实验4　微积分运算(四）　积分运算等

本文档介绍求导数的另一种方法: 直接执行菜单命令. 以及其他的微积分运算.

(1) 求函数的导函数与在特定点处的导数值:

直接对函数表达式执行菜单命令求导:

$\sin(x) \cdot e^{x} + \cos(x)^2$　　选中自变量x, 执行菜单命令symbols/Variable/Differentiate得到:

$\cos(x) \cdot \exp(x) + \sin(x) \cdot \exp(x) - 2 \cdot \cos(x) \cdot \sin(x)$　　一阶导数, 重复操作得到:

$2 \cdot \cos(x) \cdot \exp(x) + 2 \cdot \sin(x)^2 - 2 \cdot \cos(x)^2$　　二阶导数, 重复操作得到:

$-2 \cdot \sin(x) \cdot \exp(x) + 2 \cdot \cos(x) \cdot \exp(x) + 8 \cdot \sin(x) \cdot \cos(x)$　　三阶导数, 重复操作得到:

$-4 \cdot \sin(x) \cdot \exp(x) + 8 \cdot \cos(x)^2 - 8 \cdot \sin(x)^2$　　四阶导数, 重复操作得到:

定义函数, 应用Calculus运算板上的求导数按钮:

$f(x) := \sin(x) \cdot e^{x} + \cos(x)^2$　　$\frac{d}{dx} f(x) \rightarrow \cos(x) \cdot \exp(x) + \sin(x) \cdot \exp(x) - 2 \cdot \cos(x) \cdot \sin(x)$

$n := 6$　　$\frac{d^n}{dx^n} f(x) \rightarrow -8 \cdot \cos(x) \cdot \exp(x) + 32 \cdot \sin(x)^2 - 32 \cdot \cos(x)^2$

$x := \frac{\pi}{3}$　　$\frac{d}{dx} f(x) = 3.027$　　$\frac{d^5}{dx^5} f(x) = -29.427$

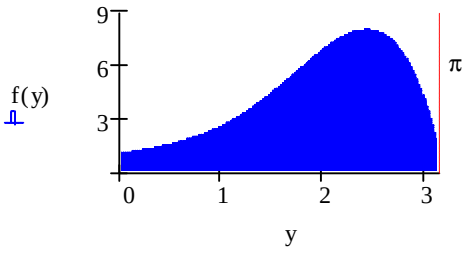
$\frac{d}{dx}\frac{d}{dx} f(x) \rightarrow 2 \cdot \cos\left(\frac{1}{3} \cdot \pi\right) \cdot \exp\left(\frac{1}{3} \cdot \pi\right) + 2 \cdot \sin\left(\frac{1}{3} \cdot \pi\right)^2 - 2 \cdot \cos\left(\frac{1}{3} \cdot \pi\right)^2$

(2) 求不定积分和定积分:

$\int f(x)\,dx \rightarrow \frac{-1}{2} \cdot \cos(x) \cdot \exp(x) + \frac{1}{2} \cdot \sin(x) \cdot \exp(x) + \frac{1}{2} \cdot \cos(x) \cdot \sin(x) + \frac{1}{2} \cdot x$

$\int_0^{\pi} f(x)\,dx \rightarrow \frac{1}{2} \cdot \exp(\pi) + \frac{1}{2} \cdot \pi + \frac{1}{2} = 13.641$

$\int \cos(x)^3\,dx \rightarrow \frac{1}{3} \cdot \cos(x)^2 \cdot \sin(x) + \frac{2}{3} \cdot \sin(x)$



$$\int x^3 \cdot \exp(x)\,dx \to x^3 \cdot \exp(x) - 3 \cdot x^2 \cdot \exp(x) + 6 \cdot x \cdot \exp(x) - 6 \cdot \exp(x)$$

$$\int \frac{1}{\sin(x)^m}\,dx \to \int \frac{1}{\sin(x)^m}\,dx \qquad \int_0^1 x \cdot \ln(x+1)\,dx \to \frac{1}{4}$$

$$\int_0^{\frac{\pi}{2}} \exp(\alpha \cdot x) \cdot \sin(\beta \cdot x)\,dx \to \exp\left(\frac{1}{2} \cdot \pi \cdot \alpha\right) \cdot \frac{\left(-\beta \cdot \cos\left(\frac{1}{2} \cdot \pi \cdot \beta\right) + \alpha \cdot \sin\left(\frac{1}{2} \cdot \pi \cdot \beta\right)\right)}{\left(\alpha^2 + \beta^2\right)} + \frac{\beta}{\left(\alpha^2 + \beta^2\right)}$$

$a := 2 \qquad b := 3 \qquad \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \cdot \sin(x)^2 + b^2 \cos(x)^2 + 1}\,dx \to \frac{1}{20} \cdot \sqrt{2} \cdot \pi$

(3) 级数展开

$\sin(x) \cdot e^x + \cos(x)^2$ **选中自变量x, 执行菜单命令symbols/Variable/Expand to Series得到:**

$$1 + 1 \cdot x + \frac{1}{3} \cdot x^3 + \frac{1}{3} \cdot x^4 - \frac{1}{30} \cdot x^5 + O\left(x^6\right)$$

$f(t) := \sin(t) \cdot e^t + \cos(t)^2$

$f(t)\ \text{series}, t, 7 \to 1 + t + \frac{1}{3} \cdot t^3 + \frac{1}{3} \cdot t^4 - \frac{1}{30} \cdot t^5 - \frac{1}{18} \cdot t^6$ **使用符号运算板上的series按钮**

(4) 求函数的极限

$$S(n) := \sum_{k=1}^{n} \frac{1}{k^2} \qquad \lim_{n \to \infty} S(n) \to \frac{1}{6} \cdot \pi^2 \qquad \sum_{k=1}^{\infty} \frac{1}{k^2} \to \frac{1}{6} \cdot \pi^2$$

$$\lim_{n \to \infty} \sqrt[n]{\frac{\prod_{k=1}^{n} (2 \cdot k - 1)}{\prod_{k=1}^{n} (2 \cdot k)}} \to 1 \qquad \lim_{n \to \infty} \frac{2^n}{n!} \to 0 \qquad \lim_{x \to \frac{\pi}{2}} \frac{\ln(\sin(x))}{(\pi - 2 \cdot x)^2} \to \frac{-1}{8}$$

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{\sqrt[3]{1+x} - 1} \to \frac{3}{2} \qquad \lim_{x \to 0} \frac{\sqrt{x \cdot \sin(x) + 1} - 1}{\exp\left(x^2\right) - 1} \to \frac{1}{2}$$

$$\lim_{x \to \frac{\pi}{2}} (1 + 3 \cdot \cot(x))^{\sec(x)} \to \exp(3) \qquad \lim_{x \to 0^+} x \cdot (\ln(x+1) - \ln(x)) \to 0$$

$$\lim_{t \to 0} \frac{\sin(t) \cdot t^2}{\sin\left(3t^3\right)} \to \frac{1}{3} \qquad \lim_{t \to 0^+} \frac{\sin(t) \cdot e^t}{\sin(3t)} \to \frac{1}{3} \qquad \lim_{t \to \infty} \left(1 - \frac{4}{t}\right)^t \to \exp(-4)$$

$$\lim_{t \to \infty} t^n \cdot e^{-t} \to 0 \qquad \lim_{t \to 0} \frac{\ln\left(3t^2 + 1\right)}{t \cdot \sin(t)} \to 3 \qquad \lim_{x \to 0} \frac{\sin(x) - x \cdot \cos(x)}{\sin(x)^3} \to \frac{1}{3}$$

$$\lim_{x \to 1} x^{\frac{1}{1-x}} \to \exp(-1) \qquad \lim_{x \to 1} (1 - x) \cdot \tan\left(\frac{\pi}{2} \cdot x\right) \to \frac{2}{\pi} \qquad \lim_{x \to 0^+} \left(\ln\left(\frac{1}{x}\right)\right)^x \to 1$$

$$\lim_{x \to \infty} \left(\frac{2}{\pi} \cdot \operatorname{atan}(x)\right)^x \to \exp\left(\frac{-2}{\pi}\right) \qquad \lim_{x \to 1^-} \left(\frac{2}{\pi} \cdot \operatorname{acos}(x)\right)^{\frac{1}{x}} \to 0$$

(5) 求弧长和旋转体体积

$$\int_0^{\pi} \sqrt{1 + f(x)^2}\, dx = 14.156$$

0 到 π 上曲线f(x)的弧长.

$$\int_0^{\pi} \pi \cdot f(x)^2\, dx \to \frac{1}{8} \cdot \pi \cdot \exp(2 \cdot \pi) + \frac{2}{5} \cdot \pi \cdot \exp(\pi) + \frac{3}{8} \cdot \pi^2 + \frac{11}{40} \cdot \pi = 243.932$$

$$S(n) := \sum_{k=1}^{n} \frac{1}{k^2} \qquad n := 10000, 50000 .. 100000$$

$\sqrt{6 \cdot S(n)} =$

3.14149716394721
3.14157355512957
3.1415820433013