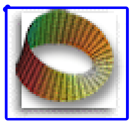

实验11 求多元函数的条件极值

利用Mathcad 的内部 maximize 和 minimize 函数可以求多元函数的条件极值.

例1 求函数 $u = x - 2y + 2z$ 在条件 $x^2 + y^2 + z^2 = 1$ 下的极值

1 定义函数 $f(x,y,z) := x - 2\cdot y + 2\cdot z$

2 为各个自变量指定猜测值 $x := 1$ $y := 0$ $z := -1$

3 将约束条件置于关键字Given之后，用maximize求极大值点.

Given

$$x^2 + y^2 + z^2 = 1$$

$$max := \text{Maximize}(f, x, y, z) \qquad max = \begin{pmatrix} 0.333 \\ -0.667 \\ 0.667 \end{pmatrix}$$

极大值为 $f(max_0, max_1, max_2) = 3$

4 将约束条件置于关键字Given之后，用minimize求极小值点.

Given

$$x^2 + y^2 + z^2 = 1$$

$$min := \text{Minimize}(f, x, y, z) \qquad min = \begin{pmatrix} -0.333 \\ 0.667 \\ -0.667 \end{pmatrix} \qquad f(min_0, min_1, min_2) = -3$$

极小值为 $f(min_0, min_1, min_2) = -3$

运用拉格郎日乘数法求解:

令: $\Phi(u,v,w,\lambda) := u - 2\cdot v + 2\cdot w + \lambda\cdot(u^2 + v^2 + w^2 - 1)$

$$\frac{d}{du}\Phi(u,v,w,\lambda) \rightarrow 1 + 2\cdot\lambda\cdot u \qquad \frac{d}{dv}\Phi(u,v,w,\lambda) \rightarrow -2 + 2\cdot\lambda\cdot v \qquad \frac{d}{dw}\Phi(u,v,w,\lambda) \rightarrow 2 + 2\cdot\lambda\cdot w$$

Given

$$1 + 2\cdot\lambda\cdot u = 0 \qquad -2 + 2\cdot\lambda\cdot v = 0 \qquad 2 + 2\cdot\lambda\cdot w = 0 \qquad u^2 + v^2 + w^2 - 1 = 0$$

$$X := \text{Find}(u, v, w, \lambda) \qquad X^T \rightarrow \begin{pmatrix} \frac{-1}{3} & \frac{2}{3} & \frac{-2}{3} & \frac{3}{2} \\ \frac{1}{3} & \frac{-2}{3} & \frac{2}{3} & \frac{-3}{2} \end{pmatrix}$$

$M := \text{submatrix}(X, 0, 2, 0, 0)$ $\quad M^T \rightarrow \begin{pmatrix} \frac{-1}{3} & \frac{2}{3} & \frac{-2}{3} \end{pmatrix}$ **驻点1**

$N := \text{submatrix}(X, 0, 2, 1, 1)$ $\quad N^T \rightarrow \begin{pmatrix} \frac{1}{3} & \frac{-2}{3} & \frac{2}{3} \end{pmatrix}$ **驻点2**

$u := \frac{1}{3}$ $\quad v := \frac{-2}{3}$ $\quad w := \frac{2}{3}$ $\quad \lambda := \frac{3}{2}$

$A := \frac{d^2}{du^2}\Phi(u,v,w,\lambda)$ $\quad B := \frac{d^2}{dv^2}\Phi(u,v,w,\lambda)$ $\quad C := \frac{d^2}{dw^2}\Phi(u,v,w,\lambda)$

$D := \frac{d}{du}\frac{d}{dv}\Phi(u,v,w,\lambda)$ $\quad E := \frac{d}{dv}\frac{d}{dw}\Phi(u,v,w,\lambda)$ $\quad F := \frac{d}{dw}\frac{d}{du}\Phi(u,v,w,\lambda)$

$A = 3$ $\quad B = 3$ $\quad C = 3$