实验25　求解常微分方程的初值问题

例　方法1　初值问题求解:

$$5\cdot\frac{d^2}{dt^2}x(t)+\frac{d}{dt}x(t)+2\cdot x(t)=10\cdot\sin(2t)\cdot e^{-3t}\qquad x(0)=0\qquad x'(0)=1$$

将微分方程改写为一阶微分方程组:

令　$x0(t)=x(t)\qquad x1(t)=\frac{d}{dt}x0(t)$

则　$\frac{d}{dt}x0(t)=x1(t)$

$$\frac{d}{dt}x1(t)=\frac{1}{5}\cdot\left(10\cdot\sin(2\cdot t)\cdot e^{-3\cdot t}-x1(t)-2\cdot x0(t)\right)$$

$$\begin{pmatrix}x0(0)\\x1(0)\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$$

$t0:=0\qquad t1:=40$　　**解区间端点**

$v:=\begin{pmatrix}0\\1\end{pmatrix}$　　**初值条件向量**

$N:=100$　　**在区间 [t0, t1] 上的解值数量**

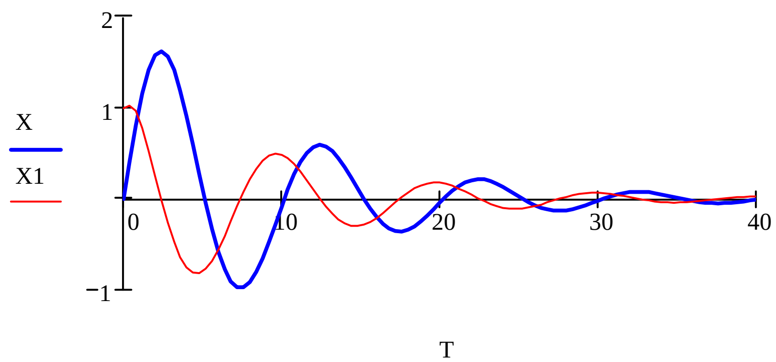
导函数:　$D(t,X):=\begin{bmatrix}X_1\\ \frac{1}{5}\cdot\left[10\cdot\sin(2t)\cdot(e)^{-3t}-X_1-2\cdot X_0\right]\end{bmatrix}$

解矩阵　$S:=\mathrm{rkfixed}(v,t0,t1,N,D)$　　**注: 函数rkfixed 也可以调用Rkadapt 或 Bulstoer 来代替**

$T:=S^{\langle 0\rangle}$　　**自变量值向量**

$X:=S^{\langle 1\rangle}$　　**解函数值向量**

$X1:=S^{\langle 2\rangle}$　　**解函数的一阶导数值向量**



$X1^T =$

	0	1	2	3	4	5	6	7	8	9
0	1	1.03	0.975	0.784	0.527	0.252	-0.016	-0.263	-0.474	-0.64

$S^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.4	0.8	1.2	1.6	2	2.4	2.8	3.2	3.6
1	0	0.403	0.809	1.164	1.428	1.584	1.63	1.573	1.425	1.2
2	1	1.03	0.975	0.784	0.527	0.252	-0.016	-0.263	-0.474	-0.64

方法2　使用laplace变换求解这个微分方程初值问题.

$$5\cdot\frac{d^2}{dt^2}x(t)+\frac{d}{dt}x(t)+2\cdot x(t)-10\cdot\sin(2t)\cdot e^{-3t}\qquad x(0)=0\qquad x'(0)=1$$

$$5\cdot s\cdot(s\cdot \mathrm{laplace}(x(t),t,s)-x(0))-5\cdot\left.\frac{d}{dt}x(t)\right|_{t\leftarrow 0}+s\cdot\mathrm{laplace}(x(t),t,s)-x(0)+2\cdot\mathrm{laplace}(x(t),t,s)-\frac{20}{\left[(s+3)^2+4\right]}$$

$$5\cdot s\cdot(s\cdot x-0)-5\cdot 1+s\cdot x-0+2\cdot x-\frac{20}{\left[(s+3)^2+4\right]}\ \text{solve}, x\ \rightarrow 5\cdot\frac{\left(17+s^2+6\cdot s\right)}{\left(s^2+6\cdot s+13\right)\cdot\left(s+2+5\cdot s^2\right)}$$

将初始条件代入上式，解出x

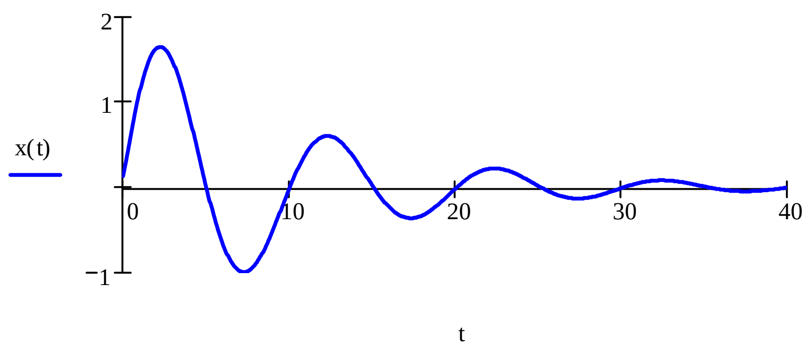
$$\frac{5\left(17+s^2+6\cdot s\right)}{\left(s^2+6\cdot s+13\right)\cdot\left(s+2+5\cdot s^2\right)}$$

执行 Symbolics\Transform\Inverse Laplace命令可以得到：

$$\frac{29}{197}\cdot\exp(-3\cdot t)\cdot\cos(2\cdot t)+\frac{12}{197}\cdot\exp(-3\cdot t)\cdot\sin(2\cdot t)-\frac{29}{197}\cdot\exp\left(\frac{-1}{10}\cdot t\right)\cdot\cos\left(\frac{1}{10}\cdot\sqrt{39}\cdot t\right)\ldots$$
$$+\frac{857}{2561}\cdot\exp\left(\frac{-1}{10}\cdot t\right)\cdot\sqrt{39}\cdot\sin\left(\frac{1}{10}\cdot\sqrt{39}\cdot t\right)$$

此式为满足初始条件的特解.

$$x(t):=\frac{29}{197}\cdot\exp(-3\cdot t)\cdot\left(\cos(2\cdot t)+\frac{12}{197}\cdot\sin(2\cdot t)\cdot\exp(-3\cdot t)-\frac{29}{197}\cdot\exp\left(\frac{-1}{10}\cdot t\right)\cdot\cos\left(\frac{1}{10}\cdot\sqrt{39}\cdot t\right)\right)\ldots$$
$$+\frac{857}{2561}\cdot\exp\left(\frac{-1}{10}\cdot t\right)\cdot\sqrt{39}\cdot\sin\left(\frac{1}{10}\cdot\sqrt{39}\cdot t\right)$$



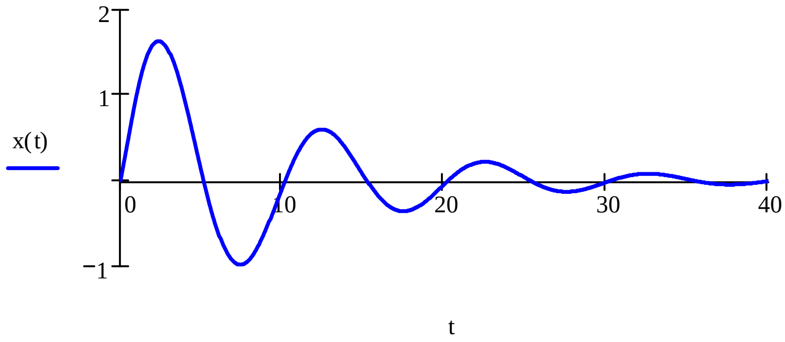
方法3　使用Given...Odesolve求解模块:

Given

$$5\cdot\frac{d^2}{dt^2}x(t)+\frac{d}{dt}x(t)+2\cdot x(t)-10\cdot\sin(2t)\cdot e^{-3t}=0 \qquad x(0)=0 \qquad x'(0)=1$$

$$x := \text{Odesolve}(t,40,100)$$

第三种方法最为简洁.



$$5\lambda^2+\lambda+2 \text{ solve},\lambda \rightarrow \begin{pmatrix} \frac{-1}{10}+\frac{1}{10}\cdot i\cdot\sqrt{39} \\ \frac{-1}{10}-\frac{1}{10}\cdot i\cdot\sqrt{39} \end{pmatrix}$$