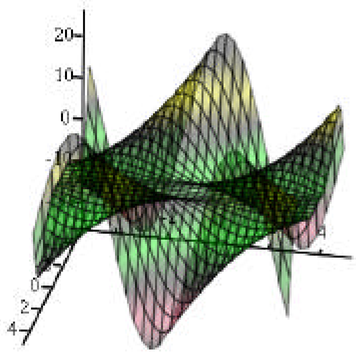
实验3 微积分运算(三) 求偏导数运算

1. 求多元函数的导数.
2. 求隐函数的导数.

(1) 设 $f(x,y) := x^2 \cdot \sin(x+y)$ ，求偏导数.



f(x,y)的图形

f

$$\frac{\partial}{\partial x} f(x,y) \to 2\cdot x\cdot \sin(x+y) + x^2\cdot \cos(x+y) \qquad \frac{\partial}{\partial y} f(x,y) \to x^2\cdot \cos(x+y)$$

$$\frac{\partial^2}{\partial x^2} f(x,y) \to 2\cdot \sin(x+y) + 4\cdot x\cdot \cos(x+y) - x^2\cdot \sin(x+y)$$

$$\frac{\partial^2}{\partial y^2} f(x,y) \to -x^2\cdot \sin(x+y) \qquad \frac{\partial}{\partial x}\frac{\partial}{\partial y} f(x,y) \to 2\cdot x\cdot \cos(x+y) - x^2\cdot \sin(x+y)$$

$$u := \frac{\pi}{3} \qquad v := \frac{\pi}{6} \qquad \frac{\partial}{\partial u} f(u,v) \to \frac{2}{3}\cdot \pi\cdot \sin\left(\frac{1}{2}\cdot \pi\right) + \frac{1}{9}\cdot \pi^2\cdot \cos\left(\frac{1}{2}\cdot \pi\right) = 2.094$$

$$\frac{\partial}{\partial v} f(u,v) \to \frac{1}{9}\cdot \pi^2\cdot \cos\left(\frac{1}{2}\cdot \pi\right) = 0$$

$$f(x,y,z) := \sqrt{\sin(x)^2 + \sin(y)^2 + \sin(z)^2}$$

$$\frac{\partial}{\partial x} f(x,y,z) \to \frac{1}{\left(\sin(x)^2 + \sin(y)^2 + \sin(z)^2\right)^{\frac{1}{2}}}\cdot \sin(x)\cdot \cos(x)$$

$$g(x,y) := \ln\left(e^x + e^y\right) \qquad \frac{\partial}{\partial x} g(x,y) - \frac{\partial}{\partial y} g(x,y) \text{ simplify } \to \frac{-(-\exp(x) + \exp(y))}{(\exp(x) + \exp(y))}$$

(2) **已知方程** $\ln\left(\sqrt{x^2+y^2}\right) = \operatorname{atan}\left(\frac{y}{x}\right)$ **确定的隐函数，求** $\frac{dy}{dx}$.

$$F(x,y) := \ln\left(\sqrt{x^2+y^2}\right) - \operatorname{atan}\left(\frac{y}{x}\right) \qquad \frac{dy}{dx} = \frac{-F_x}{F_y}$$

$$\frac{\partial}{\partial x}F(x,y) \text{ simplify } \rightarrow \frac{(x+y)}{\left(x^2+y^2\right)} \qquad \frac{\partial}{\partial y}F(x,y) \text{ simplify } \rightarrow \frac{-(-y+x)}{\left(x^2+y^2\right)}$$

$$D(x,y) := -\frac{\frac{\partial}{\partial x}F(x,y)}{\frac{\partial}{\partial y}F(x,y)} \qquad D(x,y) \text{ simplify } \rightarrow \frac{(x+y)}{(-y+x)}$$