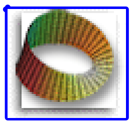

实验13　　输出特殊矩阵

本工作页练习产生元素满足一定要求的特殊矩阵：主要方法有

1. **使用内置函数 Matrix(m,n,F) 函数.**
2. **直接定义矩阵元素.**
3. **编写程序.**

例1 给出一个 4×4 阶的 Hilbert 矩阵

(1) $f(x,y) := \dfrac{1}{x+y+1}$ $\qquad$ $\text{matrix}(4,4,f) = \begin{pmatrix} 1 & 0.5 & 0.333 & 0.25 \\ 0.5 & 0.333 & 0.25 & 0.2 \\ 0.333 & 0.25 & 0.2 & 0.167 \\ 0.25 & 0.2 & 0.167 & 0.143 \end{pmatrix}$

$g(x,y) := \displaystyle\int_0^1 t^{x+y}\,dt$ $\qquad$ $\text{matrix}(4,4,g) = \begin{pmatrix} 1 & 0.5 & 0.333 & 0.25 \\ 0.5 & 0.333 & 0.25 & 0.2 \\ 0.333 & 0.25 & 0.2 & 0.167 \\ 0.25 & 0.2 & 0.167 & 0.143 \end{pmatrix}$

$$\text{matrix}(4,4,f) \rightarrow \begin{pmatrix} \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{pmatrix} = \begin{pmatrix} 0.333 & 0.25 & 0.2 & 0.167 \\ 0.25 & 0.2 & 0.167 & 0.143 \\ 0.2 & 0.167 & 0.143 & 0.125 \\ 0.167 & 0.143 & 0.125 & 0.111 \end{pmatrix}$$

注意符号运算与浮点运算结果之间的差异.

(2) $i := 0..3$ $\qquad$ $j := 0..3$ $\qquad$ $h_{i,j} := \dfrac{1}{i+j+1}$

$$h \rightarrow \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

(3)

$$\text{Hilbert} := \left|\begin{array}{l} \text{for } i \in 0..4 \\ \quad \text{for } j \in 0..4 \\ \qquad h_{i,j} \leftarrow \dfrac{1}{i+j+1} \\ h \end{array}\right.$$

$$\text{Hilbert} = \begin{pmatrix} 1 & 0.5 & 0.333 & 0.25 & 0.2 \\ 0.5 & 0.333 & 0.25 & 0.2 & 0.167 \\ 0.333 & 0.25 & 0.2 & 0.167 & 0.143 \\ 0.25 & 0.2 & 0.167 & 0.143 & 0.125 \\ 0.2 & 0.167 & 0.143 & 0.125 & 0.111 \end{pmatrix}$$

例2 给出一个 9×9 表

$$f(x,y) := x\cdot y \qquad \text{matrix}(9,9,f) \rightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 27 \\ 4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 & 36 \\ 5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 & 45 \\ 6 & 12 & 18 & 24 & 30 & 36 & 42 & 48 & 54 \\ 7 & 14 & 21 & 28 & 35 & 42 & 49 & 56 & 63 \\ 8 & 16 & 24 & 32 & 40 & 48 & 56 & 64 & 72 \\ 9 & 18 & 27 & 36 & 45 & 54 & 63 & 72 & 81 \end{pmatrix}$$

$$i := 0..8 \quad j := 0..8 \quad g_{i,j} := (i+1)\cdot(j+1) \qquad g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 27 \\ 4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 & 36 \\ 5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 & 45 \\ 6 & 12 & 18 & 24 & 30 & 36 & 42 & 48 & 54 \\ 7 & 14 & 21 & 28 & 35 & 42 & 49 & 56 & 63 \\ 8 & 16 & 24 & 32 & 40 & 48 & 56 & 64 & 72 \\ 9 & 18 & 27 & 36 & 45 & 54 & 63 & 72 & 81 \end{pmatrix}$$

$$\text{N\_NTable} := \left| \begin{array}{l} \text{for } i \in 0..8 \\ \quad \text{for } j \in 0..8 \\ \quad\quad n_{i,j} \leftarrow (i+1)\cdot(j+1) \\ n \end{array} \right.$$

$$\text{N\_NTable} \rightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 27 \\ 4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 & 36 \\ 5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 & 45 \\ 6 & 12 & 18 & 24 & 30 & 36 & 42 & 48 & 54 \\ 7 & 14 & 21 & 28 & 35 & 42 & 49 & 56 & 63 \\ 8 & 16 & 24 & 32 & 40 & 48 & 56 & 64 & 72 \\ 9 & 18 & 27 & 36 & 45 & 54 & 63 & 72 & 81 \end{pmatrix}$$

例3 如下程序可以产生奇数阶幻方

$$S(n) := \left| \begin{array}{l} \text{for } i \in 0..n-1 \\ \quad \text{for } j \in 0..n-1 \\ \quad\quad a_{i,j} \leftarrow \text{mod}\left[\left[i + j + \frac{(n+1)}{2} \right], n\right]\cdot n \ldots \\ \quad\quad\quad + \text{mod}\left[\left[i + 2\cdot j + n + \frac{(n-1)}{2} \right], n\right] \\ a \end{array} \right. \quad \text{if } \text{mod}(n,2) \neq 0$$

$$S(5) = \begin{pmatrix} 17 & 24 & 1 & 8 & 10 \\ 23 & 0 & 7 & 14 & 16 \\ 4 & 6 & 13 & 15 & 22 \\ 5 & 12 & 19 & 21 & 3 \\ 11 & 18 & 20 & 2 & 9 \end{pmatrix}$$

例3　给定 $x1, x2, ..., xk$ 的值，产生形如 $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ x1 & x2 & x3 & x4 & x5 \\ x1^2 & x2^2 & x3^2 & x4^2 & x5^2 \\ .... & .... & .... & .... & .... \\ x1^4 & x2^4 & x3^4 & x4^4 & x5^4 \end{pmatrix}$ 的范达蒙矩阵，

并计算行列式值.

```
Van(X) := | n ← length(X) − 1
          | for  i ∈ 0..n
          |   for  j ∈ 0..n
          |     | v[i,j] ← 1  if  i = 0
          |     | v[i,j] ← (X[j])^i  otherwise
          | v
```

$X := (2\ \ -3\ \ 3\ \ 1\ \ -2)^T$　　$\mathrm{Van}(X) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & -3 & 3 & 1 & -2 \\ 4 & 9 & 9 & 1 & 4 \\ 8 & -27 & 27 & 1 & -8 \\ 16 & 81 & 81 & 1 & 16 \end{pmatrix}$　　$|\mathrm{Van}(X)| = 14400$

```
Vande(X) := | s ← 1
            | n ← length(X) − 1
            | for  i ∈ 0..n
            |   for  j ∈ 0..n − 1
            |     s ← s·(X[i] − X[j])  if  i > j
            | s
```

仅计算Van de Monte行列式的值

$\mathrm{Vande}(X) = 14400$

使用如下程序生成 Van de Monte 矩阵，并计算其行列式值，当指定参数 $\alpha = 0$ 时，输出Van de Monte行列式值。

```
Van_de_Monte(X, α) := | n ← length(X) − 1
                      | for  j ∈ 0..n
                      |   v[0,j] ← 1
                      | for  i ∈ 1..n
                      |   for  j ∈ 0..n
                      |     v[i,j] ← (X[j])^i
                      | |v|  if  α = 0
                      | v  otherwise
```

$X := (-1\ \ 2\ \ 3\ \ -4)^T$

$\mathrm{Van\_de\_Monte}(X, 1) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 2 & 3 & -4 \\ 1 & 4 & 9 & 16 \\ -1 & 8 & 27 & -64 \end{pmatrix}$　　$\mathrm{Van\_de\_Monte}(X, 0) = -1512$

例5　如下程序可以产生杨辉三角形

$$\text{Yanghuiq}(n) := \left| \begin{array}{l} \text{for } i \in 0..n \\ \quad Y_{i,0} \leftarrow 1 \\ \text{for } i \in 0..n \\ \quad \text{for } j \in 1..n \\ \qquad \left| \begin{array}{l} Y_{i,j} \leftarrow 0 \ \text{ if } \ j > i \\ Y_{i,j} \leftarrow Y_{i-1,j-1} + Y_{i-1,j} \ \text{ otherwise} \end{array} \right. \\ Y \end{array} \right.$$

$$\text{Yanghuiq}(8) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 5 & 10 & 10 & 5 & 1 & 0 & 0 & 0 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 & 0 & 0 \\ 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 & 0 \\ 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 \end{pmatrix}$$