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# ZVS Frequency Analysis of a Current-Fed Resonant Converter

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**Abstract— ZVS (Zero Voltage Switching) is a basic control strategy for normal current-fed DC-AC parallel resonant converters. However, the ZVS operating frequency and how it varies with the load and system parameters remains an unsolved theoretical problem of practical importance. After identifying different types of resonant frequencies of a typical resonant tank of a current-fed resonant converter, this paper gives both accurate numerical solution and approximate analytical solution of ZVS frequencies. The analysis is based on a step current injection model in a half-switching cycle. The validity of the analysis is proven with PSpice simulation results.**

## I. INTRODUCTION

Resonant converters are becoming popular mainly for two reasons: quasi-sinusoidal waveforms can be generated with less harmonics and EMI (Electromagnetic Interference), and switching devices can be turned on or off at natural zero voltage or zero current crossing points resulting in lower switching losses and higher efficiency. For current-fed, parallel resonant DC-AC converters such as that shown in Fig. 1, an additional advantage is that the reactive power circulates inside the parallel resonant tank without going through the switching network and the DC power supply. Hence the current rating of the switching devices can be smaller and the conduction loss can be greatly reduced at a given power level. A basic requirement for this type of converter is that the switches must operate at zero voltage crossing points, otherwise shorting of the resonant capacitors will occur and a large shorting current may cause the switching devices to fail. In other words, ZVS is critical for the safe operation of these converters.

Many control techniques, such as PLL (Phase Lock Loop) controllers, current or voltage error based VCO (Voltage Controlled Oscillator) integral controllers, have been developed to achieve ZVS in the steady state [1]. Complete dynamic ZVS, including self start-up, is also achieved using initially forced DC current and instant zero voltage detection [2]. Now it is of practical and theoretical interest to know at what frequency the system will actually operate and how this is affected by load and system parameters. This knowledge is required for EMC (Electromagnetic Compatibility) analysis and can be used to improve the circuit design, such as the secondary pick-up tuning design of an IPT (Inductive Power Transfer) system [3]. It is also useful for system modeling and analysis. For example, the actual operating frequency is needed for GSSA (Generalized State Space Averaging) modeling of current-fed resonant converters, otherwise trial and error or experimental methods have to be employed [4] to approach the right results.

This paper identifies different types of resonant frequencies and analyses the steady state ZVS operating frequency of a typical current-fed parallel resonant converter.

## II. RESONANT FREQUENCIES AND ZVS FREQUENCY

### A. A Typical Current-Fed Resonant Converter

Fig. 1 shows a typical current-fed full-bridge parallel resonant converter. A series load resistor is assumed as in the load situation of IPT (Inductive Power Transfer) applications. This assumption is acceptable when an IPT system is essentially tuned and the total pick-up load is not too heavy to cause detuning problems. IPT system detuning is possible and has been observed [5]; however, the frequency shift range analysis required to account for this is beyond the scope of this paper and needs further investigation. In Fig 1, a parallel resonant tank consisting of an inductor L, a load resistor R, and a tuning capacitor C is formed. A DC inductor $L_d$ links a DC power supply $E_d$, and a full bridge inverting network comprising two switching pairs; S1, S1' and S2, S2'. The two switches in the same pair function simultaneously, when one pair is on the other pair will be off, and all such commutations are controlled to occur at the zero voltage crossing points of the resonant voltage $v_c$. As a result the switching losses are minimized and the shorting of the resonant capacitor, which is a potential danger to the switching devices, is avoided.

The resonant tank alternately receives current injection from the DC link via an inverting network. But because the average voltage across the DC inductor $L_d$ is zero in the steady state, the average resonant voltage $v_c$, over a half-switching cycle will be equal to $E_d$. Consequently, the peak value of the quasi-sinusoidal waveform of $v_c$, caused by the resonance, will be approximately $E_d$ multiplied by $\pi/2$. With this driving voltage, a quasi-sinusoidal current can be

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generated in the resonant inductor L and the power is transferred from DC supply to the load.

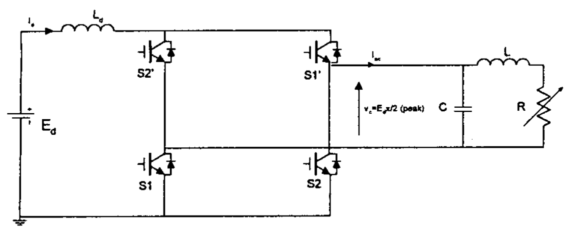


Fig. 1. A current-fed resonant converter

*B. Different Types of Resonant Frequencies*

For the series loaded parallel resonant tank as employed in Fig. 1, different resonant conditions may occur at the following frequencies.

Zero Phase Angle Frequency

If we have a pure sinusoidal voltage or current excitation at the input to the parallel resonant tank, then at a certain frequency the voltage and current will be in phase in the steady state. This frequency corresponds to zero impedance phase angle and unity power factor. The reactive power inside the resonant tank circulates only internally without exchange with the outside. This is the situation normally referred to as resonance, and this resonant frequency is only determined by circuit parameters:

$$\omega_r = \omega_0 \sqrt{1 - \tfrac{1}{Q^2}}\,, \tag{1}$$

where $\omega_0 = 1/\sqrt{LC}$, and quality factor $Q = \sqrt{\tfrac{L}{C}}/R = \omega_0 L/R$. Note here Q is in fact a defined ratio and it is not exactly the same as the circuit quality factor at resonance, which should be $\omega_r L/R$. However, they are very close when Q is high.

Maximum Inductor Current Frequency

Strictly speaking, the zero phase angle resonance does not necessarily mean that the current flowing through the resonant inductor, corresponding to the track power supply current in an IPT system [1], reaches its maximum value. For a given sinusoidal current source, it can be proven that the steady state maximum inductor current of the parallel resonant tank (series loaded ) occurs at frequency

$$\omega_{i_L \max} = \omega_0 \sqrt{1 - \tfrac{1}{2Q^2}}\,, \tag{2}$$

where $\omega_0$ and Q have the same meaning as in (1).

Natural Oscillation Frequency (Free Ringing Frequency)

If the resonant circuit has initial energy, for example the resonant inductor has an initial current of $i_L(0)$, or the capacitor has an initial voltage, then even when there is no external excitation, the resonant tank may oscillate naturally. The natural oscillation frequency, sometimes called free ringing frequency, is determined by the eigenvalues of the differential equations used to describe the resonant circuit. It can be shown that the natural frequency of the parallel resonant circuit used in Fig. 1 is:

$$\omega_f = \omega_0 \sqrt{1 - \tfrac{1}{4Q^2}}\,, \tag{3}$$

where $\omega_0$ and Q have the same meaning as in (1).

Undamped Natural Frequency

As described in [6], different resonant frequencies may be defined for one single circuit. While the three frequencies discussed above are all called resonant frequencies, the first one, zero phase angle frequency, is most commonly accepted. The first two situations are only valid for steady state sinusoidal AC analysis, while the last situation is related to the natural component of the complete system dynamic solution. One important characteristic in common is that the resonant frequencies are determined only by the system parameters and have nothing to do with the external excitations.

It is interesting to note that if Q is high the three frequencies become very close. For a lossless resonant circuit, i.e. R=0 and $Q=\infty$, all the above three frequencies converge to:

$$\omega_0 = 1/\sqrt{LC}\,. \tag{4}$$

This frequency is called undamped natural frequency.

*C. ZVS Frequency*

For the practical nonlinear current-fed resonant converter shown in Fig. 1, the commutation of the switching devices is controlled at zero voltage crossing points. In consequence, ZVS operating frequency is of primary importance.

As discussed before, if the excitation to the resonant tank is sinusoidal, then at zero phase angle frequency the voltage and current are in phase under steady state conditions. This means the ZVS frequency is the same as the zero phase angle resonant frequency $\omega_r$. But in the case of current-fed inverters, the injection current into the resonant tank is essentially square wave rather than sinusoidal in the steady state. In addition to the fundamental, it also includes higher order harmonic components. As the circuit has an unique zero phase angle resonant frequency, the fundamental and the harmonics cannot result in circuit resonance at the same time. Therefore, considering the fundamental and harmonic components together, the steady state ZVS frequency will be different from the zero phase angle resonant frequency $\omega_r$ as shown in (1).

The free ringing frequency, $\omega_f$, only reflects the natural oscillatory component of the dynamic solution; the actual ZVS frequency is determined by the complete solution of the voltage response. Therefore, a complete dynamic analysis is necessary to find the ZVS frequency.

## III. Steady State ZVS Frequency Analysis

### *A. Completete Resonant Voltage Solution*

For the converter as shown in Fig. 1, if the inductance of the DC inductor is much larger than the resonant inductor L, in the steady state the switching network approximately injects alternative square wave current into the resonant tank, where the magnitude of the of this current is determined by the load.

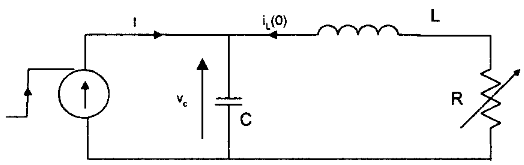


Fig. 2. Step current injection model

Fig. 2 shows a parallel resonant tank with a step current excitation. This model essentially describes the situation in a half-switching period of a practical current-fed converter. From this model, we can write the state space equations as:

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_C \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} I . \tag{5}$$

If variable $v_c$ is considered only, its second order ordinary differential equation can be written as:

$$LC\frac{d^2 v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = IR . \tag{6}$$

Considering the initial condition $v_c(0)=v_c|_{t=0}=0$, $\frac{dv_c}{dt}|_{t=0} = \frac{I+i_L(0)}{C}$, we can get the complete solution of the above equation:

$$v_c(t) = \frac{IR}{\sin\theta} e^{-t/T} \sin(\omega_f t - \theta) + IR , \tag{7}$$

where $\omega_f$ is the free ringing frequency as defined in (3), T=2L/R is the time constant, and $\theta$ is an initial phase angle:

$$\theta = \tan^{-1} \frac{\omega_f IRC}{i_L(0)+I(1-RC/T)} . \tag{8}$$

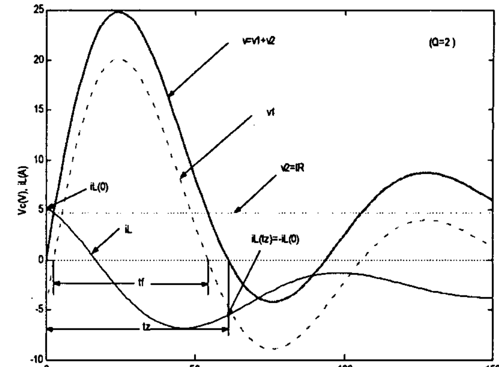


Fig. 3. Complete solution of voltage and current

Equation (7) shows that the complete solution for the resonant capacitor voltage $v_c$ has two parts, as also illustrated in Fig. 3. The first part is the transient component with oscillatory decay; the second part is the forced component, which is DC offset. Because of this offset, the time between the zero crossing points becomes larger than the free ringing half-period $t_f$, as can be seen in Fig 3.

### *B. Numerical Analysis*

As ZVS frequency is determined by the zero crossing of the capacitor voltage, so let $v_c$=0, and equation (7) becomes:

$$e^{-t_z/T} \sin(\omega_f t_z - \theta) + \sin\theta = 0 . \tag{9}$$

The solution of this equation is the ZVS half-period, $t_z$. What we need to know to solve this equation are circuit parameter L, C, load R (corresponding to a certain Q), the injection current I, and the initial current $i_L(0)$. Actually only the ratio $i_L(0)/I$, rather than I and $i_L(0)$ individually, contributes as (8) can be rewritten as:

$$\theta = \tan^{-1} \frac{\sqrt{4Q^2-1}}{2Q^2(K_i+1)-1} , \tag{10}$$

where $K_i=i_L(0)/I$, and $Q(=\omega_0 L/R)$ is the same as defined before.

Now the problem is that practically the injection current is a positive and negative square current injection in the steady state, and the initial inductor current $i_L(0)$ is a dependent value which can not be defined arbitrarily. To find this initial value, and thus the ratio $K_i$ for a given I, the following complete dynamic current analysis is done.

Similar to the voltage solution, the complete inductor current solution can be expressed as:

$$i_L(t) = \frac{I+i(0)}{\sin\theta_i} e^{-t/T} \sin(\omega_f t + \theta_i) - I\,, \tag{11}$$

where

$$\theta_i = \tan^{-1} \frac{2\omega_f L(I+i_L(0))}{R(I-i_L(0))} = \tan^{-1} \frac{(1+K_i)\sqrt{4Q^2-1}}{1-K_i}\,. \tag{12}$$

Because the process is actually repeated each half cycle with only a polarity change, in the steady state, the relationship $i_L(t_z)=-i_L(0)$ must hold (as shown in Fig. 3). This condition can be further expressed as:

$$e^{-t_z/T} \sin(\omega_f t_z - \theta_i) + \frac{K_i - 1}{K_i + 1} \sin\theta_i = 0\,. \tag{13}$$

Although accurate analytical analysis is very difficult, theoretically the solutions of $t_z$ and another variable $K_i$ are governed by (9) and (13) with $\theta$ and $\theta_i$ as interim variables which are associated with the auxiliary equations (10) and (12). With the help of modern computing techniques, numerical solutions of such problems can be implemented using many available software packages such as MATLAB and C. A computer program has been developed to do the analysis and its flow chart is shown in Fig. 4.

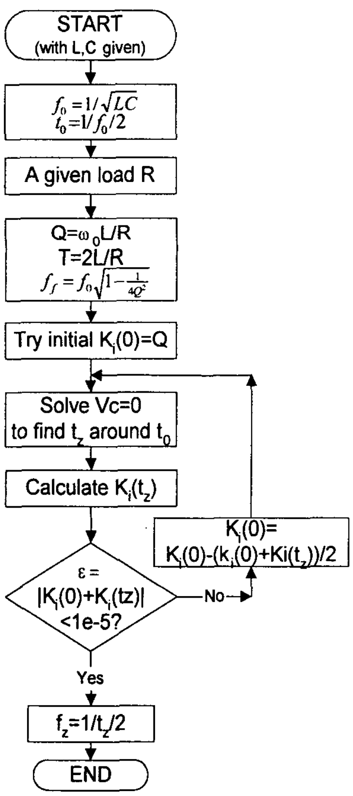


Fig. 4. Flow chart of a numerical analysis

The program starts with given circuit parameters L and C, so the undamped natural frequency $\omega_0$ and its half-period $t_0$ can be determined. Then we can choose a load R and calculate Q, time constant T, as well as the free ringing frequency $f_f$. Then comes the most important part of the algorithm. We have an initial try $K_i(0)=Q$ to start the iteration. With $K_i(0)$ known, a numerical solution $t_z$ can be obtained by finding the zero of equation (13) around $t=t_0$, then with this $t_z$ known we can calculate $K_i(t_z)=i_L(t_z)/I$ using (11). The next step is to check whether $K_i(t_z)$ and $-K_i(0)$ have converged to a given error limit, e.g. $\varepsilon=10^{-5}$. If the answer is YES, the program terminates with ZVS frequency $f_z$ calculated; otherwise, the iteration repeats with $K_i(0)$ updated at half an error step each time until a converged solution is obtained.

### *C. Approximate Analytical Analysis*

An approximate analytical result can be very helpful in providing a better starting guess of the initial values and the range of the final solutions for the numerical analysis. To achieve such a result, equation (9) can be rewritten in the following format:

$$\sin(\omega_f t - \theta) = -\sin\theta e^{t/T}\,, \tag{14}$$

so that we can see that the solution is the intersection point of a sine function curve and an exponential function curve. Using Tailor's series and ignoring the high order items: $\sin\theta = \theta$ and $e^{t/T} = 1 + t/T$, we can get the following approximate analytical solution:

$$t_z = \frac{\pi + 2\theta}{\omega_f - \theta/T}\,. \tag{15}$$

Thus, the ZVS frequency can be expressed as:

$$\omega_z = \frac{\omega_f - \theta/T}{1 + 2\theta/\pi}\,. \tag{16}$$

Furthermore, by considering the input and output power balance, the current ratio can be estimated as:

$$K_i = \frac{4}{\pi}\sqrt{Q^2 - 1}\,. \tag{17}$$

With this estimation, equation (10) becomes:

$$\theta = \tan^{-1} \frac{\pi\sqrt{4Q^2-1}}{8Q^3\sqrt{Q^2-1}+\pi(2Q^2-1)}\,. \tag{18}$$

Equation (16) and its auxiliary equation (18) give direct analytical ZVS frequency solution without iterative numeric computation.

## IV. Analysis Results And Discussions

Fig. 5 shows the relationship of ZVS frequency versus Q. It can be seen that the numerical results are quite accurate as they are very close to the direct waveform measurements from element level PSpice simulations which have been found to be in agreement with practical experiments.

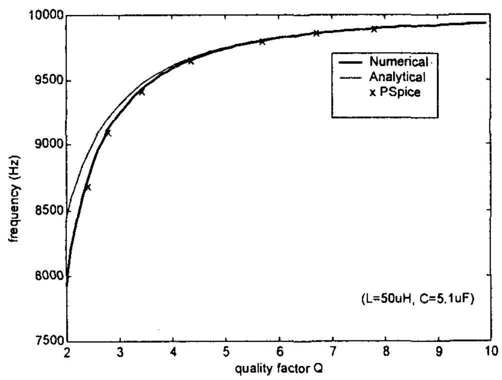


Fig. 5. ZVS frequency results

Fig. 5 also shows that the analytical results are quite good for large Q's, but the error becomes larger when Q is smaller. There are two main reasons for this: the assumption made for Tailor's series for solving the nonlinear equation (14), and the estimation of $K_i$ as shown in (17). Both procedures are only valid for large Q's. Hence the error becomes larger as Q reduces.

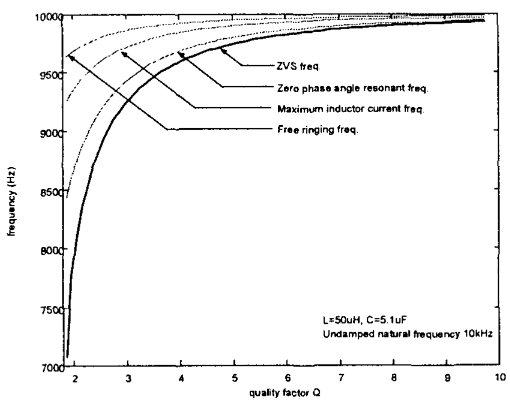


Fig. 6. Different resonant frequencies

Fig. 6 compares the different resonant frequencies in the same graph. It can be seen that all the resonant frequencies tend to converge to the undamped natural frequency 10kHz when Q is large. However, it is clear that the differences are quite large for small Q's, ZVS frequency being the lowest and free ringing frequency the highest. The maximum difference can as high as 30%. An important aspect is that the critical load conditions of the resonant situations are different. From (1) to (3), we can see that the minimum requirement of Q for free ringing, maximum inductor current resonance, and zero phase angle resonance is respectively **0.5**, **0.707**, and **1.** Due to the DC offset and the decay in a half-switching cycle (as shown in Fig. 3), a higher Q is required to maintain the ZVS condition. It is found from the numerical analysis that when Q is smaller than **1.86**, there will be no zero crossing solution, which means a steady state ZVS condition does not exist at all.

## V. Conclusion

Four definitions of resonant frequencies and their critical resonant conditions available for a parallel resonant tank have been identified in this paper. The natural frequency, also called free the ringing frequency, reflects the natural oscillatory property of the electric and magnetic energy circulation in a LC circuit; while the zero phase angle resonant frequency and the maximum inductor current resonant frequency are both based on the steady state solution with sinusoidal excitation. At zero phase angle resonant frequency, the reactive energy circulates inside the resonant tank only and the terminal voltage and current are in phase. Meanwhile, at the maximum inductor current resonant frequency, the inductor current reaches its maximum magnitude in the steady state.

For a current-fed resonant converter, the operating frequency is actually the ZVS frequency, which is different from other resonant frequencies. Based on a complete dynamic analysis of a step current injection model in a half-switching period, a numerical computation algorithm and an approximate analytical equation have been obtained for the ZVS frequency analysis. It has been shown that the numerical solutions are quite accurate as they are almost the same as the component level PSpice simulation results, and the analytical analysis gives good solutions for large Q's.

## References

[1] J. T. Boys, and A.W. Green, "Inductively coupled power transmission – concept, design and application," *IPENZ Trans.*, 1995, 22, (1) EMCH, pp. 1-9.

[2] A. P. Hu, and J. T. Boys, "Dynamic ZVS direct on-line start up of current-fed resonant converter using initially forced DC current," 2000 IEEE International Symposium on Industrial Electronics, 4-8 December 2000, in press.

[3] J. T. Boys, G. A. Covic, and A. W. Green, "Stability and control of inductively coupled power transfer system," *IEE Proc. Electric. Power Appl.*, 2000, 147, (1), pp. 37-43.

[4] A. W. Green, "Modelling a push-pull parallel resonant converter using generalised state-space averaging," *IEE Proc. B*, 1993, 140, (6), pp. 350-356.

[5] A. W. Green, and J.T. Boys, "10kHz Inductively coupled power transfer – concept and control," *IEE Power electronics and variable speed drives conference*, PEVD, 1994, 399, pp. 694-699.

[6] H. H. Skilling, *Electrical Engineering Circuits*, New York: John Wiley and Sons, INC., 1961.