A UNIFIED MODEL OF CURRENT FEEDBACK IN SWITCH MODE CONVERTERS

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Abstract. The mechanism of duty cycle generation in Current Mode (CM) and Average Current Mode (ACM) PWM converters was examined and found to be identical except for the functional relationship of the controlling parameters. This result was used to develop a SPICE compatible and topology independent Generic Current Mode (GCM) Model. The proposed universal model can be applied without further mathematical derivation to explore the large and small signal performance of CM, ACM and other realizations of current feedback in PWM converters.

I. INTRODUCTION

Current Mode (CM) control [1] was shown to be a useful design technique for improving the dynamic response of PWM converters. The basic feature of this approach is the introduction of a second, 'inner' feedback loop, which is a function of the inductor current. The additional loop complements the main, or 'outer' voltage loop. Recently, an alternative realization of CM, the Average Current Mode (ACM), has been proposed [2,3].The main difference between CM and ACM is that in the former current feedback is proportional to the peak inductor current, but in the latter it is mainly proportional to the average inductor current.

Analytical evaluation of the dynamic response of current mode converters is rather complex since the expressions that one gets in the symbolic derivations are difficult to handle and even to comprehend. This is especially true when a realistic configuration, which might include an input filter, is studied [4,5]. Consequently, there is currently an urgent need to develop analysis and design tools that will free the researchers and engineers from the computational chores involved in the analysis and design of CM and ACM systems. One of the possible approaches for the realization of such tools are SPICE [6] compatible models which emulate the average response of the system under study. As it has been already shown, the basic Switch Inductor Model (SIM) [7] can be expanded to include the CM feature [8]. These SPICE compatible models were recently shown to be effective and simple to apply tools for studying the dynamic behavior of CM systems equipped with an input filter [4]. The purpose of the present study was to develop a unified SPICE compatible model of PWM converters with current feedback, for which the CM and ACM realizations are private cases.

II. THE GENERIC CURRENT MODE MODEL

A close examination of the fundamental features of the CM realization and the ACM realization (Fig. 1) reveals that they differ mainly in one aspect: The frequency response of the current feedback loop. In the CM realization this response is , to a first order approximation, frequency independent ('k_1' in Fig. 1). In the ACM, on the other hand, the response is shaped by the Z_1 and Z_2 networks of the feedback amplifier A_1. It has been proposed [2] that this response should follow the general shape depicted in Fig. 2. The main features of this transmission are high gain at low frequency, flat portion around the switching frequency ($f_S=1/T_S$) and a lowpass behavior at higher frequencies. The purpose of the flat portion is to retain the triangular shape of the inductor current. In the followings it will be therefore assumed that the output signal of amplifier A_1 (Fig. 1b) includes the triangular waveform of the inductor current multiplied by a constant ('k_1' in Fig. 2), along with low frequency signals which are a function of the average inductor current and output voltage.

The mechanism that determines the duty cycle (D) in the CM realization can be followed by examining the trigonometrical relationship of the waveforms shown in Fig. 3. The output of the error amplifier (V_c), modified by a 'compensation slope' (M_C) [1] is compared to the inductor current (I_L) which is characterized by a rising slope (k_1m_1), a falling slope (k_1m_2) and low frequency component (A). The average magnitude of (A), per cycle, is denoted (I_{av}). The PWM switch is turned 'off' when the amplified inductor current reaches the momentary value of (V_c).

In the ACM (Fig. 4), the switch is turned 'off' when the magnitude of the sawtooth rising slope (S_C) reaches the output voltage of amplifier (A_1). The output of this amplifier (V_A) includes a remnant of the inductor current waveform with slopes (S_1) and (S_2), and a low frequency component (A) which is a function of the inductor current

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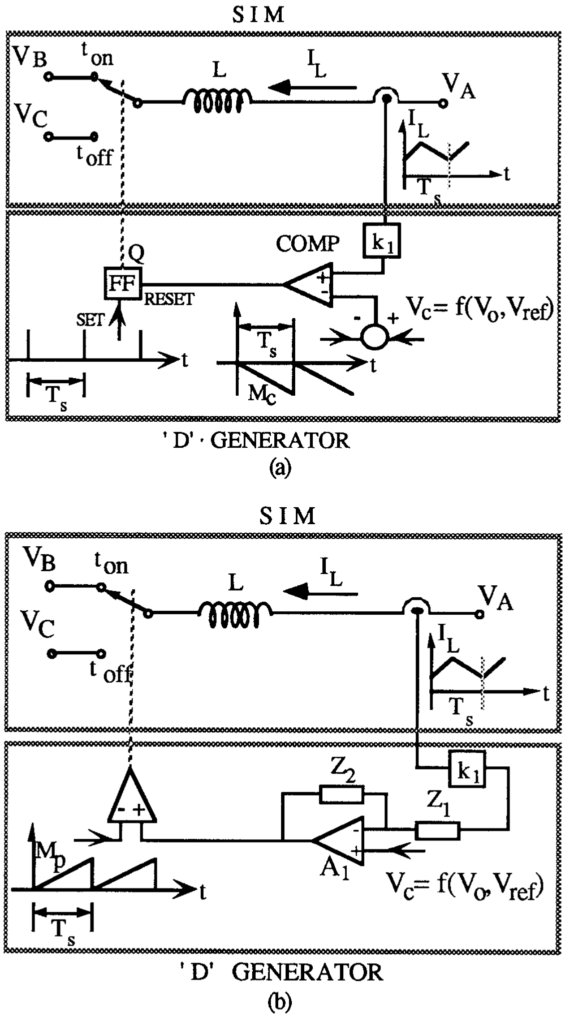

Fig.1 Generic features of Current Mode (CM) (a) and Average Current Mode (ACM) (b) PWM converters.

and output voltage. The average value of (A), per cycle, is denoted (I).

An examination of the waveforms of Figs. 3 and 4 discloses the interesting fact that despite the difference in realization, the mechanism of 'D' generation in the two cases is remarkably similar. It is evident that in both cases, the T_{ON} duration is determined by the time it takes two sloped signals, with a given initial separation, to intersect. That is, the two methods are in fact special cases of a general model except for the functional relationship between the parameters of the converter and the initial separation and slopes of these signals. It is thus conceivable that a Generic Current Mode (GCM) model can be used to describe both the CM and ACM, as well as

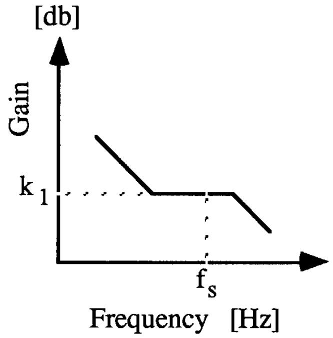

Fig. 2 The general waveshape of the current loop response in ACM.

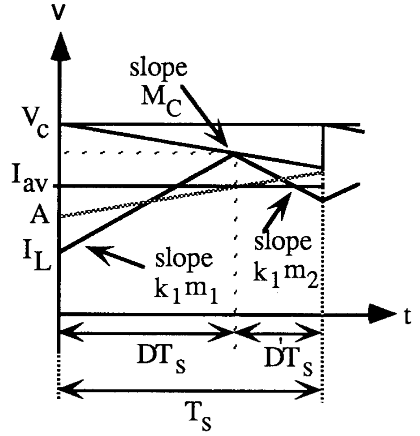

Fig. 3 Basic CM waveforms. See text for notations.

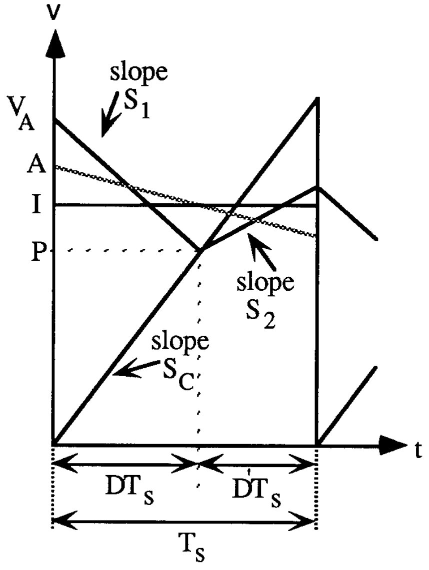

Fig. 4 Basic ACM waveforms. See text for notations.

many other possible realizations. This conclusion is the underlining premise of the proposed model.

The duty cycle function of the GCM Model. The basic duty cycle (D) function of the GCM is developed as a function of the switching period, the closing slopes (S_C) and (S_1) and the per cycle average (I) of the signal which comprises a falling slope (S_1) and a rising slope (S_2). The intersection at (DT_S) determines the duty cycle (D). Defining (P) as the magnitude of the rising slope (S_C) at the intersection time (DT_S) we find:

$$P = S_C \, D \, T_S \qquad (1)$$

The distance (I-P) in Fig. 4 is the average value of the two triangles formed by slopes (S_1) and (S_2):

$$P\text{-}I = \frac{\dfrac{(S_1 \, D \, T_S) \, D \, T_S}{2} + \dfrac{(S_2 \, D' \, T_S) \, D' \, T_S}{2}}{T_S} \qquad (2)$$

where D' = 1-D and the slopes (S_1) and (S_2) are taken in absolute value.

By combining (1) and (2) one gets [9]:

$$I = S_c \, D \, T_s + 0.5 \, S_1 \, D^2 \, T_s + 0.5 \, S_2 \, (1\text{-}D)^2 \, T_s \qquad (3)$$

This second order equation in (D) can now be used to calculate the duty cycle for a given set of parameters and variables (I, S_C, S_1, S_2, T_S). Furthermore, continuous SPICE simulation of the average behavior can be carried out by emulating the functional relationship of (3) by an equivalent circuit as reported earlier [8].

The ACM case. In the private case of ACM, the variable (I) represent the per cycle average of the amplifier's (A_1) output voltage (Fig 1b). Hence:

$$I_{ACM} = F_c \, V_c - \bar{I}_L \, k_1 \, F_I \qquad (4)$$

where (refer to Figs. 1b and 2) k_1 is the gain of the current loop at (f_s), $\bar{I}_L$ is the average inductor current, V_c is the voltage feedback signal and F_I and F_c are the transfer functions of the current and voltage paths:

$$F_I = \frac{Z_2}{Z_1} \qquad (5)$$

$$F_c = \frac{Z_1 + Z_2}{Z_1} \qquad (6)$$

The slopes of the ACM are defined as follows:

$$S_1 = F_I k_1 m_1 \qquad (7)$$

$$S_2 = F_I k_1 m_2 \qquad (8)$$

$$m_1 = \left| \frac{V_A - V_B}{L} \right| \qquad (9)$$

$$m_2 = \left| \frac{V_A - V_C}{L} \right| \qquad (10)$$

$$S_c = M_p \qquad (11)$$

where (V_A) and (V_B), are the switched inductor voltages as defined in Fig. 1b, (L) is the inductance of the main inductor and (M_p) is the slope of the PWM slope (Fig. 1b).

The CM case. In this case we still find :

$$I_{CM} = F_c \, V_c - \bar{I}_L \, k_1 \, F_I \qquad (12)$$

$$S_1 = F_I \, k_1 m_1 \qquad (13)$$

$$S_2 = F_I \, k_1 m_2 \qquad (14)$$

But:

$$F_c = F_I = 1 \qquad (15)$$

and

$$S_C = M_C \qquad (16)$$

where M_C is the compensation slope (Figs. 1a and 3).

III. RESULTS AND CONCLUSIONS

Following the simulation procedure suggested earlier [4, 7, 8]. We developed a SPICE input file that describes a general equivalent circuit of the switched inductor model (SIM) [7] and the GCM model presented here. The general form of the SPICE compatible equivalent circuit of the GCM is given in Fig. 5. The simulation was carried by HSPICE (Meta Software Inc., Campbel, CA, USA) on a VAX (Digital Equipment Corp., Marlboro, Mass.) computer. Typical simulation results of small signal responses for Buck Converter are shown in Fig 6. The simulation parameters were as follows:

Power stage [3]: V_{in} =14 volt, V_{out}=5 volt, L=37.5 µH, C (output capacitor) = 380 µF, R_{out} (load)=1 Ω, R_c (capacitor's ESR)= 20 mΩ, f_s=50 kHz.

Current loop parameters: Average: k_1=0.1, Z_1=R_1=2.2 KΩ, Z_2=C_{fp}‖(R_f+C_{fz}), C_{fp}=220 pF, C_{fz}=5.8 nF and R_f=30.5 KΩ, Slope Compensation (Mp=M_C)=2.5 A/µsec.

Peak: k_1=0.33.

An important conclusion of the present study is that the CM and ACM are private cases of a family of converters in which the voltage feedback loop is supplemented by a current feedback loop. In CM the transfer function of the current loop is frequency independent. In the ACM realization the current loop gain

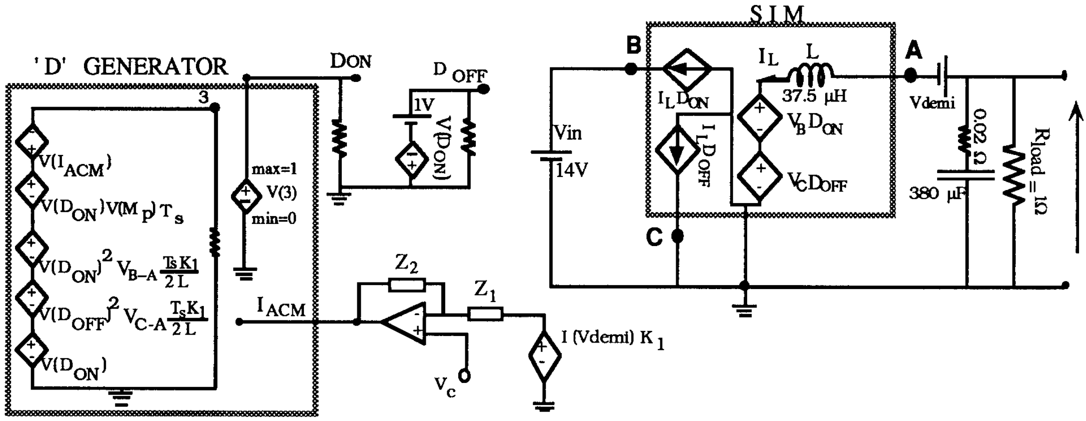

Fig. 5 SPICE compatible equivalent circuit of the GCM model

is enhanced at low frequencies. Obviously, many other realizations are possible and may have advantages in specific applications. It is of interest to note both the CM and ACM realization may be subjected to sub-harmonic oscillations [1] if:

$$|S_c| < |S_1| \qquad (17)$$

Since the SIM and GCM models are topology independent they can be prepared as a library subcircuit and be called upon for any given case. The ability to carry out both small and large signal simulations without the need for any further mathematical derivations could make the proposed models a viable research and engineering tool.

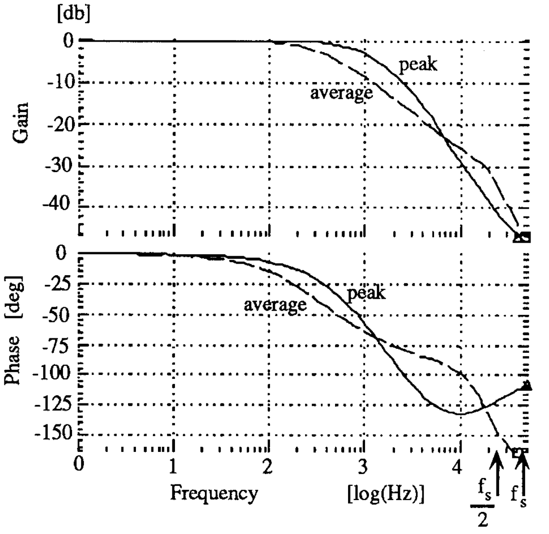

Fig.6 Buck converter control to output transfer function with a closed inner loop for Average Current Mode (ACM) [3] and peak Current Mode (CM) with the same power stage parameters.

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