Modeling of Air Venting in Pressure Die Casting Process

In this study an analytical model has been developed to describe air pressure and residual air mass variations in pressure die casting for atmospheric venting. During injection of molten metal into a die cavity, air is evacuated from the cavity through vents. In this study, the influences of air velocity and friction factor due to temperature dependent viscosity and vent roughness change have been investigated. The results of the model show that there is a critical area ratio over which a quasi steady state is reached, therefore, the air pressure in the cavity remains constant. In addition, for each area ratio there is a critical/ minimum time ratio below which outlet Mach number is not large enough to create choked flow. In this case, the rate of outflow air mass is not maximum. Finally, the results of the model addresses that the friction factor depends on hydraulic diameter of the vent and assuming a constant value for it is not valid. [DOI: 10.1115/1.1767187]

1 Introduction

Porosity is a major issue in die casting. It causes common defects such as pores and reduced thermal and mechanical properties such as yield strength, ductility and modulus of elasticity of parts produced in the pressure die casting. The porosity can be resulted from air entrainment or/and shrinkage of liquid metal during the solidification process.

In high-pressure die casting during die filling process, molten metal is injected into a die cavity at a high velocity by a die casting machine. The air inside the die cavity is compressed and high pressure can be created if there is not a sufficient venting area or if the die vents are sealed off before the die cavity is completely filled up. Air pressure thus caused may be several times atmospheric pressure [1–3]. The increased air pressure could increase the amount of air dissolving into the molten metal and cause porosity in the pressure die casting [4,5]. In ideal conditions the outgoing air volume should be equal to the incoming molten metal volume to insure that the cavity is completed full of molten metal when metal flow stops. All the air inside the cavity should be vented before solidification of the molten metal to avoid defects.

Bar-Meir et al. [6] indicated that air porosity contributes a significant part of the total porosity. Draper [7] reported that the porosity level of a cast part decreases with an increase of the vent area for atmospheric venting. Moreover, he found that porosity decreases with increasing vent area or filling time until the vent area or filling time reaches or exceeds a critical value after which the porosity remains constant. Lindsey and Wallace [8], who neglected the friction in the venting system, have shown that the porosity decreases with an increase of evacuated volume. These important findings demonstrate that the critical vent area is crucial to the reduction of porosity.

Lee and Lu [9] proposed a new mathematical model for the calculation of the induced air pressure in the cavity. In this model, air discharge through rectangular vents is modeled by Poiseuille flow. Venting capability is directly proportional to the width and inversely proportional to the length of the vent. The vent thickness exerts the greatest influence on the venting capability. They concluded that there is a considerable difference between their model and the traditional model in the calculation of the cavity pressure. In order to have an optimum venting condition, the vent thickness should be close to the possible maximum in the vent design of the die casting dies.

Karni [10] performed an analytical approach to describe the pressure variations in the cavity for atmospheric venting taking into account the flow resistance in the venting system. The temperature in the cavity and the flow in the vent are assumed to be constant and adiabatic respectively. In these calculations the conditions in the cavity are determined from the conditions at the exit for unchoked as well for choked flow.

Sachs [11] developed a quasi steady model for the maximum mass flow rate from a die cavity based on an isentropic process in the die cavity. The only resistance to the gas flow was assumed to occur at the entrance of the vent. He indicated that for a choked flow, the pressure ratio was about two between the cavity and the vent exit. Veinik [12] developed a similar model for an instantaneously choked flow. He assumed that the lowest pressure in the cavity is equal to about two atmospheres, and then presented the calculation of the vent area as a ratio of the air volume to the air velocity and filling time. Veinik [13] also introduced the friction in the venting system with the lowest pressure of two atmospheres in the die cavity.

Bennett [14] developed a model to calculate the vent area for a cavity evacuated to the atmosphere under unchoked flow and also a constant pressure in the cavity during filling time. He calculated the vent area from the flow through an orifice by assuming that the air volume flow rate at the vent is equal to the liquid metal flow rate at the gate.

The objective of this study is to develop a transient model for air flow through exhaust vents under choked and unchoked flows. The corresponding ordinary differential equation is satisfied through a variable area ratio, inlet Mach number due to the friction factor and air velocity changes. The area ratio is equal to the vent area times filling time and sonic velocity at initial temperature divided by initial volume of air in the cavity. It should be noted that these effects have not been analyzed before in the literature.

2 Mathematical Formulation

A schematic diagram of a commercial die casting system with an air vent is depicted in Fig. 1. Traditionally, the die filling process in a pressure die casting is usually divided into three stages. In the first stage, the plunger advances slowly until the base of the shot sleeve is covered fully with molten metal. In the second stage, the die cavity being filled at a virtually constant pressure and air is expelled through a vent to the atmosphere. The last stage is typified by a high rise in packing pressure when the die cavity is fully filled. Lui et al. [15] divided the die filling process into five stages. In this process the first and second stages remain intact. Yet the third stage breaks down into two and a fifth residual.
stage is added. There seems to be agreement that the second stage is the most important and that it plays a key role in completing the whole function of a pressure die casting process.

Because of relatively small resistance to the air flow in the unfilled shot sleeve, runner and cavity compared with the resistance to the air flow in the venting system the following model is proposed to simplify the process. The unfilled shot sleeve, runner and die cavity are combined and called the cylinder cavity and as such the pressure in the cylinder is assumed to be uniform.

Kaye [16] stated that for more than 95 percent of the plunger stroke, the speed is almost constant. Therefore, the time variation of air volume in the cylinder cavity can be written as:

\[ V_c(t) = V_c(0) \left(1 - \frac{t}{t_{\text{max}}} \right) \]  

(1)

where \( V_c(0) \) is the initial volume of air in the cylinder cavity and \( t_{\text{max}} \) is the filling time for liquid metal to reach vent. The filling time of the die cavity is equal to the volume of cavity to be filled divided by the flow rate of the molten metal driven by the plunger. On the other hand, Allsop [17] recommended \( t_{\text{max}} \) \( = \frac{40}{\text{average thickness}} \) [mm] for aluminum solidification process. So that, the average wall thickness can be determined as a ratio of the volume of cavity to be filled to the average of the inside and outside surface areas of the part produced.

Applying the ideal gas equation, the air mass contained in the cylinder cavity is

\[ m_c(t) = \frac{P_c(t)V_c(t)}{RT_c(t)} \]  

(2)

where \( P_c(t) \) and \( T_c(t) \) are pressure and temperature of the air in the cylinder cavity respectively. If the air undergoes an isentropic process during an expansion or compression process we have

\[ T_c(t) = T_c(0) \left(\frac{P_c(t)}{P_c(0)}\right)^{(K-1)/K} \]  

(3)

Introducing Eqs. (1) and (3) into Eq. (2), yields

\[ m_c(t) = m_c(0) \left(\frac{V_c(t)}{V_c(0)}\right)^{(K-1)/K} \left(\frac{P_c(t)}{P_c(0)}\right) \]  

(4)

\[ \text{where } m_c(0) = P_c(0)V_c(0)/RT_c(0) \text{ is the initial air mass in the cylinder cavity.} \]

The air mass flow rate through the vent is:

\[ m_{\text{out}} = \rho AV_c \left(\frac{P_c(t)}{RT_c(t)}\right) A_M \sqrt{KRT_i} \]  

(5)

where \( \rho, A, V \) are density, vent area and velocity of the air, \( M \) is Mach number, \( K \) and \( R \) are the specific heat ratio and air constant respectively. Subscript \( i \) indicates the conditions at the vent inlet.

Considering an isentropic process for the air flow between the cavity as a stagnation state and the vent inlet as a nozzle, we have

\[ T_i(t) = T_i(0) \left(\frac{P_i(t)}{P_i(0)}\right)^{(K-1)/K} \]  

(6)

Introducing Eq. (6) for \( T_i \) and \( P_i \), Eq. (5) yields the air mass flow rate through the nozzle as follows.

\[ m_{\text{out}} = \frac{M_i A_P c(0) \sqrt{K}}{RT_i(0)} \left[ \frac{P_c(t)}{P_c(0)} \right]^{(K+1)/2K} \left[ 1 + K - 1 \right] \left[ \frac{M_i^2}{2} \right] \]  

(7)

The rate of change of the air mass in the cylinder cavity is:

\[ \frac{dm_c}{dt} + m_{\text{out}} = 0 \]  

(8)

Exerting Eqs. (4) and (7) into Eq. (8), the air pressure differential in the cavity as a function of dimensionless parameters: \( t^* = t/t_{\text{max}} \), \( P_c^* = P_c(t)/P_c(0) \) and \( A^* = A_{\text{max}} \sqrt{KRT_i(0)/V_c(0)} \) under the initial condition \( P_c^*(0) = 1 \) will be.

\[ \frac{d}{dt^*} \left[ 1 - t^* \right] P_c^* = 1 \]  

(9)

Consider now a one-dimensional steady flow with friction through a constant area vent (or duct). For the case of adiabatic flow with no external work, termed Fanno line flow, the Mach number as a function of the vent length can be written as [18].

\[ \frac{4fL}{D} = 1 \left[ \frac{1}{K} \right] \left[ \frac{1}{M_i^2} - 1 \right] + K + 1 \]  

(10)

Similarly to the Eq. (10), a relationship is obtained between the Mach number and air pressure in the cylinder cavity by the following equation.

\[ \frac{P_c(t)}{P_e} = \frac{M_e}{M_i} \left[ \frac{K - 1}{2} \right]^{0.5} \times \left[ 1 + \frac{K - 1}{2} \right] \left[ \frac{M_i^2}{2} \right] \]  

(11)

where \( P_e \) is the air pressure at the vent exit plane. Subscripts \( i \) and \( e \) indicate conditions at the inlet and exit planes of the vent respectively. \( f \) and \( D_H \) are the mean friction factor and hydraulic diameter of the vent respectively. The friction factor depends on the Reynolds number and vent roughness as [18].

\[ f = \left[ \frac{1}{K} \right] \frac{0.0625 \log \left[ \frac{e}{3.7A} \right]}{\left( \frac{K}{\text{Re}^{0.5}} \right)} \]  

(12)

\[ f = \begin{cases} 0.0625 & \text{if } \text{Re} < 2300 \\ 0.40 & \text{if } \text{Re} \geq 2300 \end{cases} \]  

(13)

\[ f = \frac{\mu}{\mu_{\text{ref}}} = \left( \frac{T}{T_{\text{ref}}} \right)^{n} \]  

(14)

where \( \mu \) is the average of the air viscosity. The value of viscosity varies somewhat with temperature and for gases being approximately described by:

\[ \mu = \left( \frac{T}{T_{\text{ref}}} \right)^{n} \]  

(15)
where $T$ is the absolute static temperature at the particular point in the vent and for common gases $0.5 < n < 0.8$. Because Fanno line flow is being considered, there are no changes in stagnation temperature in the vent and as such $T = T_i/ [1 + 0.5(K-1)M^2]$. Applying $T_{ref} = T_i(0)$ as a reference temperature and substituting Eq. (3) into Eq. (15), the result is:

$$\frac{\mu}{\mu[T_i(0)]} = \left[ \frac{p_e^{n(k-1)/k}}{1 + \frac{k-1}{2} M^2} \right]^n$$  (16)

Applying this equation at the inlet and exit plane of the vent, the mean viscosity will be:

$$\mu = \frac{1}{2} (\mu_i + \mu_e) = \frac{1}{2} \mu[T_i(0)]P_e^{n(k-1)/k} \left[ \left( 1 + \frac{K-1}{2} M_e^2 \right)^{-n} + \left( 1 + \frac{K-1}{2} M_i^2 \right)^{-n} \right]$$  (17)

The mean Reynolds number in the vent is obtained by introducing Eq. (17) in Eq. (14).

$$Re = \frac{2m_{inj}D_H}{\mu[T_i(0)]A P_e^{n(k-1)/k} \left[ \left( 1 + \frac{K-1}{2} M_e^2 \right)^{-n} + \left( 1 + \frac{K-1}{2} M_i^2 \right)^{-n} \right]}$$  (18)

### 3 Solution Algorithm

Before starting the solution procedure it is necessary to describe the conditions under which the system is choked or unchoked by the vent (or duct). Let us first consider how the exit pressure and Mach number vary as the upstream static pressure is increased. Depending on the upstream pressure, the minimum pressure point or point of maximum Mach number can occur at the vent exit. When the upstream pressure is low, the exit Mach number is less than one and the exit pressure is equal to the atmospheric pressure. Then Eqs. (10,11) are solved simultaneously for two unknown variables $M_i$ and $M_e$. In the event that the upstream static pressure is high enough that the exit Mach number is one, the exit pressure would be equal to or greater than the atmospheric pressure and the vent chokes the system. This means that the maximum flow can be passed through the vent. In this case $M_e = 1$ and Eqs. (10,11) are solved simultaneously for two other unknown variables i.e., $M_i$ and $P_e$.

To start the numerical method of Eq. (9), first the time dependent inlet and outlet Mach numbers are required. If the filling process is supposed to be completed in a number of very small time intervals, the Mach numbers can nearly be assumed to be invariant. In this case, first some mean values for $M_i$ and $M_e$ are guessed in each time step $\Delta t^*$ (= 0.0005) and then Eq. (9) integrated numerically by the fourth-order Runge-Kutta method. Applying the value of $P_e^*$ into Eq. (7) and then $m_{inj}$ into Eq. (18), the final result yields the Reynolds number. Using the value of the Reynolds number and invoking Eq. (12) or (13), the mean friction factor is calculated. The next step would solve Eqs. (10,11) simultaneously for $M_i$ and $M_e$. If the difference between the calculated value and the initial guess for $M_i$ and $M_e$ are not sufficiently small, the initial guesses for $M_i$ and $M_e$ are updated and the above calculations will be repeated until a specified convergence tolerance ($< 10^{-6}$) is established. In the next time step, again some new mean values for $M_i$ and $M_e$ are guessed and the above procedure is repeated until $t^* = 1$ is reached.

### 4 Results and Discussion

The following results have been obtained for a venting system under unchoked flow conditions by a rectangular vent. The input parameters required for calculations of data are: vent length 0.2 m, vent roughness $e = 2 \times 10^{-6}$ m, hydraulic diameters $D_H = 0.2$ and 2 mm, initial temperature and pressure of the air in the cylinder cavity 300 K° and 1 atm respectively for five values of area ratio $A^* = (0.1, 0.5, 1.5, 15, 25)$.

Figure 2 illustrates the Mach number at the inlet and outlet of the vent as a function of normalized time for various values of area ratio, all for hydraulic diameter of 2 mm. The area ratio is equal to the area times filling time and sonic velocity divided by the initial value of the cylinder cavity. The change from a low exit Mach number to a choked flow occurs within a large range of time. For example, choked flow occurs at $t^* = 0.55$ and 0.73 for $A^* = 0$ and 1 respectively. For a large area ratio the flow is not choked even for a large time as sufficient pressure difference would not be attained. The variation of the inlet Mach number changes gradually and the influence of area ratio on the exit Mach number in contrast to the exit Mach number is quite small.

Figure 3 shows the variation of pressure ratio in the cylinder cavity versus normalized time with different areas. The pressure ratio is equal to the air pressure in the cavity divided by its initial value. The maximum rate of change of pressure and the maximum final pressure occur for curve of $A^* = 0$. In this case, no air can escape through the vent as the air inside the cylinder cavity is adiabatically compressed. Nearly, a quasi steady is reached for curve of $A^* = 5$. This quasi steady state is achieved when the outgoing volumetric air flow rate is equal to the incoming molten volume pushed by the piston.

A comparison of air pressure and residual air mass in the cylinder cavity between the Bar-Meir et al. [6] model and the present model at $t^* = 0.9$ are shown in Fig. 4 for various values of $A^*$ and $4fL/D_H = 5$. Note that in the Bar-Meir model, the area ratio has been defined as $A^* (M)_{max}$ where $(M)_{max} = (0.306552)$ is the maximum entrance Mach number for a specific vent determined based on $4fL/D_H = 5$. The results of the Bar-Meir model design...
nated by the markers show a very good agreement with the present model prediction designated by the lines. The reason of this agreement is that for $4fL/D_H=5$ the inlet Mach number is constant (see Figs. 6, 3, 2). In this case, the residual air mass and air pressure in the cavity obtained by Eqs. (4, 9) are only functions of area ratio. Therefore, the results of two models are identical.

Figure 5 describes dimensionless air mass remaining in the cylinder cavity versus normalized time for various area ratios obtained by Eq. (4). The residual air mass decreases as area increases. Two interesting limiting cases are found when the vents are sealed and when $A^*$ is larger than 5. When $A^*$ is small there is a large resistance to the air flow and relatively little air leaves the cylinder cavity. For $A^*>5$ a quasi steady state is reached and pressure is constant (see Fig. 3). The quasi steady state is observed when the volumetric air flow rate out is equal to the volume pushed by the piston. The pressure and air mass flow rate are maintained constant once this state is reached. The other curves lie between the two extreme cases.

Figure 6 exhibits the variations of $4fL/D_H$ as a function of normalized time. The ordinate is a dimensionless function, where $f$ is the fanning friction factor and $L$ is the length of the vent. One interesting finding is that $4fL/D_H$ in the vent drops to a very low value after a short time. For example, for $A^*<15$, the parameter $4fL/D_H$ falls to less than 70 percent of its initial value after 12 percent of time. This result is in contrast to Sachs [11], Draper [7], Veinik [12], Lindsey and Wallace [8] and Bar-Meir model [19].

The models published by these authors assumed that the friction factor is constant or at most can range between 3 and 7.

The influence of area ratio on friction factor of the vent is significant and depends on the hydraulic diameter of the duct. Figure 7 shows that when the hydraulic diameter changes from 2 mm to 0.2 mm, the rate of change of the friction factor becomes more tangible. The minimum points on lines $A^*=1$ and 0 indicate the transition condition from the laminar to turbulent flow regime. To prove this point, the friction factor is plotted against Reynolds number in Fig. 8. The figure demonstrates that the location of transition occurs at Re=2300 corresponding to the minimum point in Fig. 7 where the flow regime changes from laminar to turbulent flow.

5 Conclusions
A transient model for the calculation of the air pressure and residual air mass in the cylinder cavity is proposed which involves the influence of inlet Mach number, roughness and temperature dependent viscosity through the friction factor. Computations are carried out numerically by the fourth-order Runge-Kutta method for a vent with different sizes. The results of the computation were analyzed and some useful findings are obtained. There is a critical time ratio below which the Mach number is not large enough to create choked flow. In addition, for $A^*>5$ a quasi steady state is reached and the air pressure is constant. The quasi steady state is observed when the volumetric air flow rate out is equal to the...
The variation range of the friction factor depends on the hydraulic diameter and assuming a constant value for it is not valid.

Nomenclature

\[ A = \text{vent area} \]
\[ A^* = A_{max} \sqrt{KRT_c(0)/\mathcal{V}_c(0)} \]
\[ D = \text{diameter} \]
\[ f = \text{fanning friction factor} \]
\[ K = \text{specific heat ratio} \]
\[ L = \text{vent length} \]
\[ m = \text{mass} \]
\[ M = \text{Mach number} \]
\[ P = \text{pressure} \]
\[ P^* = P(t)/P(0) \]
\[ R = \text{air constant} \]
\[ \text{Re} = \text{Reynolds number}, \rho V D_H/\mu \]
\[ t = \text{time} \]
\[ t^* = t/t_{max} \]

Greek Symbols

\[ \Delta = \text{difference} \]
\[ \varepsilon = \text{vent roughness} \]
\[ \mu = \text{viscosity} \]
\[ \rho = \text{density} \]

Subscripts

\[ c = \text{cylinder cavity} \]
\[ e = \text{exit} \]
\[ H = \text{hydraulic} \]
\[ i = \text{inlet} \]
\[ \text{max} = \text{maximum} \]
\[ \text{out} = \text{outlet} \]
\[ \text{ref} = \text{reference} \]
\[ 0 = \text{initial condition} \]

References