实验 2　　微积分运算(二)　　求导数运算

本文档用 Mathcad 作求导数的运算:

1. **求一元函数的导数, 求高阶导数.**
2. **求由参数方程确定的函数的导数.**

求导数的基本操作方法 :

- **定义函数f(x).**
- **使用热键Shift+/输入** $\frac{d}{dx}f(x)$ **或** $\frac{d^k}{dx^k}f(x)$ **, 在右边占位符处输入f(x).**
- **使用Ctrl+>执行符号运算, 如果输出结果较复杂, 可点击Symbolc板上的simplify按钮, 使得结果尽可能得到简化.**

1. (1) $f(x) := e^{x^2}\cdot\cos\left(e^{-2\cdot x}\right)$

$$\frac{d}{dx}f(x)\ \text{simplify}\ \rightarrow 2\cdot x\cdot\exp\left(x^2\right)\cdot\cos(\exp(-2\cdot x)) + 2\cdot\sin(\exp(-2\cdot x))\cdot\exp[x\cdot(x-2)]$$

$$h(x) := \sqrt{x\cdot\sin(x)\cdot\sqrt{1-\exp(x)}}$$

$$\frac{d}{dx}h(x)\ \text{simplify}\ \rightarrow \frac{-1}{4}\cdot\frac{(-2\cdot\sin(x) + 2\cdot\sin(x)\cdot\exp(x) - 2\cdot x\cdot\cos(x) + 2\cdot x\cdot\cos(x)\cdot\exp(x) + x\cdot\sin(x)\cdot\exp(x))}{\left[x\cdot\sin(x)\cdot(1-\exp(x))^{\frac{1}{2}}\right]^{\frac{1}{2}}\cdot(1-\exp(x))^{\frac{1}{2}}}$$

$$\frac{d}{dx}\ln(f(x))\ \text{simplify}\ \rightarrow 2\cdot\frac{(x\cdot\cos(\exp(-2\cdot x)) + \sin(\exp(-2\cdot x))\cdot\exp(-2\cdot x))}{\cos(\exp(-2\cdot x))}$$

$$\frac{d^2}{dx^2}\ln(f(x))\ \text{simplify}\ \rightarrow -2\cdot\frac{\left(-\cos(\exp(-2\cdot x))^2 + 2\cdot\cos(\exp(-2\cdot x))\cdot\sin(\exp(-2\cdot x))\cdot\exp(-2\cdot x) + 2\cdot\exp(-4\cdot x)\right.}{\cos(\exp(-2\cdot x))^2}$$

(2)　$g(x) := x^2\cdot\sin(2\cdot x)$　　$\frac{d}{dx}g(x) \rightarrow 2\cdot x\cdot\sin(2\cdot x) + 2\cdot x^2\cdot\cos(2\cdot x)$

$h(x) := \ln(1+x)$　　$\frac{d^{10}}{dx^{10}}h(x) \rightarrow \frac{-362880}{(x+1)^{10}}$　　$\frac{d^{15}}{dx^{15}}h(x) \rightarrow \frac{87178291200}{(x+1)^{15}}$

1. **如果要求给定函数在某点处的导数值, 在当前工作页内, 换名定义局部变量, 并赋值.**
2. **然后执行求导运算即可.**

$u := 0$　　$\frac{d^5}{du^5}g(u) \rightarrow -160\cdot\cos(0) = -160$　　$\frac{d^{10}}{du^{10}}h(u) \rightarrow -362880$

2　$x(t) := \frac{\cos(t)}{1+\sin(t)^2}$　　$y(t) := \frac{\sin(t)\cdot\cos(t)}{1+\sin(t)^2}$　　**双纽线**

$$\frac{d}{dx}y(x) = \frac{\frac{d}{dt}y(t)}{\frac{d}{dt}x(t)} \qquad \frac{\frac{d}{dt}y(t)}{\frac{d}{dt}x(t)} \text{ simplify } \rightarrow \frac{-\left(3\cdot\cos(t)^2 - 2\right)}{\sin(t)\cdot\left(2 + \cos(t)^2\right)}$$

$x(a,t) := a\cdot(t - \sin(t))$ $y(a,t) := a\cdot(1 - \cos(t))$ **摆线**

$$\frac{\frac{d}{dt}y(a,t)}{\frac{d}{dt}x(a,t)} \rightarrow \frac{\sin(t)}{(1 - \cos(t))}$$