实验36　　计算概率举例

问题1： 设随机向量 (ξ,η) 的概率密度函数为 ：

$$p(x,y) := \begin{cases} \left(\frac{1}{2}\cdot\sin(x+y)\right) & \text{if } 0 \le x \le \frac{\pi}{2},\ 0 \le y \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$p(0.5,0.5) = 0.421$$

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \frac{1}{2}\cdot\sin(x+y)\,dx\,dy = 1 \qquad \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} p(x,y)\,dx\,dy = 1$$

$$p1(x) := \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin(x+y)\,dy \qquad p2(y) := \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin(x+y)\,dx$$

$$p1(x) \rightarrow \frac{1}{2}\cdot\sin(x) + \frac{1}{2}\cdot\cos(x) \qquad p2(y) \rightarrow \frac{1}{2}\cdot\sin(y) + \frac{1}{2}\cdot\cos(y)$$

于是随机变量 ξ 的概率密度函数为:　　　**类似地随机变量 η 的概率密度函数为：**

$$p\xi(x) := \begin{cases} p1(x) & \text{if } 0 \le x \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases} \qquad p\eta(y) := \begin{cases} p2(y) & \text{if } 0 \le y \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$E\xi := \int_0^{\frac{\pi}{2}} x\cdot p1(x)\,dx \qquad E\xi = 0.785 \qquad E\xi \rightarrow \frac{1}{4}\cdot\pi \qquad E\eta := E\xi$$

$$E\xi\eta := \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \frac{1}{2}\cdot x\cdot y\cdot\sin(x+y)\,dx\,dy \qquad E\xi\eta = 0.571 \qquad E\xi\eta \rightarrow \frac{1}{2}\cdot\pi - 1$$

$$cov\xi\eta := E\xi\eta - E\xi\cdot E\eta \qquad cov\xi\eta \rightarrow \frac{1}{2}\cdot\pi - 1 - \frac{1}{16}\cdot\pi^2 = -0.046$$

$$E\xi\xi := \int_0^{\frac{\pi}{2}} x^2\cdot p1(x)\,dx \rightarrow \frac{1}{8}\cdot\pi^2 + \frac{1}{2}\cdot\pi - 2$$

$D\xi := E\xi\xi - E\xi \cdot E\xi \quad D\xi \rightarrow .80449687693107 - \frac{1}{16} \cdot \pi^2 \qquad D\eta := D\xi \quad D\xi = 0.188$

$$\rho := \frac{cov\xi\eta}{\sqrt{D\xi \cdot D\eta}} \qquad \rho \rightarrow \frac{\left(\frac{1}{2}\cdot\pi - 1 - \frac{1}{16}\cdot\pi^2\right)}{\left(.80449687693107 - \frac{1}{16}\cdot\pi^2\right)} = -0.245$$

问题2： 设随机向量 (x,h) 的概率密度函数为：

$$f(x,y) := \begin{cases} \frac{2\cdot\exp(-y+1)}{x^3} & \text{if } \begin{cases} x \ge 1 \\ y \ge 1 \end{cases} \\ 0 & \text{otherwise} \end{cases} \qquad \int_1^\infty \int_1^\infty \frac{2\cdot\exp(-y+1)}{x^3}\,dx\,dy \rightarrow \exp(1)\cdot\exp(-1) = 1$$

$$f1(x) := \int_1^\infty \frac{2\cdot\exp(-y+1)}{x^3}\,dy \rightarrow \frac{2}{x^3}\cdot\exp(1)\cdot\exp(-1) \qquad ff1(x) := \begin{cases} \frac{2}{x^3} & \text{if } x \ge 1 \\ 0 & \text{otherwise} \end{cases}$$

$$g1(y) := \int_1^\infty \frac{2\cdot\exp(-y+1)}{x^3}\,dx \rightarrow \exp(-y+1) \qquad gg1(y) := \begin{cases} \exp(-y+1) & \text{if } y \ge 1 \\ 0 & \text{otherwise} \end{cases}$$

$$E\xi := \int_1^\infty x\cdot\frac{2}{x^3}\,dx \rightarrow 2 \qquad E\xi\xi := \int_1^\infty x^2\cdot\frac{2}{x^3}\,dx \rightarrow \infty$$

所以ξ的方差不存在.

$$E\eta := \int_1^\infty y\cdot\exp(-y+1)\,dy \rightarrow 2\cdot\exp(1)\cdot\exp(-1) \qquad E\eta = 2$$

$$E\eta\eta := \int_1^\infty y^2\cdot\exp(-y+1)\,dy \rightarrow 5\cdot\exp(1)\cdot\exp(-1) \qquad E\eta\eta = 5$$

$D\eta := E\eta\eta - E\eta\cdot E\eta \qquad D\eta = 1$

问题3：求N(0,1)分布的各阶矩.　设k为自然数,求N(0,1)分布的各阶矩

,由于N(0,1)分布的密度为偶函数, 所以它的奇数阶矩为0。

$$E\xi := \int_{-\infty}^\infty \frac{x}{\sqrt{2\pi}}\cdot\exp\left(\frac{-x^2}{2}\right)dx \rightarrow 0 \qquad E\xi2 := \int_{-\infty}^\infty \frac{x^2}{\sqrt{2\pi}}\cdot\exp\left(\frac{-x^2}{2}\right)dx \rightarrow 1$$

$$E\xi4 := \int_{-\infty}^\infty \frac{x^4}{\sqrt{2\pi}}\cdot\exp\left(\frac{-x^2}{2}\right)dx \rightarrow 3 \qquad E\xi6 := \int_{-\infty}^\infty \frac{x^6}{\sqrt{2\pi}}\cdot\exp\left(\frac{-x^2}{2}\right)dx \rightarrow 15$$

$$E\xi8 := \int_{-\infty}^{\infty} \frac{x^8}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2}\right) dx \rightarrow 105 \qquad E\xi10 := \int_{-\infty}^{\infty} \frac{x^{10}}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2}\right) dx \rightarrow 945$$

$n := 1, 2 .. 8$

$$E\xi_n := \frac{(2n)!}{2^n \cdot n!}$$

$E\xi_n =$

1
3
15
105
945
10395
135135
2027025

$$E\xi(k) := \int_{-\infty}^{\infty} \frac{x^k}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2}\right) dx$$

$E\xi(1) \rightarrow 0$ $\quad E\xi(2) \rightarrow 1$ $\quad E\xi(3) \rightarrow 0$ $\quad E\xi(4) \rightarrow 3$

$E\xi(5) \rightarrow 0$ $\quad E\xi(6) \rightarrow 15$ $\quad E\xi(7) \rightarrow 0$ $\quad E\xi(8) \rightarrow 105$

$E\xi(9) \rightarrow 0$ $\quad E\xi(10) \rightarrow 945$ $\quad E\xi(11) \rightarrow 0$ $\quad E\xi(12) \rightarrow 10395$

$$E\Xi(n) := \prod_{k=1}^{n} (2 \cdot k - 1) \qquad n := 1 .. 12$$

$E\Xi(n) =$

1
3
15
105
945
10395

1.351?0^{5}
2.027?0^{6}
3.446?0^{7}
6.547?0^{8}
1.375?0^{10}
3.162?0^{11}