A Unified Approach to Teaching Feedback in Electronic Circuits Courses

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Abstract—**A generalized negative feedback model is proposed as a vehicle for teaching feedback amplifiers in engineering schools. The model applies to both discrete and operational amplifier configurations and does not dictate the use of specific gain definitions (such as transconductance) for analyzing a given negative feedback amplifier. This paper presents the main features of the suggested methodology and gives some examples for quick evaluation of closed-loop gain, frequency response, and input and output impedances of negative feedback amplifiers. The proposed method has been successfully tested in the classroom over the past two years.**

Introduction

THE concept of negative feedback (henceforth referred to as feedback) and its application in electronic circuit analysis and design is widely recognized as one of the most important subjects in electronic curricula. In many university programs, basic feedback theory is first introduced in preceding courses such as "signals and systems," "dynamic systems," "linear systems," "introduction to control theory," and the like. However, notwithstanding the importance of basic feedback theory to the curricula, teachers of "analog electronic circuits" courses must still devote considerable time to developing the analytical tools required for handling feedback amplifiers. The problems on hand are the practical aspects of feedback amplifiers which do not show up at the block diagram level of feedback systems. These include input and output impedances, loading effects, direct paths from input to output, the distinction between current and voltage feedback, serial and parallel connection at the error summing point, and other related subjects.

The feedback concept in electronic circuits is usually first introduced in connection with discrete amplifiers. A comprehensive treatment will be given at this stage, deriving both the basic mathematical relationships and procedures for approximate analysis of feedback amplifiers using the feedback terminology and features. Surprisingly, however, this wealth of knowledge and experience gained by the students is put to little use, if any, when covering at a later stage the subject of operational amplifiers which are, in fact, the ultimate feedback devices. As an example of this lack of generality, consider the nonideal operational amplifier in the noninverting configuration. Experience shows that few students, and for that matter even experienced electronic engineers, can analyze it in terms of a feedback amplifier. The reason for this shortcoming is the absence of a comprehensible linkage between the treatment of feedback in discrete amplifiers and the operational amplifier configuration. This dificiency can be traced to the lack of generality in the treatment of feedback amplifiers. In fact, this lack of generality is a serious handicap to students when attempting to analyze operational amplifier based systems in the nonideal operating regions. Also, the unfamiliarity with a generalized feedback model makes it difficult to analyze feedback amplifiers which do not match the classical topologies usually considered in textbooks.

The introduction of practical, user friendly, and powerful computer-aided electronic circuit simulators is another reason for the need for reexamination of the electronic circuit courses curricula. Simulation tools, such as SPICE, enable instructors to introduce rather complex feedback configurations which can be easily tested numerically by the studies. Contrary perhaps to common belief, this development calls for a deeper understanding on the part of the students as a prerequisite for intelligent and efficient use of simulation. Furthermore, analytical comprehension of basic concepts and strategies such as feedback is indispensable for good circuit design.

This paper presents a unified approach to the subject of feedback in electronic circuits that was developed while teaching courses in "analog electronic circuits" and "linear integrated circuits." The proposed approach was developed to meet the following three main objectives:

1) to present a unified feedback model that is applicable to all classes of circuits, including operational amplifier based configurations;

2) to develop system insight to feedback amplifier analysis, and hence to facilitate the use of results from general feedback systems theory;

3) to develop a unified and systematic procedure for approximate analysis of feedback amplifiers emphasizing the central role of the "loop gain" concept and devising guidelines for its straightforward evaluation. (Accurate analysis can of course by easily performed by CAD tools.)

This paper covers only the outline of the teaching methodology developed to meet the above objectives. It mainly

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emphasizes the aspect of the unified approach which is believed to be novel. Of course, numerous textbooks and papers cover the subject of feedback amplifiers. Many of them, for example [1]–[3], include elements of the proposed approach but fail to present a generalized model to treat all discrete and operational amplifier configurations.

Since the main objective of this paper is to present the proposed unified approach, it does not cover the subject matter of feedback in full, nor does it discuss aspects of homework assignments and application of computer-aided electronic circuit simulation. The proposed methodology is discussed and exemplified in terms of small signal steady-state analysis assuming sinusoidal excitation.

The Generalized Feedback Model

One of the reasons for the difficulty which students face when analyzing feedback amplifiers is the fact that they are usually familiar with only the truncated model of feedback systems (Fig. 1). Embedded in this simplified model are the following three basic assumptions:

1) in the absence of feedback ($\beta = 0$) the input signal is fed directly to input ports of the amplifier;

2) that the feedback signal originates at the system's output;

3) that there is no direct path between input and output (other than through A);

4) that the feedback network does not load the amplifier's input and output nodes.

As an example of the discrepancy between the classical canonical model of Fig. 1 and real electronic circuits, consider the simple discrete feedback amplifier of Fig. 2. For this circuit, none of the assumptions apply. Consequently, application of the elementary feedback model of Fig. 1 to this case requires considerable preparation and a host of rules of thumb as a prerequisite for even an approximate analysis. These problems can be circumvented if the extended, generalized feedback model of Fig. 3 is used. The generalized model includes blocks which are usually found in practical electronic circuits. G denotes the transfer ratio between the input ports of the system and the amplifier's (dependent source) control terminals (ϵ). A represents the main amplifier gain and Q is the transfer ratio between the signal return point ($S_{o'}$) and output terminals (S_o). D denotes the direct path between input and output. The connection on the output side could be either to the feedback sensing point (D') or directly to the output terminals (D). The signals S (assumed here for simplicity to be sinusoidal perturbations) could be current or voltage signals or a *combination* of both. In the latter case the overall transfer function block will not be dimensionless.

It should be noted that the blocks of the generalized model (Fig. 3) represent unidirectional transfer functions which include all input and output loadings. Furthermore, the blocks are not necessarily a one-to-one translation of the original electronic circuit topology. For example, (R_f) in Fig. 3 does not correspond to any particular block (such

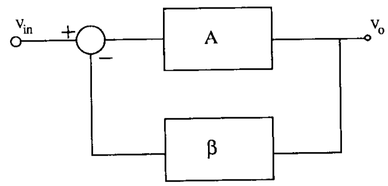


Fig. 1. The classical negative feedback model.

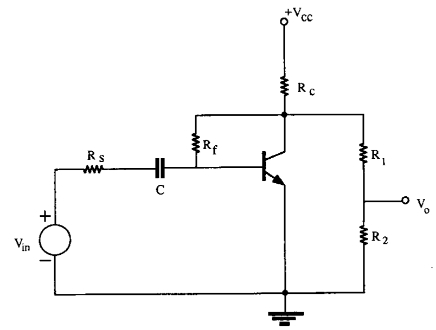


Fig. 2. An example of a feedback amplifier which is incompatible with the model of Fig. 1.

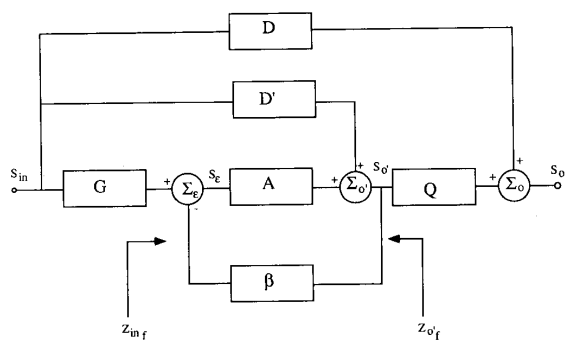


Fig. 3. Proposed generalized model of negative feedback amplifiers.

as β). Rather, (R_f) will be included in the expression for say, (β) and (D).

Defining (A_c) as the gain of the dependent source, the transfer functions of the generalized feedback model (Fig. 3) are formally defined as follows:

$$G = \left.\frac{S_\epsilon}{S_{in}}\right|_{AC=0} \tag{1}$$

$$A = \left.\frac{S_{o'}}{S_\epsilon}\right|_{S_{in}=0} \tag{2}$$

$$\beta = \left.\frac{S_{o'}}{S_\epsilon}\right|_{AC=0,\,S_{in}=0} \tag{3}$$

$$Q = \left.\frac{S_o}{S_{o'}}\right|_{AC=0,\,S_{in}=0} \tag{4}$$

$$D = \frac{S_o}{S_{in}}\bigg|_{Ac=0, S_{in}=0} \quad (5)$$

$$D' = \frac{S_{o'}}{S_{in}}\bigg|_{Ac=0, S_{in}=0}. \quad (6)$$

Note that the definitions of the transfer functions include the normal loading of the actual circuit. The open-loop relationships are obtained by zeroing the controlled gain of the dependent source (A_c), a procedure which does not affect input or output loading conditions. The transfer functions (D) and (D') represent the feedforward paths that are present even when the gain of the dependent source (A_c) is zero.

It can easily be shown that the expression for the closed-loop gain (A_{CL}) of the system represented by the model of Fig. 3 is

$$A_{CL} \triangleq \frac{S_o}{S_{in}} = \frac{(GA + D')}{1 + \beta A} Q + D. \quad (7)$$

Clearly, not all blocks, even if present, are significant. In particular, QD' and D are negligibly small in most practical circuits. However, the inclusion of these often neglected blocks serves an important educational purpose. First, it emphasizes the fact that direct paths do exist in the general case, and hence one would usually expect some output signal even if the gain of the dependent source drops to zero. The second reason for inclusion of the direct paths in the model is to help explain discrepancies between accurate analysis (say by solving the network equations) and approximate analyses. Only by inclusion of the direct path can one obtain accurate solutions of even simple circuits such as a BJT follower (see below).

The generalized closed-loop gain expression (7) emphasizes the following two major and fundamental corollaries.

1) βA is dimensionless, whereas GAQ has the same dimensions as the desired transfer ratio S_o/S_{in}.

2) Evaluation of the expression for the individual transfer function blocks (A, β, etc.) is really not necessary when deriving the closed-loop gain (A_{CL}), or for that matter input and output impedances, of feedback amplifiers. For all practical purposes it is sufficient to evaluate the expression of combined blocks, i.e., βA, GAQ, and $GD'Q$.

Based on these premises, the generalized procedure for deriving the approximate (or accurate) closed-loop gain will be as follows.

1) Identify and define the dependent source and the summing nodes of the systems. In particular, identify (A_c) (S_ϵ) and ($S_{o'}$) and define their nature (voltages or currents and gain type) in the most convenient way.

2) Evaluate βA by setting $S_{in} = 0$ and forcing the control signal of the dependent source to be unity. The return ratio (S_ϵ) is βA.

3) Evaluate GAQ by a two-step process. First evaluate G by forcing the dependent source to be zero (short if a voltage source and open circuit for a current source) and deriving the transfer ratio between the input (S_{in}) and the control port of the dependent source (S_ϵ). AQ is then derived by expressing the output (S_o) as a function of a unity signal to the dependent sources input terminals (S_ϵ).

4) Evaluate $D'Q$ and D when the dependent source is zeroed.

4) Evaluate $D' \cdot Q$ and D when the dependent source is zeroed.

It is important to reemphasize the point that the evaluations of the transfer functions are carried out by zeroing independent and dependent sources but without shorting and/or disconnecting any of the circuit's branches. By this, all actual loadings are taken into account.

As an example to the procedure suggested above, consider the amplifier of Fig. 2. Starting with the simplified (h) parameter small-signal model [Fig. 4(a)], we identify Σ_ϵ as a parallel summation and arbitrarily assume that it is a current summation [Fig. 4(b)]. If a hybrid π model would have been used, parallel voltage summation (by superposition) could have been chosen. The loop gain ($\beta A = -I_b|_{S_{in}=0}$) is easily evaluated from the equivalent circuit of Fig. 4(c) by shorting the input source and setting the magnitude of the controlled source to ($h_{fe} \cdot 1$). Next, G is evaluated from the equivalent circuit of Fig. 4(d) ($G = I_b|_{Ac=0}$) by setting $V_{in} = 1$ and zeroing the dependent source. $A \cdot Q$ is evaluated from Fig. 4(e) ($A \cdot Q = V_o|_{S_{in}=0}$) by setting $I_b = 1$ and zeroing the input. Finally, $D'Q$ is derived using the equivalent circuit of Fig. 4(d). In this particular case $D = 0$ since there is no direct bypass to V_o except via D' which is taken into account when deriving $D'Q$. For midband analysis, $X_c \to 0$ and the expression will be manageable. If $D'Q << GAQ$ the direct path can, of course, be neglected.

It is evident that the procedure suggested above does not dictate the use of specific dependent sources. The analyses can be carried out in terms of a voltage-to-voltage dependent source, a mutual conductance source, or any other which may be convenient.

Input and Output Impedances

It is important to emphasize the fact that the familiar expression for input and output impedances of negative feedback amplifiers applies only to the impedances of the summing points Σ_ϵ and $\Sigma_{o'}$ (Fig. 3). The input and output impedances of the complete system can be derived by first evaluating $Z_{in'f}$ and $Z_{o'f}$ (Fig. 3) and then adding the contributions of the auxiliary G and Q branches.

For a parallel connection at the input [Figs. 3 and 5(a)]

$$G_{in'f} = G_\epsilon(1 + \beta A), \quad (8)$$

for a series connection [Figs. 3 and 5(b)]

$$Z_{in'f} = Z_\epsilon(1 + \beta A), \quad (9)$$

for voltage feedback [Figs. 3 and 5(c)]

$$Z_{o'f} = \frac{Z_{o'}}{1 + \beta A}; \qquad G_{o'f} = G_{o'}(1 + \beta A), \quad (10)$$

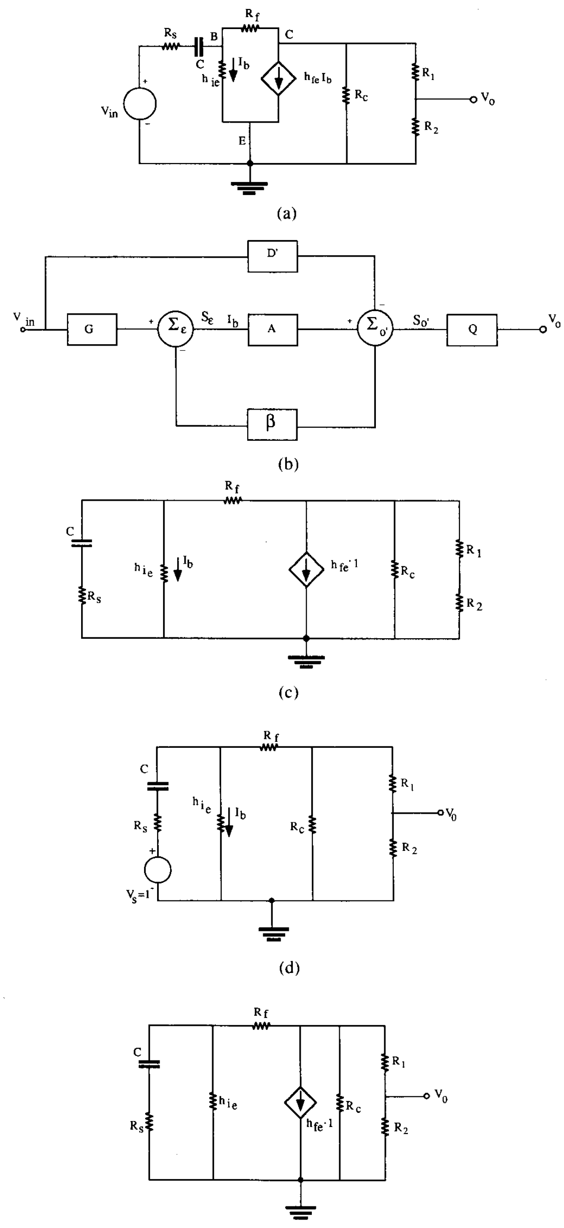


Fig. 4. Systematic analysis of the negative feedback amplifier of Fig. 2. (a) Small-signal model. (b) Feedback model. (c) βA derivation. (d) Evaluation of G. (e) Evaluation of AQ and D'.

and for current feedback [Figs. 3 and 5(d)]

$$Z_{o'f} = Z_{o'}(1 + \beta A) \tag{11}$$

where Z_ϵ, G_ϵ, and $Z_{o'}$, $G_{o'}$ are the impedances and admittances (including all loadings) seen into the summation junctions, when both the input and the dependent sources are zeroed.

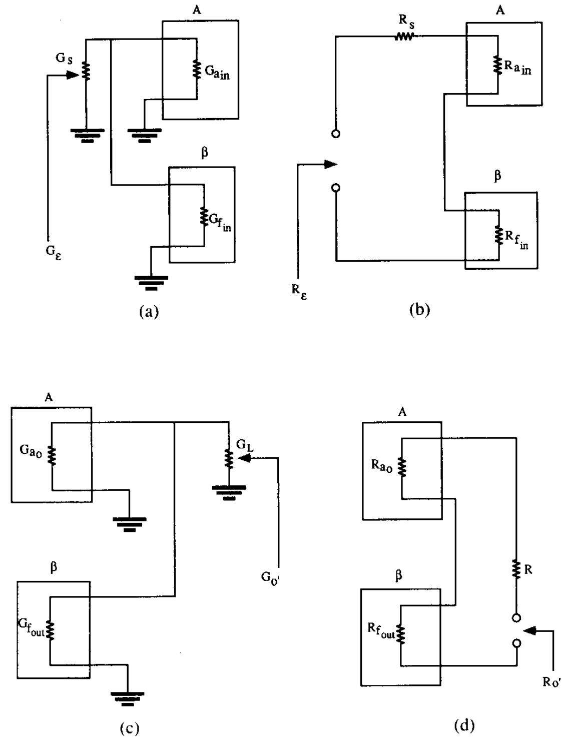


Fig. 5. Input and output connections of negative feedback amplifiers. (a) Parallel connection input. (b) Series connection at input. (c) Voltage feedback. (d) Current feedback.

As an example, consider the voltage follower of Fig. 6. βA is evaluated from the circuit of Fig. 6(b) to be

$$\beta A|_{v_s=0} = -\frac{h_{fe}R_e}{R_e + h_{ie} + R_s} \tag{12}$$

whereas

$$Z_\epsilon|_{h_{fe}=0} = R_s + h_{ie} + R_e. \tag{13}$$

Hence

$$Z_{in'f} = (R_s + h_{ie} + R_e)\left(1 + \frac{h_{fe}R_e}{R_s + h_{ie} + R_e}\right) \tag{14}$$

$$R_{inf} = Z_{in'f} = h_{ie} + R_s + (h_{fe} + 1)R_e. \tag{15}$$

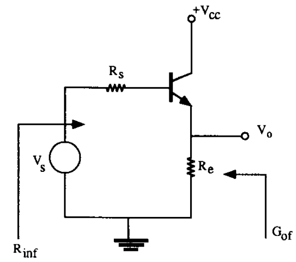
For the output impedance [Fig. 5(c)]

$$Z_{o'} = R_e \| (h_{ie} + R_s) \tag{16}$$

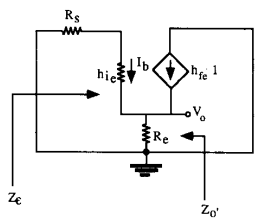
$$Z_{o'f} = \frac{R_e \| (h_{ie} + R_s)}{1 + \dfrac{h_{fe}R_e}{R_s + h_{ie} + R_e}} = \left(\frac{h_{ie} + R_s}{h_{fe} + 1}\right) \| R_e. \tag{17}$$

Similarly, for the inverting amplifier [Fig. 7(a) and (b)] the input configuration is a parallel connection [Fig. 5(a)]. Assuming $R_o << R_f, R_L$ [Fig. 5(a)]

$$G_\epsilon = G_s + G_d + G_f \tag{18}$$

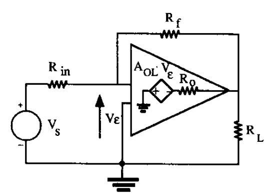


(a)

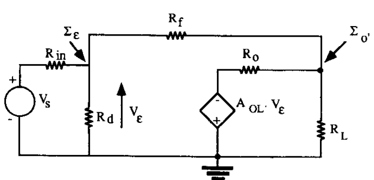


(b)

Fig. 6. (a) A BJT voltage follower and (b) small-signal equivalent circuit used in proposed analysis procedure.



(a)



(b)

Fig. 7. (a) An inverting amplifier and (b) its equivalent circuit used for evaluating input and output impedances.

$$\beta A = A_{OL} \frac{G_f}{G_f + G_s + G_d} \tag{19}$$

$$G_{in'_f} = (G_s + G_f + G_d)\left(\frac{A_{OL} G_f}{G_f + G_s + G_d}\right) \tag{20}$$

$$G_{in'_f} = G_s + G_d + G_f(A_{OL} + 1). \tag{21}$$

For the output impedance

$$G_{o'} \cong G_o \tag{22}$$

$$G_{o'f} = G_o\left(1 + \frac{A_{OL} \cdot G_f}{G_f + G_s + G_d}\right). \tag{23}$$

High-Loop Gain

The cases for which $|\beta A| >> 1$ are of special interest. In these cases the closed-loop gain asymptotically approaches the expression

$$\left.\frac{S_o}{S_{in}}\right|_{|\beta A| \to \infty} = \frac{GQ}{\beta} + D. \tag{24}$$

On the other hand, when $|\beta A| << 1$

$$\left.\frac{S_o}{S_{in}}\right|_{|\beta A| << 1} = GAQ + D' \cdot Q + D. \tag{25}$$

When D' and D are negligibly small

$$\left.\frac{S_o}{S_{in}}\right|_{|\beta A| >> 1} \cong \frac{GQ}{\beta} \tag{26}$$

$$\left.\frac{S_o}{S_{in}}\right|_{|\beta A| << 1} \cong GAQ. \tag{27}$$

As an example, consider the inverting amplifier of Fig. 7(a). If $R_o << R_f, R_L$

$$D' = D \cong 0; \quad Q \cong 1 \tag{28}$$

$$G = \frac{R_f}{R_f + R_{in}} \Rightarrow GA = -\frac{A_{OL} R_f}{R_f + R_{in}} \tag{29}$$

$$\beta A = A_{OL} \frac{R_{in}}{R_f + R_{in}}. \tag{30}$$

Hence

$$\left.\frac{V_o}{V_s}\right|_{|\beta A| >> 1} = -\frac{GA}{\beta A} = -\frac{R_f}{R_{in}}. \tag{31}$$

Frequency Response

Approximate closed-loop Bode plots can be easily derived by considering the two regions $|\beta A| >> 1$ and $|\beta A| << 1$ (we assume here $D' = D = 0$, $Q = 1$). The method will be demonstrated by considering the inverting amplifier of Fig. 7(a). Since G/β is the closed-loop gain for $|\beta A| >> 1$ it corresponds to the closed-loop gain of an ideal amplifier. Hence, the transfer function can be found by assuming a virtual ground at the negative input terminal. At high frequencies when A_{OL} decreases, βA_{OL} will drop below unity and the overall gain will approach GA_{OL}. Consequently, the procedure for deriving the closed-loop frequency response for operational amplifier based systems will be as follows (Fig. 8).

1) Draw the closed-loop response assuming that the operational amplifier is ideal. This corresponds to $(G/\beta)(f)$.

2) Draw A_{OL} of the operational amplifier.

3) Draw $G(f)$ for the high frequency range.

4) Multiply $A_{OL}(f)$ by $G(f)$ at high frequencies to obtain the intersection of GA_{OL} with G/β. Note that this shifts $A_{OL}(f)$ by $G(f)$ downward since G represents a divider, and hence $|G| < 1$.

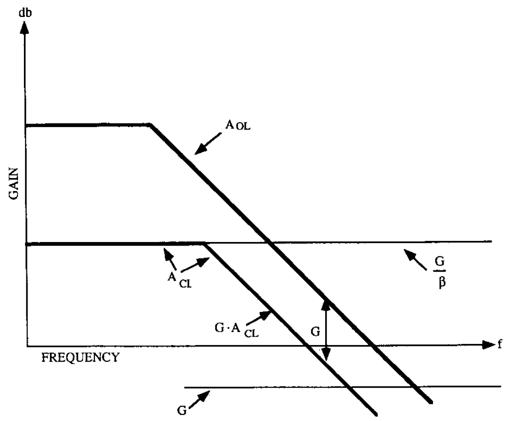


Fig. 8. Graphical representation of the proposed method for evaluating closed-loop response of negative feedback amplifiers.

Discussion and Conclusions

The main feature of the generalized feedback model presented here is its ability to cover most, if not all, practical feedback amplifier configurations. It is believed that little, if any, advantage is gained by using recipes which rigidly dictate the use of specific amplifier types (e.g., transconductance) for each given configuration. The fact that βA is dimensionless permits the use of any model to describe the dependent source, be it a voltage, current transconductance, or mutual resistance amplifier. Obviously, β will always have the reciprocal dimensions of the conveniently defined A. And as already shown, there is really no need to derive A or β individually. This can save considerable effort and ambiguity. Even for asymptotic expressions when $|\beta A| >> 1$, it is usually easier to derive GA and βA rather than to try to evaluate each transfer ratio separately.

The inclusion of the Q transfer ratio in the model (Fig. 3) serves an important educational purpose. It emphasizes the fact that feedback signals are not always taken from the output terminal. To illustrate this point, consider the amplifier of Fig. 9. Despite its simplicity, it would be rather cumbersome to derive the expression of the closed-loop gain if the truncated feedback model of Fig. 1 is used. Using the unified theory proposed here, the closed-loop response can be derived quickly and accurately. Choosing the control signal (S_ϵ) to be I_b and the output ($S_{o'}$) to be I_c one obtains

$$GAQ = \frac{V_o}{V_s} = -\frac{R_L h_{fe}}{R_s + h_{ie} + R_e} \qquad (32)$$

and

$$\beta A = I_b = \frac{h_{fe} \cdot R_e}{R_e + h_{ie} + R_s} \qquad (33)$$

hence

$$A_{OL} = \frac{GAQ}{1 + \beta A} = -\frac{R_L h_{fe}}{h_{ie} + R_s + R_e(h_{fe} + 1)}. \qquad (34)$$

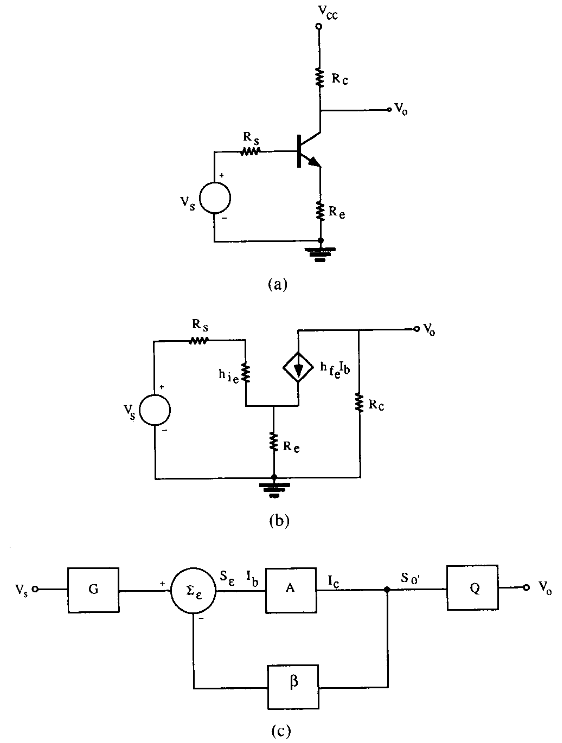


Fig. 9. A negative feedback amplifier in which the feedback signal is not taken from: (a) output, (b) its small-signal equivalent circuit, and (c) feedback model.

As demonstrated above, the proposed approach is applicable to both discrete and operational amplifier based systems. This is facilitated by the introduction of G which is fundamental to the inverting amplifier topology. Without its inclusion in the model, analysis of the inverting amplifier using negative feedback tools is far from being straightforward. An important corollary to the proposed unified approach is the immediate conclusion that βA is identical for both the noninverting and inverting amplifiers. This implies that both amplifiers will have identical closed-loop output impedances if the feedback components are the same. Indeed, when the input signals are zeroed, the amplifiers are identical. This simple yet important conclusion cannot be easily reached by previous analytical techniques whch are based on the truncated feedback model of Fig. 1.

The proposed unified approach was class-tested in the course sequence "Linear Electronic Circuits" and "Linear Integrated Circuits" which are mandatory courses for EE&CE students at the Ben-Gurion University of the Negev. The method was first introduced about ten years ago in the latter course to help bridge the gap between OA based amplifiers and discrete feedback amplifiers. When first introduced, the author was teaching only the second course of the series. The method was developed after finding out that the rigid feedback analysis schemes and rules of thumb, found in most textbooks, deprive students of the opportunity to gain an intuitive feeling for the general aspects of negative amplifiers. As a result, the students were lost when they encountered a feedback circuit which did not match a familiar type. If these statements sound too strong to teachers who read this paper, try to challenge your students with the example given here and ask for *accurate* solutions which are based on feedback analysis (rather than general nodal analysis).

The unified feedback analysis approach was further expanded two years ago, when the author was assigned to teach both courses of the sequence. The adopted pedagogical objective was to strengthen the theoretical and intuitive aspects, supplemented by SPICE simulation. The experience gained in actual classwork seem to indicate that the proposed unified feedback model is an extremely useful tool for the students to reach the objectives set forth. The model is quickly mastered by students and they enjoy the way that all the pieces fit together to yield the desired results, which are in good agreement with nodal analysis and simulation. In fact, the initiative to include (D) and (D') in the general model (Fig. 3) was a student complaint that SPICE simulation is inconsistent with the analysis of a BJT emitter follower which has been analyzed by using a textbook procedure.

The main difficulty that has been encountered with the proposed feedback model approach is the lack of textbooks which follow this concept. To remedy this, we have duplicated the lecture notes of one of the (better organized) students and made them available at the University book store.

For the sake of brevity, the present paper was confined to single amplifier and single feedback loop systems. With a little effort, the proposed approach can be expanded to multiamplifier and multifeedback loop systems. The objective will be again, to reach a system perspective by identifying the signal summing nodes and the transfer ratios of all paths.

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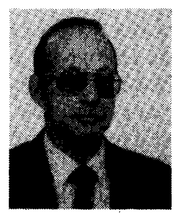

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