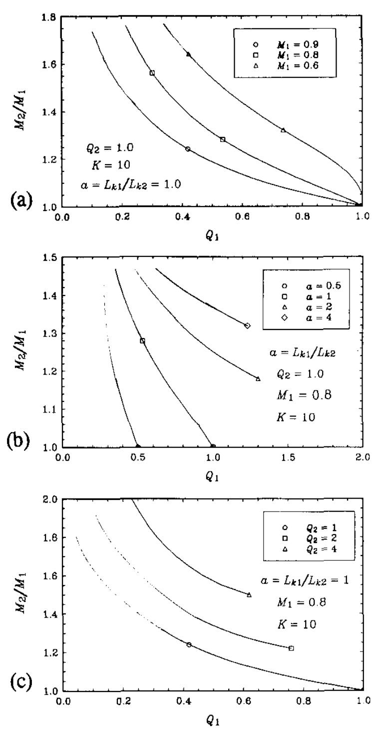


Fig. 6 Cross regulation characteristics of SRC-CF with (a) $M_1$ as the parameter, (b) $L_{k1}/L_{k2}$ as the parameter, and (c) $Q_2$ as the parameter.

The cross regulation characteristics can be plotted as $M_2$ versus $Q_1$ with $M_1$ as parameter. According to this formula, for the ideal cross regulation performance the characteristic curves must have zero slopes and zero spread with respect to the parameter $M_1$.

The cross regulation curves for SRC with capacitive filter are shown in Fig. 6 (a), (b) and (c). Fig. 6 (a) provides the effect of variation of converter gain $M_1$ on the cross regulation performance. Since the output voltage in output 1 is regulated, the variation in $M_1$ gives the effect of change in the line voltage of the power supply. From Fig. 6 (a), we observe from the slopes of these curves that the percentage cross regulation in output 2 is smaller at higher values of converter gain $M_1$. Hence for low percentile cross regulation, the transformer must have higher turn ratios. The transformer with high turn ratios do not have high bandwidth. Therefore, the converter can not be operated at high switching frequencies.

Fig. 6 (b) provides the cross regulation curves with the ratio of leakage inductances ($a = L_{k1}/L_{k2}$) as a parameter. These curves provide the effect of manufacturing tolerances in the leakage inductances of the transformer. The slope of the characteristic curves is smaller for higher values of a, which means that either $L_{k1}$ must have a large value or the value of $L_{k2}$ must be small. In any case, the cross regulation is highly dependent on the leakage inductances and ,therefore, the manufacturing tolerances for the transformers are tight.

Fig. 6 (c) shows the cross regulation characteristic curves with the normalized load resistance in the output 2, $Q_2$, as a parameter. As $Q_2$ increases, which means a lower current in output 2, the ratio $M_2/M_1$ increases. The percentile cross regulation or the slope of the curves do not change appreciably. Hence the conclusion is that the cross regulation in output 2 is at its worst when it has a light load (low output current in output 2). Furthermore, the range of variation of load resistance $Q_2$ must be small in order to avoid large changes in converter gain $M_2$, which means that the post regulation will be more difficult.

### 3.3.2 Control Characteristics

The second output in the converter has a significant effect on the control characteristics of the main output (output 1 ) which is regulated by feedback. The contol characteristics of the converter are plotted as the normalized switching frequency $f_{ns}$ versus normalized load resistance $Q_1$, as shown in Fig. 7 (a), (b) and (c). In Fig. 7 (a), the curves, $f_{ns}$ vs. $Q_1$ are plotted with $M_1$ as the parameter. In the previous section, it was shown that the cross regulation performance is better at high values of $M_1$. However, the converter operation at high values of $M_1$ requires higher switching frequencies and the transformer with a higher turn ratio. The transformer with a high turn and high bandwidth is difficult to design.

Fig. 7 (b) shows the control characteristic curves with the ratio of leakage inductances ($a = L_{k1}/L_{k2}$) as the parameter. Again, the switching frequencies vary widely for different values of a. Furthermore, it is evident from these curves that the minimum current in output 1 (the highest value of load resistance $Q_1$) required to maintain the Converter operation in CCM increases significantly for the small values of $L_{k1}/L_{k2}$.

Fig. 7 (c) shows the control characteristic curves with $Q_2$ as the parameter. It can be seen that the switching frequency varies significantly for different values of $Q_2$. Higher switching frequencies are required when $Q_2$ is decreased (the current is increased on output 2). Furthermore, it is evident from these curves that the minimum current in output 1 (the highest value of load resistance $Q_1$ ) required to maintain the Converter operation in CCM increases significantly for the small values of $Q_2$. In other words, the ratios of load resistances in the two outputs $Q_1/Q_2$ has to be much smaller as the second output is operated at smaller currents. For example, at $Q_2 = 1.0$ the ratio $Q_1/Q_2 = 1.0$ whereas at $Q_2 = 4.0$ the ratio $Q_1/Q_2$ drops to 0.15.

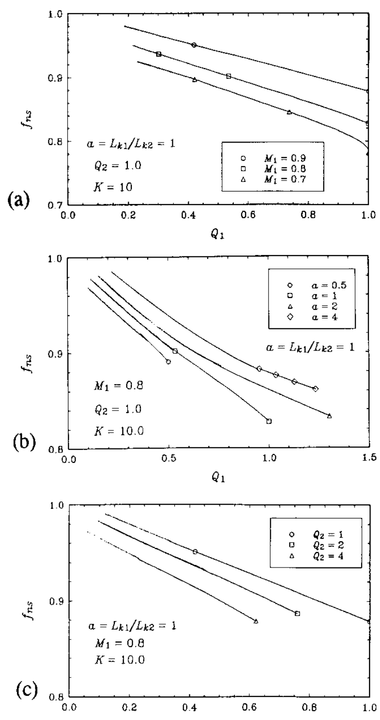


Fig. 7 Control characteristics of SRC-CF with (a) $M_1$ as the parameter, (b) $L_{k1}/L_{k2}$ as the parameter, and (c) $Q_2$ as the parameter.

## IV. STEADY STATE PERFORMANCE OF SRC WITH INDUCTIVE FILTER

In this section, we consider the steady state performance of SRC with two inductive filter outputs, operating in the Continuous Conduction Mode (CCM). The equivalent circuit of the converter is shown in Fig. 8. It has been assumed that the output $V_{o1}$ is feedback regulated and its output current $I_{o1}$ is higher than the output current $I_{o2}$ of the other output.

The output circuit with the inductive filter is modeled by a 3-state voltage sink $I_{Ei}$ as given below,

$$I_{Ei} = \frac{v_{oi}}{|v_{oi}|} I_{oi} \qquad i = 1,2 \qquad v_{oi} \neq 0$$

$$= short \quad (very\ low\ impedence) \qquad i_{si} < I_{oi} \qquad (11)$$

where $I_{oi}$ and $v_{oi}$ are indicated in Fig. 8. $i_{si}$ is the current in the secondary of the output transformer.

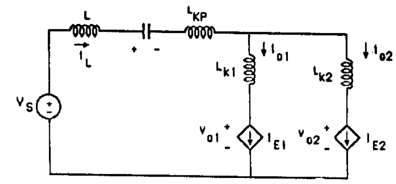


Fig. 8 Two-output SRC with inductive filter (SRC-IF)

### 4.1 Converter Operation

The converter operation over a half switching period can be described by a mode sequence, $m_1 \rightarrow m_2 \rightarrow m_3 \rightarrow m_4$, as shown in Fig. 9. In circuit mode $m_1$, current $i_L$ is clamped at $(I_{o1} + I_{o2})$. Since the currents in $L_{k1}$ and $L_{k2}$ are constant and equal to $+I_{o1}$ and $+I_{o2}$, respectively , hence the voltages on $L_{k1}$ and $L_{k2}$ and on L are all zero. The resonant capacitor is charged from $-V_g$ to $+V_g$ by current $( I_{o1} + I_{o2} )$. Circuit mode $m_1$ ends and $m_2$ begins when the capacitor voltages reaches $+V_g$ and $v_{o1}$ and $v_{o2}$ reverse their polarities. With both current sinks switched to "short" state, the currents in $L_{k1}$ and $L_{k2}$ are unclamped . In circuit mode $m_3$, the current in $L_{k2}$ is clamped at $-I_{o2}$ while the current in $L_{k1}$ is still varying. This mode ends and circuit mode $m_4$ begins when the current in $L_{k1}$ is clamped at $-I_{o1}$. Circuit mode $m_4$ ends when source $V_S$ is switched to $-V_g$.

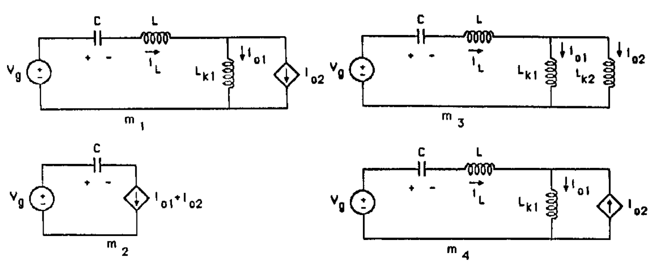


Fig. 9 Circuit modes over a half switching period

#### 4.1.1 Comparision of Converter Operation in SRC-CF and SRC-IF

Unlike in SRC-CF, the currents in the leakage inductances in the two secondaries, and hence the leakage fluxes, in SRC-IF are constant over most of the switching period. They vary over a small fraction of the switching period when the current sinks are going through the change of state. Hence, the average differential rate of change of leakage fluxes, which is the cause of cross regulation in output 2, is much smaller in SRC-IF than in SRC-CF. Thus, the leakage inductances have little effect on the cross regulation performance of SRC with inductive filter.

### 4.2 Steady State Analysis

The steady state response of the converter is represented by the state-plane diagram given in Fig. 10 (a). The voltage waveforms for $v_{o1}$ and $v_{o2}$ can be deduced from this diagram and are shown in Fig.10 (b).

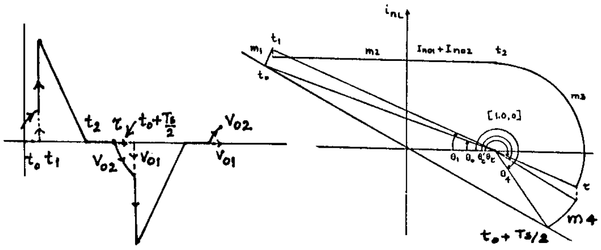


Fig. 10 (b) Voltage waveforms Fig. 10 (a) Steady state trajectory

The normalized average voltage on current sink $I_{E2}$ can be calculated from,

$$M_2 = \frac{2}{T_S}\left[\int_{t_0}^{t_1} |v_{no2}(t)|\,dt + \int_{\tau}^{t_2} |v_{no2}(t)|\,dt + \int_{t_2}^{t_0+T_S/2} |v_{no2}(t)|\,dt\right] \quad (12)$$

and the normalized average current from the source can be determined from,

$$I_{ng} = \frac{2}{T_S}\left[\int_{t_0}^{t_1}(I_{no1}+I_{no2})dt + \int_{t_1}^{\tau}|i_{nL}(t)|\,dt + \int_{\tau}^{t_2}|i_{nL}(t)|\,dt - \int_{t_2}^{t_0+T_S/2}(I_{no1}+I_{no2})dt\right] \quad (13)$$

Eqn. (12) and (13) are used for deriving the cross regulation characteristics as will be shown in the next section.

### 4.3 Performance Characteristics

Using the results from the steady-state analysis and the power conservation principles, we can determine the performance characteristics of the converter which can be categorized into the cross regulation characteristics and the control characteristics.

#### 4.3.1 Cross Regulation Characteristics

The converter gain $M_2$ can be determined as a function of normalized load resistances $Q_1$, $Q_2$ and converter gain $M_1$. The cross regulation characteristics curves are plotted as $M_2/M_1$ versus $Q_1$ with $M_1$, $L_{k1}/L_{k2}$ and $Q_2$ as parameters.

The cross regulation curves for SRC with inductive filter are shown in Fig. 11 (a), (b) and (c). From Fig. 11 (a) we see that the slope of the curves and consequently the percentage cross regulation in output 2 is smaller at lower values of converter gain $M_1$. Hence for low percentile cross regulation, the transformer must have a smaller turn ratio. Transformer with a low turn ratio has high bandwidth. Therefore, the converter can be operated at high switching frequencies. The slope of the curves do not differ appreciably for different values of $M_1$. The highest value of load resistance $Q_1$ for which the converter operation in CCM is possible, varies widely at different values of $M_1$. According to the curve˜rs given in Fig. 11 (a), the curve with $M_1 = 0.4$ is the only curve with low slope, hence the choice of converter gain $m_1$ for the small percentile cross regulation is very restric˜ed.

Fig. 11(b) shows the cross regulation curves with the ratio of leakage inductances ($a = L_{k1}/L_{k2}$) as a parameter. The slope of the characteristic curves is smaller for the smaller values of a, which means that $L_{k2}$ may have a large value. Hence the the transformer design becomes less critical.

Fig. 11 (c) shows the cross regulation characteristic curves with the normalized load resistance $Q_2$ as a parameter. At high values of $Q_2$, which means the low current in output 2, the converter gain $M_2$ is high. The percentile cross regulation or the slope of the curves do not change appreciably with $Q_2$. Therefore, the larger range of variation of load resistance $Q_2$ is permitted and hence the design of the post regulation will be easier.

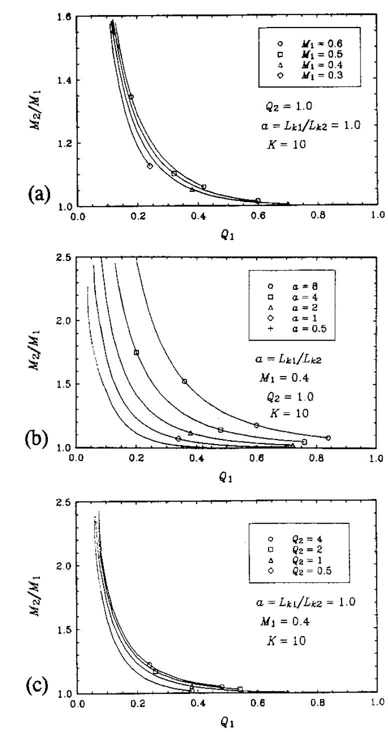


Fig. 11 Cross regulation characteristics of SRC-IF with (a) $M_1$ as the parameter, (b) $L_{k1}/L_{k2}$ as the parameter, and (c) $Q_2$ as the parameter.

#### 4.3.2 Control Characteristics

The effect of the second output in the series resonant converter with inductive filter is depicted in Fig. 12 (a), (b) and (c). The contol characteristics of the converter are plotted as the normalized switching frequency $f_{ns}$ versus normalized load resistance in $Q_1$. In Fig. 12 (a), the curves are plotted with $M_1$ as the parameter. Lower switching frequencies are required for operation at small values of $M_1$, which became necessary to minimize the percentile cross regulation in output 2. The control will be easier.

Fig. 12 (b) shows the control characteristic curves with the ratio of leakage inductances ($a = L_{k1}/L_{k2}$) as the parameter. The switching frequencies do not vary appreciably for different values of a. Hence the control performance of SRC is not affected by the addition of of a second output.

Fig. 12 (c) shows the control characteristic curves with $Q_2$ as the parameter. frequencies vary significantly for different values of $Q_2$. Higher switching frequencies are required when $Q_2$ is decreased (the current is increased on output 2).

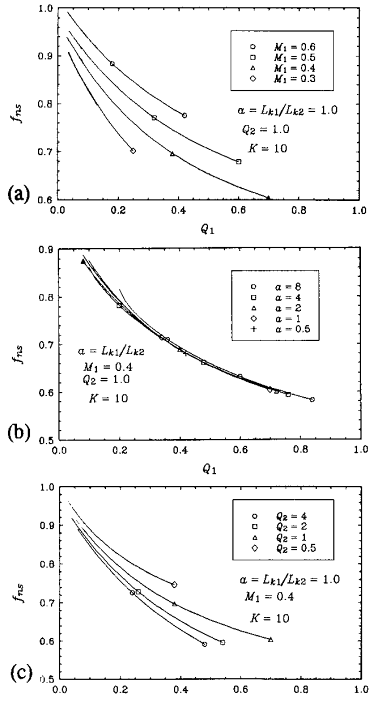


Fig. 12 Control characteristics of SRC-IF with (a) $M_1$ as the parameter, (b) $L_{k1}L_{k2}$ as the parameter, and (c) $Q_2$ as the parameter.

## V. COMPARISION OF CROSS REGULATION IN SRC-CF AND SRC-IF

A comparision of cross regulation characteristics between SRC-CF and SRC-IF can be made from Fig. 6 (a) and 11 (a). In SRC-CF, the curve for $M_1 = 0.9$ has the lowest slope and consequently the lowest percentile cross regulation. The curve for lowest slope in SRC-IF is at $M_1 = 0.4$. The lower converter gain $M_1$ will enable the transformer design with low turn ratios which will have higher bandwidths. Thus, the transformer will not restrain the converter operation at high switching frequencies. Secondly, The slope of the best curve in SRC-IF is lower than that in SRC-CF which translates into the smaller percentile cross regulation in the former. However, the large variation in $M_1$ is not possible in SRC-IF which means that large fluctuations in the line voltage are not permissible.

Next, compare the cross regulation curves in Fig. 6 (b) and Fig. 11 (b). The cross regulation curves in SRC-CF are highly dependent on the ratio of leakage inductances as compared to those in SRC-IF. The cross regulation curves in Fig. 6 (c) and Fig. 11 (c) show the contrast in the effect of load resistance $Q_2$. The cross regulation curves in SRC-IF are relatively independent of loading on the second output in comparison to the high dependence in SRC-CF.

Judging by cross regulation performance curves of Fig 6 (a)-(c) and Fig. 11 (a)-(c), it can be concluded that SRC with inductive filter is a better topology for multi-output converters.

## VI. SIMULATION RESULTS

The theoretical results have been verified by computer simulation. Fig. 13 shows the current waveforms in the two-output SRC with capacitive filter having the following component values:

$$L = L_{kp} = 6\ uH,\ L_{k1} = 1\ uH,\ L_{k2} = 3\ uH,\ C = 15\ nH,$$

$$V_g = 75\ v,\ R_{L1} = 20\ ohms,\ R_{L2} = 100\ ohms,\ f_S = 330\ KHz$$

The current waveforms of Fig. 13 are similar to the predicted waveforms from the steady state analysis which are given in Fig. 5.

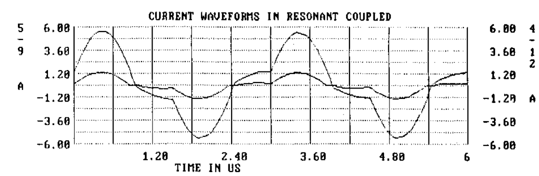


Fig. 13 Simulation results of SRC with capacitive filter.

## VII. CONCLUSION

From the performance characteristic curves of SRC with capacitive and inductive output filter, it can be concluded that the cross regulation performance of the later is much improved. The cross regulation in SRC-IF is less dependent on the variations in the line voltage and the manufacturing tolerances of the leakage inductances in the transformers, and the output current levels in the unregulated outputs. Furthermore, the unregulated outputs in SRC-IF have minimal effect on the control characteristics of the main output which is regulated by feedback.

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