



实验21

求方程根的函数

本文档将给出用Mathcad计算方程根的方法.

求方程 $x^3 - 17x + 32 = 0$ 在区间 $(-2, -1)$ 、 $(1, 2)$ 、 $(16, 17)$ 中的根.

方法1: 调用 $\text{root}(f(x), x, [a, b])$ 函数直接计算

$$g(x) := x^3 - 17x^2 + 32$$

$$\text{root}(g(x), x, -2, -1) = -1.32158 \quad \text{root}(g(x), x, 1, 2) = 1.43378 \quad \text{root}(g(x), x, 16, 17) = 16.8878$$

方法2: 首先设置猜测值

$$x1 := -2 \quad x2 := 1 \quad x3 := 17$$

$$M := \begin{pmatrix} \text{root}(g(x1), x1) \\ \text{root}(g(x2), x2) \\ \text{root}(g(x3), x3) \end{pmatrix} \quad M = \begin{pmatrix} -1.3215968 \\ 1.43378414 \\ 16.88779713 \end{pmatrix}$$

方法3: 调用 $\text{polyroots}(v)$ 函数计算, 其中 v 为多项式的系数向量.

$$x^3 - 17x^2 + 32 \text{ coeffs, } x \rightarrow \begin{pmatrix} 32 \\ 0 \\ -17 \\ 1 \end{pmatrix} \quad v := \begin{pmatrix} 32 \\ 0 \\ -17 \\ 1 \end{pmatrix} \quad \text{polyroots}(v) = \begin{pmatrix} -1.32158 \\ 1.43378 \\ 16.8878 \end{pmatrix}$$

方法4: 调用 Given...Find 求解模块, 调用之前首先设置猜测值

$$x := -2 \quad \text{Given} \quad x^3 - 17x^2 + 32 = 0 \quad \text{Find}(x) = -1.32158$$

$$x := 1 \quad \text{Given} \quad x^3 - 17x^2 + 32 = 0 \quad \text{Find}(x) = 1.43378292$$

$$x := 17 \quad \text{Given} \quad x^3 - 17x^2 + 32 = 0 \quad \text{Find}(x) = 16.88779712$$

$$\begin{aligned} \text{roots}(a, b, c) &:= \begin{cases} \text{if } b^2 \geq 4 \cdot a \cdot c \\ \quad \left| \begin{array}{l} x1 \leftarrow \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \\ x2 \leftarrow \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array} \right. \\ \text{error}(\text{"根为复数"}) \text{ otherwise} \\ \quad \begin{pmatrix} x1 \\ x2 \end{pmatrix} \end{cases} \\ \text{roots}(2, 5, 3) &= \begin{pmatrix} -1 \\ -1.5 \end{pmatrix} \\ \text{roots}(2, 2, 3) &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{aligned}$$

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roots(a, b, c) :=
| break if a = 0
|  $\Delta \leftarrow b^2 - 4 \cdot a \cdot c$ 
|  $x1 \leftarrow \frac{-b + \sqrt{\Delta}}{2 \cdot a}$ 
|  $x2 \leftarrow \frac{-b - \sqrt{\Delta}}{2 \cdot a}$ 
| (x1 x2)

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$\text{roots}(4, 8, 3) = (-0.5 \quad -1.5)$ $\text{roots}(2, 1, 3) \rightarrow \left(\frac{-1}{4} + \frac{1}{4} \cdot i \cdot \sqrt{23} \quad \frac{-1}{4} - \frac{1}{4} \cdot i \cdot \sqrt{23} \right)$

$\text{roots}(0, 3, 1) = \blacksquare$

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This is invalid. If you are using conditional statements in a Mathcad program, make sure all cases are accounted for.