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# 1

# Passive Components

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## 1.1 Resistors

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The resistor is an electrical device whose primary function is to introduce resistance to the flow of electric current. The magnitude of opposition to the flow of current is called the resistance of the resistor. A larger resistance value indicates a greater opposition to current flow.

The resistance is measured in ohms. An ohm is the resistance that arises when a current of one ampere is passed through a resistor subjected to one volt across its terminals.

The various uses of resistors include setting biases, controlling gain, fixing time constants, matching and loading circuits, voltage division, and heat generation. The following sections discuss resistor characteristics and various resistor types.

## Resistor Characteristics

### Voltage and Current Characteristics of Resistors

The resistance of a resistor is directly proportional to the **resistivity** of the material and the length of the resistor and inversely proportional to the cross-sectional area perpendicular to the direction of current flow. The resistance $R$ of a resistor is given by

$$R = \frac{\rho l}{A} \tag{1.1}$$

where $\rho$ is the resistivity of the resistor material ($\Omega \cdot \text{cm}$), $l$ is the length of the resistor along direction of current flow (cm), and $A$ is the cross-sectional area perpendicular to current flow ($\text{cm}^2$) (Fig. 1.1). Resistivity is an inherent property of materials. Good resistor materials typically have resistivities between $2 \times 10^{-6}$ and $200 \times 10^{-6}\ \Omega \cdot \text{cm}$.

The resistance can also be defined in terms of sheet resistivity. If the sheet resistivity is used, a standard sheet thickness is assumed and factored into resistivity. Typically, resistors are rectangular in shape; therefore the length $l$ divided by the width $w$ gives the number of squares within the resistor (Fig. 1.2). The number of squares multiplied by the resistivity is the resistance.

$$R_{\text{sheet}} = \rho_{\text{sheet}} \frac{l}{w} \tag{1.2}$$

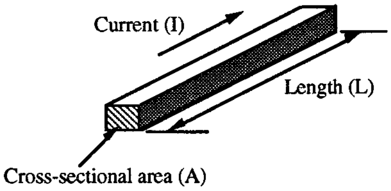


**FIGURE 1.1** Resistance of a rectangular cross-section resistor with cross-sectional area $A$ and length $L$.

where $\rho_{\text{sheet}}$ is the sheet resistivity (Ω/square), $l$ is the length of resistor (cm), $w$ is the width of the resistor (cm), and $R_{\text{sheet}}$ is the sheet resistance (Ω).

The resistance of a resistor can be defined in terms of the **voltage drop** across the resistor and current through the resistor related by Ohm's law,

$$R = \frac{V}{I} \tag{1.3}$$

where $R$ is the resistance (Ω), $V$ is the voltage across the resistor (V), and $I$ is the current through the resistor (A). Whenever a current is passed through a resistor, a voltage is dropped across the ends of the resistor. Figure 1.3 depicts the symbol of the resistor with the Ohm's law relation.

All resistors dissipate power when a voltage is applied. The power dissipated by the resistor is represented by

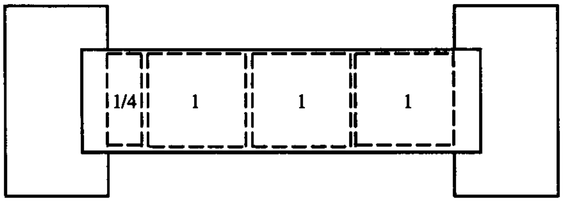
$$P = \frac{V^2}{R} \tag{1.4}$$

where $P$ is the power dissipated (W), $V$ is the voltage across the resistor (V), and $R$ is the resistance (Ω). An ideal resistor dissipates electric energy without storing electric or magnetic energy.

## Resistor Networks

Resistors may be joined to form networks. If resistors are joined in series, the effective resistance ($R_T$) is the sum of the individual resistances (Fig. 1.4).

$$R_T = \sum_{i=1}^{n} R_i \tag{1.5}$$



THE ABOVE RESISTOR IS 3.25 SQUARES
IF ρ = 100 Ω/□, THEN R = 3.25□ X 100 Ω/□ = 325 Ω

**FIGURE 1.2** Number of squares in a rectangular resistor.

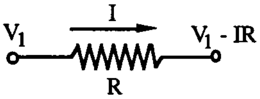


**FIGURE 1.3** A resistor with resistance $R$ having a current $I$ flowing through it will have a voltage drop of $IR$ across it.

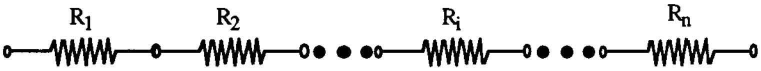


**FIGURE 1.4** Resistors connected in series.

If resistors are joined in parallel, the effective resistance ($R_T$) is the reciprocal of the sum of the reciprocals of individual resistances (Fig. 1.5).

$$\frac{1}{R_T} = \sum_{i=1}^{n} \frac{1}{R_i} \tag{1.6}$$

### Temperature Coefficient of Electrical Resistance

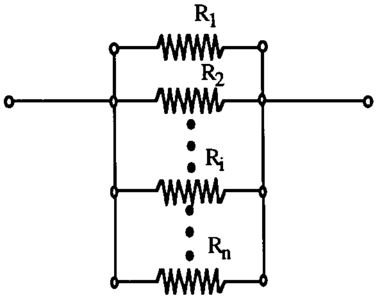


**FIGURE 1.5** Resistors connected in parallel.

The resistance for most resistors changes with temperature. The temperature coefficient of electrical resistance is the change in electrical resistance of a resistor per unit change in temperature. The **temperature coefficient of resistance** is measured in Ω/°C. The temperature coefficient of resistors may be either positive or negative. A positive temperature coefficient denotes a rise in resistance with a rise in temperature; a negative temperature coefficient of resistance denotes a decrease in resistance with a rise in temperature. Pure metals typically have a positive temperature coefficient of resistance, while some metal alloys such as constantin and manganin have a zero temperature coefficient of resistance. Carbon and graphite mixed with binders usually exhibit negative temperature coefficients, although certain choices of binders and process variations may yield positive temperature coefficients. The temperature coefficient of resistance is given by

$$R(T_2) = R(T_1)[1 + \alpha_{T1}(T_2 - T_1)] \tag{1.7}$$

where $\alpha_{T1}$ is the temperature coefficient of electrical resistance at reference temperature $T_1$, $R(T_2)$ is the resistance at temperature $T_2$ (Ω), and $R(T_1)$ is the resistance at temperature $T_1$ (Ω). The reference temperature is usually taken to be 20°C. Because the variation in resistance between any two temperatures is usually not linear as predicted by Eq. (1.7), common practice is to apply the equation between temperature increments and then to plot the resistance change versus temperature for a number of incremental temperatures.

### High-Frequency Effects

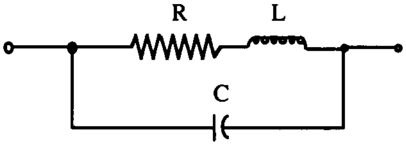


**FIGURE 1.6** Equivalent circuit for a resistor.

Resistors show a change in their resistance value when subjected to ac voltages. The change in resistance with voltage frequency is known as the *Boella effect.* The effect occurs because all resistors have some inductance and capacitance along with the resistive component and thus can be approximated by an equivalent circuit shown in Fig. 1.6. Even though the definition of useful frequency range is application dependent, typically, the useful range of the resistor is the highest frequency at which the impedance differs from the resistance by more than the tolerance of the resistor.

The frequency effect on resistance varies with the resistor construction. Wire-wound resistors typically exhibit an increase in their impedance with frequency. In composition resistors the capacitances are formed by the many conducting particles which are held in contact by a dielectric binder. The ac impedance for film resistors remains constant until 100 MHz (1 MHz = $10^6$ Hz) and then decreases at higher frequencies (Fig. 1.7). For film resistors, the decrease in dc resistance at higher frequencies decreases with increase in resistance. Film resistors have the most stable high-frequency performance.

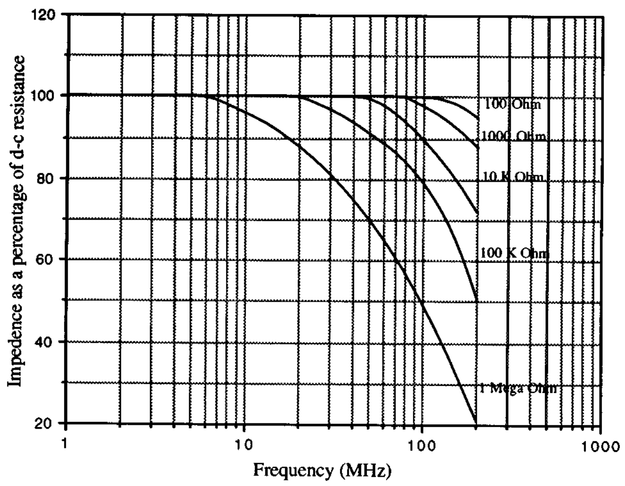


**FIGURE 1.7** Typical graph of impedance as a percentage of dc resistance versus frequency for film resistors.

The smaller the diameter of the resistor the better is its frequency response. Most high-frequency resistors have a length to diameter ratio between 4:1 to 10:1. Dielectric losses are kept to a minimum by proper choice of base material.

## Voltage Coefficient of Resistance

Resistance is not always independent of the applied voltage. The **voltage coefficient of resistance** is the change in resistance per unit change in voltage, expressed as a percentage of the resistance at 10% of rated voltage. The voltage coefficient is given by the relationship

$$\text{Voltage coefficient} = \frac{100(R_1 - R_2)}{R_2(V_1 - V_2)} \tag{1.8}$$

where $R_1$ is the resistance at the rated voltage $V_1$ and $R_2$ is the resistance at 10% of rated voltage $V_2$.

## Noise

Resistors exhibit electrical noise in the form of small ac voltage fluctuations when dc voltage is applied. Noise in a resistor is a function of the applied voltage, physical dimensions, and materials. The total noise is a sum of Johnson noise, current flow noise, noise due to cracked bodies, and loose end caps and leads. For variable resistors the noise can also be caused by the jumping of a moving contact over turns and by an imperfect electrical path between the contact and resistance element.

The Johnson noise is temperature-dependent thermal noise (Fig. 1.8). Thermal noise is also called "white noise" because the noise level is the same at all frequencies. The magnitude of thermal noise, $E_{\text{RMS}}$ (V), is dependent on the resistance value and the temperature of the resistance due to thermal agitation.

$$E_{\text{RMS}} = \sqrt{4kRT\Delta f} \tag{1.9}$$

where $E_{\text{RMS}}$ is the root-mean-square value of the noise voltage (V), $R$ is the resistance (Ω), $K$ is the Boltzmann constant ($1.38 \times 10^{-23}$ J/K), $T$ is the temperature (K), and $\Delta f$ is the bandwidth (Hz) over which the noise energy is measured.

Figure 1.8 shows the variation in current noise versus voltage frequency. Current noise varies inversely with frequency and is a function of the current flowing through the resistor and the value of the resistor. The magnitude of current noise is directly proportional to the square root of current. The current noise magnitude is usually expressed by a noise index given as the ratio of the root-mean-square current noise voltage ($E_{\text{RMS}}$)

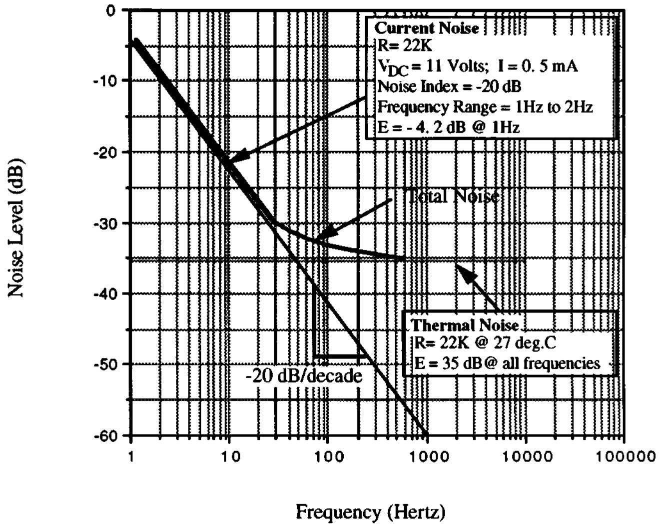


**FIGURE 1.8** The total resistor noise is the sum of current noise and thermal noise. The current noise approaches the thermal noise at higher frequencies. (*Source:* Phillips Components, Discrete Products Division, *1990–91 Resistor/Capacitor Data Book,* 1991. With permission.)

over one decade bandwidth to the average voltage caused by a specified constant current passed through the resistor at a specified hot-spot temperature [Phillips, 1991].

$$\text{N.I.} = 20\ \log_{10}\left(\frac{\text{Noise voltage}}{\text{dc voltage}}\right) \tag{1.10}$$

$$E_{\text{RMS}} = V_{\text{dc}} \times 10^{\text{N.I.}/20}\sqrt{\log\left(\frac{f_2}{f_1}\right)} \tag{1.11}$$

where N.I. is the noise index, $V_{\text{dc}}$ is the dc voltage drop across the resistor, and $f_1$ and $f_2$ represent the frequency range over which the noise is being computed. Units of noise index are µV/V. At higher frequencies, the current noise becomes less dominant compared to Johnson noise.

Precision film resistors have extremely low noise. Composition resistors show some degree of noise due to internal electrical contacts between the conducting particles held together with the binder. Wire-wound resistors are essentially free of electrical noise unless resistor terminations are faulty.

## Power Rating and Derating Curves

Resistors must be operated within specified temperature limits to avoid permanent damage to the materials. The temperature limit is defined in terms of the maximum power, called the *power rating,* and derating curve. The power rating of a resistor is the maximum power in watts which the resistor can dissipate. The maximum power rating is a function of resistor material, maximum voltage rating, resistor dimensions, and maximum allowable hot-spot temperature. The maximum hot-spot temperature is the temperature of the hottest part on the resistor when dissipating full-rated power at rated ambient temperature.

The maximum allowable power rating as a function of the ambient temperature is given by the derating curve. Figure 1.9 shows a typical power rating curve for a resistor. The derating curve is usually linearly drawn from the full-rated load temperature to the maximum allowable no-load temperature. A resistor may be operated at ambient temperatures above the maximum full-load ambient temperature if operating at lower than full-rated power capacity. The maximum allowable no-load temperature is also the maximum storage temperature for the resistor.

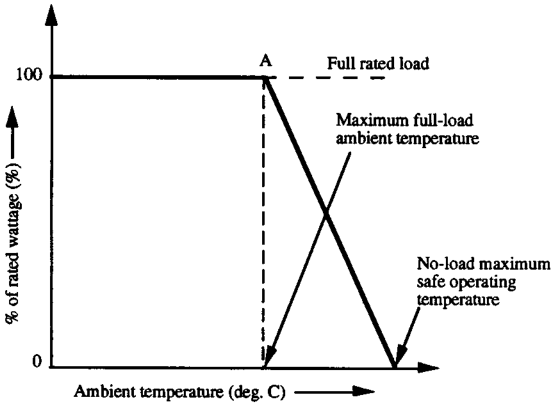


**FIGURE 1.9** Typical derating curve for resistors.

## Voltage Rating of Resistors

The maximum voltage that may be applied to the resistor is called the **voltage rating** and is related to the power rating by

$$V = \sqrt{PR} \tag{1.12}$$

where $V$ is the voltage rating (V), $P$ is the power rating (W), and $R$ is the resistance (Ω). For a given value of voltage and power rating, a critical value of resistance can be calculated. For values of resistance below the critical value, the maximum voltage is never reached; for values of resistance above the critical value, the power dissipated is lower than the rated power (Fig. 1.10).

## Color Coding of Resistors

Resistors are generally identified by color coding or direct digital marking. The color code is given in Table 1.1. The color code is commonly used in composition resistors and film resistors. The color code essentially consists of four bands of different colors. The first band is the most significant figure, the second band is the second significant figure, the third band is the multiplier or the number of zeros that have to be added after the first two significant figures, and the fourth band is the tolerance on the resistance value. If the fourth band is not present, the resistor tolerance is the standard 20% above and below the rated value. When the color code is used on fixed wire-wound resistors, the first band is applied in double width.

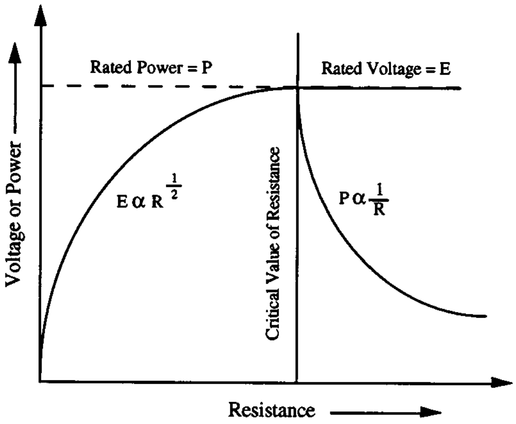


**FIGURE 1.10** Relationship of applied voltage and power above and below the critical value of resistance.

**TABLE 1.1** Color Code Table for Resistors

| Color | First Band | Second Band | Third Band | Fourth Band Tolerance, % |
|---|---|---|---|---|
| Black | 0 | 0 | 1 | |
| Brown | 1 | 1 | 10 | |
| Red | 2 | 2 | 100 | |
| Orange | 3 | 3 | 1,000 | |
| Yellow | 4 | 4 | 10,000 | |
| Green | 5 | 5 | 100,000 | |
| Blue | 6 | 6 | 1,000,000 | |
| Violet | 7 | 7 | 10,000,000 | |
| Gray | 8 | 8 | 100,000,000 | |
| White | 9 | 9 | 1,000,000,000 | |
| Gold | | | 0.1 | 5% |
| Silver | | | 0.01 | 10% |
| No band | | | | 20% |

Blanks in the table represent situations which do not exist in the color code.

## Resistor Types

Resistors can be broadly categorized as fixed, variable, and special-purpose. Each of these resistor types is discussed in detail with typical ranges of their characteristics.

### Fixed Resistors

The fixed resistors are those whose value cannot be varied after manufacture. Fixed resistors are classified into composition resistors, wire-wound resistors, and metal-film resistors. Table 1.2 outlines the characteristics of some typical fixed resistors.

***Wire-Wound Resistors.*** Wire-wound resistors are made by winding wire of nickel-chromium alloy on a ceramic tube covering with a vitreous coating. The spiral winding has inductive and capacitive characteristics that make it unsuitable for operation above 50 kHz. The frequency limit can be raised by noninductive winding so that the magnetic fields produced by the two parts of the winding cancel.

***Composition Resistors.*** Composition resistors are composed of carbon particles mixed with a binder. This mixture is molded into a cylindrical shape and hardened by baking. Leads are attached axially to each end, and the assembly is encapsulated in a protective encapsulation coating. Color bands on the outer surface indicate the resistance value and tolerance. Composition resistors are economical and exhibit low noise levels for resistances above 1 MΩ. Composition resistors are usually rated for temperatures in the neighborhood of 70°C for power ranging from 1/8 to 2 W. Composition resistors have end-to-end shunted capacitance that may be noticed at frequencies in the neighborhood of 100 kHz, especially for resistance values above 0.3 MΩ.

***Metal-Film Resistors.*** Metal-film resistors are commonly made of nichrome, tin-oxide, or tantalum nitride, either hermetically sealed or using molded-phenolic cases. Metal-film resistors are not as stable as the

**TABLE 1.2** Characteristics of Typical Fixed Resistors

| Resistor Types | Resistance Range | Watt Range | Operating Temp. Range | α, ppm/°C |
|---|---|---|---|---|
| Wire-wound resistor | | | | |
| Precision | 0.1 to 1.2 MΩ | 1/8 to 1/4 | –55 to 145 | 10 |
| Power | 0.1 to 180 kΩ | 1 to 210 | –55 to 275 | 260 |
| Metal-film resistor | | | | |
| Precision | 1 to 250 MΩ | 1/20 to 1 | –55 to 125 | 50–100 |
| Power | 5 to 100 kΩ | 1 to 5 | –55 to 155 | 20–100 |
| Composition resistor | | | | |
| General purpose | 2.7 to 100 MΩ | 1/8 to 2 | –55 to 130 | 1500 |

wire-wound resistors. Depending on the application, fixed resistors are manufactured as precision resistors, semiprecision resistors, standard general-purpose resistors, or power resistors. Precision resistors have low voltage and power coefficients, excellent temperature and **time stabilities**, low noise, and very low reactance. These resistors are available in metal-film or wire constructions and are typically designed for circuits having very close resistance tolerances on values. Semiprecision resistors are smaller than precision resistors and are primarily used for current-limiting or voltage-dropping functions in circuit applications. Semiprecision resistors have long-term temperature stability. General-purpose resistors are used in circuits that do not require tight resistance tolerances or long-term stability. For general-purpose resistors, initial resistance variation may be in the neighborhood of 5% and the variation in resistance under full-rated power may approach 20%. Typically, general-purpose resistors have a high coefficient of resistance and high noise levels. Power resistors are used for power supplies, control circuits, and voltage dividers where operational stability of 5% is acceptable. Power resistors are available in wire-wound and film constructions. Film-type power resistors have the advantage of stability at high frequencies and have higher resistance values than wire-wound resistors for a given size.

### Variable Resistors

***Potentiometers.*** The potentiometer is a special form of variable resistor with three terminals. Two terminals are connected to the opposite sides of the resistive element, and the third connects to a sliding contact that can be adjusted as a voltage divider.

Potentiometers are usually circular in form with the movable contact attached to a shaft that rotates. Potentiometers are manufactured as carbon composition, metallic film, and wire-wound resistors available in single-turn or multiturn units. The movable contact does not go all the way toward the end of the resistive element, and a small resistance called the *hop-off* resistance is present to prevent accidental burning of the resistive element.

***Rheostat.*** The rheostat is a current-setting device in which one terminal is connected to the resistive element and the second terminal is connected to a movable contact to place a selected section of the resistive element into the circuit. Typically, rheostats are wire-wound resistors used as speed controls for motors, ovens, and heater controls and in applications where adjustments on the voltage and current levels are required, such as voltage dividers and bleeder circuits.

### Special-Purpose Resistors

***Integrated Circuit Resistors.*** Integrated circuit resistors are classified into two general categories: semiconductor resistors and deposited film resistors. Semiconductor resistors use the bulk resistivity of **doped** semiconductor regions to obtain the desired resistance value. Deposited film resistors are formed by depositing resistance films on an insulating substrate which are etched and patterned to form the desired resistive network. Depending on the thickness and dimensions of the deposited films, the resistors are classified into thick-film and thin-film resistors.

Semiconductor resistors can be divided into four types: diffused, bulk, pinched, and ion-implanted. Table 1.3 shows some typical resistor properties for semiconductor resistors. Diffused semiconductor resistors use resistivity of the diffused region in the semiconductor substrate to introduce a resistance in the circuit. Both *n*-type and *p*-type diffusions are used to form the diffused resistor.

A bulk resistor uses the bulk resistivity of the semiconductor to introduce a resistance into the circuit. Mathematically the sheet resistance of a bulk resistor is given by

$$R_{\text{sheet}} = \frac{\rho_e}{d} \tag{1.13}$$

where $R_{\text{sheet}}$ is the sheet resistance in ($\Omega$/square), $\rho_e$ is the sheet resistivity ($\Omega$/square), and $d$ is the depth of the *n*-type **epitaxial layer**.

Pinched resistors are formed by reducing the effective cross-sectional area of diffused resistors. The reduced cross section of the diffused length results in extremely high sheet resistivities from ordinary diffused resistors.

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**TABLE 1.3** Typical Characteristics of Integrated Circuit Resistors

| Resistor Type | Sheet Resistivity (per square) | Temperature Coefficient (ppm/°C) |
|---|---|---|
| Semiconductor | | |
| Diffused | 0.8 to 260 Ω | 1100 to 2000 |
| Bulk | 0.003 to 10 kΩ | 2900 to 5000 |
| Pinched | 0.001 to 10 kΩ | 3000 to 6000 |
| Ion-implanted | 0.5 to 20 kΩ | 100 to 1300 |
| Deposited resistors | | |
| Thin-film | | |
| Tantalum | 0.01 to 1 kΩ | ∓100 |
| $SnO_2$ | 0.08 to 4 kΩ | –1500 to 0 |
| Ni-Cr | 40 to 450 Ω | ∓100 |
| Cermet (Cr-SiO) | 0.03 to 2.5 kΩ | ∓150 |
| Thick-film | | |
| Ruthenium-silver | 10 Ω to 10 MΩ | ∓200 |
| Palladium-silver | 0.01 to 100 kΩ | –500 to 150 |

Ion-implanted resistors are formed by implanting ions on the semiconductor surface by bombarding the silicon lattice with high-energy ions. The implanted ions lie in a very shallow layer along the surface (0.1 to 0.8 μm). For similar thicknesses ion-implanted resistors yield sheet resistivities 20 times greater than diffused resistors. Table 1.3 shows typical properties of diffused, bulk, pinched, and ion-implanted resistors. Typical sheet resistance values range from 80 to 250 Ω/square.

***Varistors.*** Varistors are voltage-dependent resistors that show a high degree of nonlinearity between their resistance value and applied voltage. They are composed of a nonhomogeneous material that provides a rectifying action. Varistors are used for protection of electronic circuits, semiconductor components, collectors of motors, and relay contacts against overvoltage.

The relationship between the voltage and current of a varistor is given by

$$V = kI^{\beta} \tag{1.14}$$

where $V$ is the voltage (V), $I$ is the current (A), and $k$ and $\beta$ are constants that depend on the materials and manufacturing process. The electrical characteristics of a varistor are specified by its $\beta$ and $k$ values.

*Varistors in Series.* The resultant $k$ value of $n$ varistors connected in series is $nk$. This can be derived by considering $n$ varistors connected in series and a voltage $nV$ applied across the ends. The current through each varistor remains the same as for $V$ volts over one varistor. Mathematically, the voltage and current are expressed as

$$nV = k_1 I^{\beta} \tag{1.15}$$

Equating the expressions (1.14) and (1.15), the equivalent constant $k_1$ for the series combination of varistors is given as

$$k_1 = nk \tag{1.16}$$

*Varistors in Parallel.* The equivalent $k$ value for a parallel combination of varistors can be obtained by connecting $n$ varistors in parallel and applying a voltage $V$ across the terminals. The current through the varistors will still be $n$ times the current through a single varistor with a voltage $V$ across it. Mathematically the current and voltage are related as

$$V = k_2(nI)^{\beta} \tag{1.17}$$

From Eqs. (1.14) and (1.17) the equivalent constant $k_2$ for the series combination of varistors is given as

$$k_2 = \frac{k}{n^{\beta}} \tag{1.18}$$

***Thermistors.*** Thermistors are resistors that change their resistance exponentially with changes in temperature. If the resistance decreases with increase in temperature, the resistor is called a negative temperature coefficient (NTC) resistor. If the resistance increases with temperature, the resistor is called a positive temperature coefficient (PTC) resistor.

NTC thermistors are ceramic semiconductors made by sintering mixtures of heavy metal oxides such as manganese, nickel, cobalt, copper, and iron. The resistance temperature relationship for NTC thermistors is

$$R_T = A e^{B/T} \tag{1.19}$$

where $T$ is temperature (K), $R_T$ is the resistance (Ω), and $A$, $B$ are constants whose values are determined by conducting experiments at two temperatures and solving the equations simultaneously.

PTC thermistors are prepared from $BaTiO_3$ or solid solutions of $PbTiO_3$ or $SrTiO_3$. The resistance temperature relationship for PTC thermistors is

$$R_T = A + Ce^{BT} \tag{1.20}$$

where $T$ is temperature (K), $R_T$ is the resistance (Ω), and $A$, $B$ are constants determined by conducting experiments at two temperatures and solving the equations simultaneously. Positive thermistors have a PTC only between certain temperature ranges. Outside this range the temperature is either zero or negative. Typically, the absolute value of the temperature coefficient of resistance for PTC resistors is much higher than for NTC resistors.

## Defining Terms

**Doping:** The intrinsic carrier concentration of semiconductors (e.g., Si) is too low to allow controlled charge transport. For this reason some impurities called dopants are purposely added to the semiconductor. The process of adding dopants is called doping. Dopants may belong to group IIIA (e.g., boron) or group VA (e.g., phosphorus) in the periodic table. If the elements belong to the group IIIA, the resulting semiconductor is called a *p*-type semiconductor. On the other hand, if the elements belong to the group VA, the resulting semiconductor is called an *n*-type semiconductor.

**Epitaxial layer:** Epitaxy refers to processes used to grow a thin crystalline layer on a crystalline substrate. In the epitaxial process the wafer acts as a seed crystal. The layer grown by this process is called an epitaxial layer.

**Resistivity:** The resistance of a conductor with unit length and unit cross-sectional area.

**Temperature coefficient of resistance:** The change in electrical resistance of a resistor per unit change in temperature.

**Time stability:** The degree to which the initial value of resistance is maintained to a stated degree of certainty under stated conditions of use over a stated period of time. Time stability is usually expressed as a percent or parts per million change in resistance per 1000 hours of continuous use.

**Voltage coefficient of resistance:** The change in resistance per unit change in voltage, expressed as a percentage of the resistance at 10% of rated voltage.

**Voltage drop:** The difference in potential between the two ends of the resistor measured in the direction of flow of current. The voltage drop is $V = IR$, where $V$ is the voltage across the resistor, $I$ is the current through the resistor, and $R$ is the resistance.

**Voltage rating:** The maximum voltage that may be applied to the resistor.

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## Related Topics

22.1 Physical Properties • 25.1 Integrated Circuit Technology • 51.1 Introduction

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# 1.2 Capacitors and Inductors

*Glen Ballou*

## Capacitors

If a potential difference is found between two points, an electric **field** exists that is the result of the separation of unlike charges. The strength of the field will depend on the amount the charges have been separated.

*Capacitance* is the concept of energy storage in an electric field and is restricted to the area, shape, and spacing of the **capacitor** plates and the property of the material separating them.

When electrical current flows into a capacitor, a force is established between two parallel plates separated by a **dielectric.** This energy is stored and remains even after the input is removed. By connecting a **conductor** (a resistor, hard wire, or even air) across the capacitor, the charged capacitor can regain electron balance, that is, discharge its stored energy.

The value of a parallel-plate capacitor can be found with the equation

$$C = \frac{x\epsilon[(N - 1)A]}{d} \times 10^{-13} \tag{1.21}$$

where $C$ = capacitance, F; $\epsilon$ = dielectric constant of insulation; $d$ = spacing between plates; $N$ = number of plates; $A$ = area of plates; and $x = 0.0885$ when $A$ and $d$ are in centimeters, and $x = 0.225$ when $A$ and $d$ are in inches.

The work necessary to transport a unit charge from one plate to the other is

$$e = kg \tag{1.22}$$

where $e$ = volts expressing energy per unit charge, $g$ = coulombs of charge already transported, and $k$ = proportionality factor between work necessary to carry a unit charge between the two plates and charge already transported. It is equal to $1/C$, where $C$ is the capacitance, F.

The value of a capacitor can now be calculated from the equation

$$C = \frac{q}{e} \tag{1.23}$$

where $q$ = charge (C) and $e$ is found with Eq. (1.22).

The energy stored in a capacitor is

$$W = \frac{CV^2}{2} \tag{1.24}$$

where $W$ = energy, J; $C$ = capacitance, F; and $V$ = applied voltage, V.

The **dielectric constant** of a material determines the electrostatic energy which may be stored in that material per unit volume for a given voltage. The value of the dielectric constant expresses the ratio of a capacitor in a vacuum to one using a given dielectric. The dielectric of air is 1, the reference unit employed for expressing the dielectric constant. As the dielectric constant is increased or decreased, the capacitance will increase or decrease, respectively. Table 1.4 lists the dielectric constants of various materials.

The dielectric constant of most materials is affected by both temperature and frequency, except for quartz, Styrofoam, and Teflon, whose dielectric constants remain essentially constant.

The equation for calculating the *force of attraction* between two plates is

$$F = \frac{AV^2}{k(1504S)^2} \tag{1.25}$$

**TABLE 1.4** Comparison of Capacitor Dielectric Constants

| Dielectric | $K$ (Dielectric Constant) |
|---|---|
| Air or vacuum | 1.0 |
| Paper | 2.0–6.0 |
| Plastic | 2.1–6.0 |
| Mineral oil | 2.2–2.3 |
| Silicone oil | 2.7–2.8 |
| Quartz | 3.8–4.4 |
| Glass | 4.8–8.0 |
| Porcelain | 5.1–5.9 |
| Mica | 5.4–8.7 |
| Aluminum oxide | 8.4 |
| Tantalum pentoxide | 26 |
| Ceramic | 12–400,000 |

*Source:* G. Ballou, *Handbook for Sound Engineers, The New Audio Cyclopedia,* Carmel, Ind.: Macmillan Computer Publishing Company, 1991. With permission.

where $F$ = attraction force, dyn; $A$ = area of one plate, cm$^2$; $V$ = potential energy difference, V; $k$ = dielectric coefficient; and $S$ = separation between plates, cm.

The $Q$ for a capacitor when the resistance and capacitance is in series is

$$Q = \frac{1}{2\pi fRC} \tag{1.26}$$

where $Q$ = ratio expressing the factor of merit; $f$ = frequency, Hz; $R$ = resistance, Ω; and $C$ = capacitance, F.

When capacitors are connected in *series,* the total capacitance is

$$C_T = \frac{1}{1/C_1 + 1/C_2 + \cdots + 1/C_n} \tag{1.27}$$

and is always less than the value of the smallest capacitor.

When capacitors are connected in *parallel*, the total capacitance is

$$C_T = C_1 + C_2 + \cdots + C_n \tag{1.28}$$

and is always larger than the largest capacitor.

When a voltage is applied across a group of capacitors connected in series, the voltage drop across the combination is equal to the applied voltage. The drop across each individual capacitor is inversely proportional to its capacitance.

$$V_C = \frac{V_A C_X}{C_T} \tag{1.29}$$

where $V_C$ = voltage across the individual capacitor in the series ($C_1$, $C_2$, ...,$C_n$), V; $V_A$ = applied voltage, V; $C_T$ = total capacitance of the series combination, F; and $C_X$ = capacitance of individual capacitor under consideration, F.

In an ac circuit, the **capacitive reactance,** or the **impedance**, of the capacitor is

$$X_C = \frac{1}{2\pi f C} \tag{1.30}$$

where $X_C$ = capacitive reactance, Ω; $f$ = frequency, Hz; and $C$ = capacitance, F. The current will lead the voltage by 90° in a circuit with a pure capacitor.

When a dc voltage is connected across a capacitor, a time $t$ is required to charge the capacitor to the applied voltage. This is called a **time constant** and is calculated with the equation

$$t = RC \tag{1.31}$$

where $t$ = time, s; $R$ = resistance, Ω; and $C$ = capacitance, F.

In a circuit consisting of pure resistance and capacitance, the *time constant t* is defined as the time required to charge the capacitor to 63.2% of the applied voltage.

During the next time constant, the capacitor charges to 63.2% of the remaining difference of full value, or to 86.5% of the full value. The charge on a capacitor can never actually reach 100% but is considered to be 100% after five time constants. When the voltage is removed, the capacitor discharges to 63.2% of the full value.

*Capacitance* is expressed in microfarads (μF, or $10^{-6}$ F) or picofarads (pF, or $10^{-12}$ F) with a stated accuracy or tolerance. Tolerance may also be stated as GMV (guaranteed minimum value), sometimes referred to as MRV (minimum rated value).

All capacitors have a *maximum working voltage* that must not be exceeded and is a combination of the dc value plus the peak ac value which may be applied during operation.

### Quality Factor (*Q*)

**Quality factor** is the ratio of the capacitor's **reactance** to its resistance at a specified frequency and is found by the equation

$$\begin{aligned} Q &= \frac{1}{2\pi f C R} \\ &= \frac{1}{PF} \end{aligned} \tag{1.32}$$

where $Q$ = quality factor; $f$ = frequency, Hz; $C$ = value of capacitance, F; $R$ = internal resistance, Ω; and PF = power factor

### Power Factor (PF)

**Power factor** is the preferred measurement in describing capacitive losses in ac circuits. It is the fraction of input volt-amperes (or power) dissipated in the capacitor dielectric and is virtually independent of the capacitance, applied voltage, and frequency.

### Equivalent Series Resistance (ESR)

**Equivalent series resistance** is expressed in ohms or milliohms (Ω, mΩ) and is derived from lead resistance, termination losses, and dissipation in the dielectric material.

### Equivalent Series Inductance (ESL)

The *equivalent series inductance* can be useful or detrimental. It reduces high-frequency performance; however, it can be used in conjunction with the internal capacitance to form a resonant circuit.

### Dissipation Factor (DF)

The **dissipation factor** in percentage is the ratio of the effective series resistance of a capacitor to its reactance at a specified frequency. It is the reciprocal of *quality factor* ($Q$) and an indication of power loss within the capacitor. It should be as low as possible.

### Insulation Resistance

*Insulation resistance* is the resistance of the dielectric material and determines the time a capacitor, once charged, will hold its charge. A discharged capacitor has a low insulation resistance; however once charged to its rated value, it increases to megohms. The leakage in electrolytic capacitors should not exceed

$$I_L = 0.04C + 0.30 \tag{1.33}$$

where $I_L$ = leakage current, μA, and $C$ = capacitance, μF.

### Dielectric Absorption (DA)

The *dielectric absorption* is a reluctance of the dielectric to give up stored electrons when the capacitor is discharged. This is often called “memory” because if a capacitor is discharged through a resistance and the resistance is removed, the electrons that remained in the dielectric will reconvene on the electrode, causing a voltage to appear across the capacitor. DA is tested by charging the capacitor for 5 min, discharging it for 5 s, then having an open circuit for 1 min after which the recovery voltage is read. The percentage of DA is defined as the ratio of recovery to charging voltage times 100.

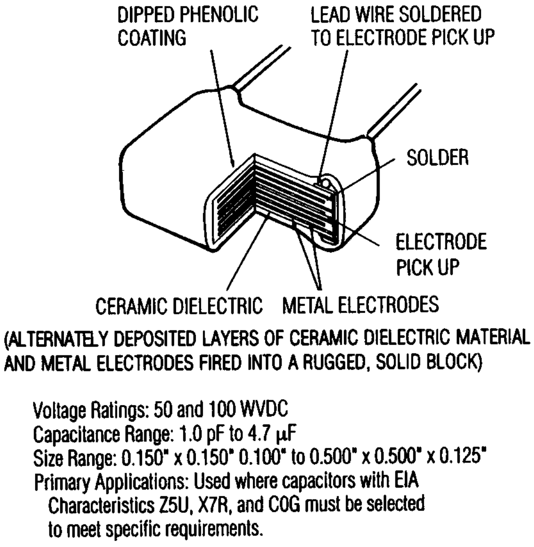
## Types of Capacitors

Capacitors are used to filter, couple, tune, block dc, pass ac, bypass, shift phase, compensate, feed through, isolate, store energy, suppress noise, and start motors. They must also be small, lightweight, reliable, and withstand adverse conditions.

Capacitors are grouped according to their dielectric material and mechanical configuration.

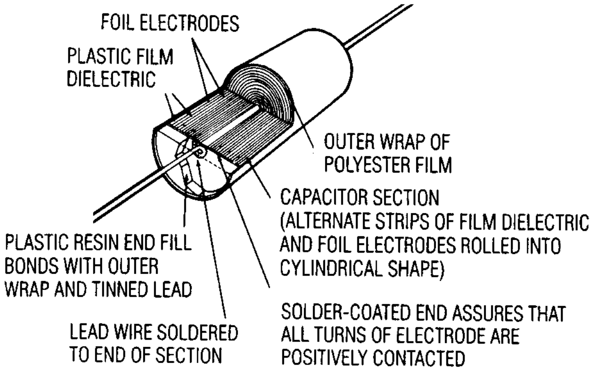
### Ceramic Capacitors

*Ceramic capacitors* are used most often for bypass and coupling applications (Fig. 1.11). Ceramic capacitors can be produced with a variety of $K$ values (dielectric constant). A high $K$ value translates to small size and less stability. High-$K$ capacitors with a dielectric constant >3000 are physically small and have values between 0.001 to several microfarads.



Voltage Ratings: 50 and 100 WVDC
Capacitance Range: 1.0 pF to 4.7 μF
Size Range: 0.150" x 0.150" 0.100" to 0.500" x 0.500" x 0.125"
Primary Applications: Used where capacitors with EIA Characteristics Z5U, X7R, and COG must be selected to meet specific requirements.

**FIGURE 1.11** Monolythic® multilayer ceramic capacitors. (Courtesy of Sprague Electric Company.)



Voltage Range: 50 to 1000 WVDC
Capacitance Range: 0.001 to 1.0 μF
Size Range: 0.190" dia. x 0.450" long to 0.890" dia. x 2.125" long
Primary Applications: Radio, communications gear, and other commercial equipment where high IR, size, weight, and cost are important considerations.

**FIGURE 1.12** Film-wrapped film capacitors. (Courtesy of Sprague Electric Company.)

Good temperature stability requires capacitors to have a $K$ value between 10 and 200. If high $Q$ is also required, the capacitor will be physically larger. Ceramic capacitors with a zero temperature change are called **negative-positive-zero (NPO)** and come in a capacitance range of 1.0 pF to 0.033 μF.

An N750 temperature-compensated capacitor is used when accurate capacitance is required over a large temperature range. The 750 indicates a 750-ppm decrease in capacitance with a 1°C increase in temperature (750 ppm/°C). This equates to a 1.5% decrease in capacitance for a 20°C temperature increase. N750 capacitors come in values between 4.0 and 680 pF.

### Film Capacitors

*Film capacitors* consist of alternate layers of metal foil and one or more layers of a flexible plastic insulating material (dielectric) in ribbon form rolled and encapsulated (see Fig. 1.12).

### Mica Capacitors

*Mica capacitors* have small capacitance values and are usually used in high-frequency circuits. They are constructed as alternate layers of metal foil and mica insulation, which are stacked and encapsulated, or are silvered mica, where a silver electrode is screened on the mica insulators.

### Paper-Foil-Filled Capacitors

*Paper-foil-filled capacitors* are often used as motor capacitors and are rated at 60 Hz. They are made of alternate layers of aluminum and paper saturated with oil that are rolled together. The assembly is mounted in an oil-filled, hermetically sealed metal case.

### Electrolytic Capacitors

**Electrolytic capacitors** provide high capacitance in a tolerable size; however, they do have drawbacks. Low temperatures reduce performance, while high temperatures dry them out. The **electrolytes** themselves can leak and corrode the equipment. Repeated surges above the rated working voltage, excessive ripple currents, and high operating temperature reduce performance and shorten capacitor life.

Electrolytic capacitors are manufactured by an electrochemical formation of an oxide film on a metal surface. The metal on which the oxide film is formed serves as the **anode** or positive terminal of the capacitor; the oxide film is the dielectric, and the **cathode** or negative terminal is either a conducting liquid or a gel.

The equivalent circuit of an electrolytic capacitor is shown in Fig. 1.13, where $A$ and $B$ are the capacitor terminals, $C$ is the effective capacitance, and $L$ is the self-inductance of the capacitor caused by terminals, electrodes, and geometry.

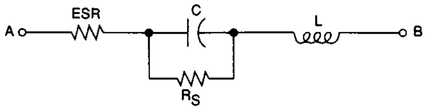


**FIGURE 1.13** Simplified equivalent circuit of an electrolytic capacitor.

The shunt resistance (insulation resistance) $R_s$ accounts for the dc leakage current. Heat is generated in the ESR from ripple current and in the shunt resistance by voltage. The ESR is due to the spacer-electrolyte-oxide system and varies only slightly except at low temperature, where it increases greatly.

The *impedance* of a capacitor (Fig. 1.14) is frequency-dependent. The initial downward slope is caused by the capacitive reactance $X_C$. The trough (lowest impedance) is almost totally resistive, and the upward slope is due to the capacitor's self-inductance $X_L$. An ESR plot would show an ESR decrease to about 5–10 kHz, remaining relatively constant thereafter.

**Leakage current** is the direct current that passes through a capacitor when a correctly polarized dc voltage is applied to its terminals. It is proportional to temperature, becoming increasingly important at elevated **ambient temperatures**. Leakage current decreases slowly after voltage is applied, reaching steady-state conditions in about 10 min.

If a capacitor is connected with reverse polarity, the oxide film is forward-biased, offering very little resistance to current flow. This causes overheating and self-destruction of the capacitor.

The total heat generated within a capacitor is the sum of the heat created by the $I_{\text{leakage}} \times V_{\text{applied}}$ and the $I^2R$ losses in the ESR.

The ac **ripple current** rating is very important in filter applications because excessive current produces temperature rise, shortening capacitor life. The maximum permissible rms ripple current is limited by the internal temperature and the rate of heat dissipation from the capacitor. Lower ESR and longer enclosures increase the ripple current rating.

Capacitor life expectancy is doubled for each 10°C decrease in operating temperature, so a capacitor operating at room temperature will have a life expectancy 64 times that of the same capacitor operating at 85°C (185°F).

The *surge voltage* specification of a capacitor determines its ability to withstand high transient voltages that generally occur during the starting up period of equipment. Standard tests generally specify a short on and long off period for an interval of 24 h or more, and the allowable surge voltage levels are generally 10% above the rated voltage of the capacitor.

Figure 1.15 shows how temperature, frequency, time, and applied voltage affect electrolytic capacitors.

***Aluminum Electrolytic Capacitors.*** *Aluminum electrolytic capacitors* use aluminum as the base material (Fig. 1.16). The surface is often etched to increase the surface area as much as 100 times that of unetched foil, resulting in higher capacitance in the same volume.

Aluminum electrolytic capacitors can withstand up to 1.5 V of reverse voltage without detriment. Higher reverse voltages, when applied over extended periods, lead to loss of capacitance. Excess reverse voltages applied for short periods cause some change in capacitance but not to capacitor failure.

Large-value capacitors are often used to filter dc power supplies. After a capacitor is charged, the rectifier stops conducting and the capacitor discharges into the load, as shown in Fig. 1.17, until the next cycle. Then the capacitor recharges again to the peak voltage. The $\Delta e$ is equal to the total peak-to-peak ripple voltage and is a complex wave containing many harmonics of the fundamental ripple frequency, causing the noticeable heating of the capacitor.

***Tantalum Capacitors.*** *Tantalum electrolytics* are the preferred type where high reliability and long service life are paramount considerations.

Tantalum capacitors have as much as three times better capacitance per volume efficiency than aluminum electrolytic capacitors, because tantalum pentoxide has a dielectric constant three times greater than that of aluminum oxide (see Table 1.4).

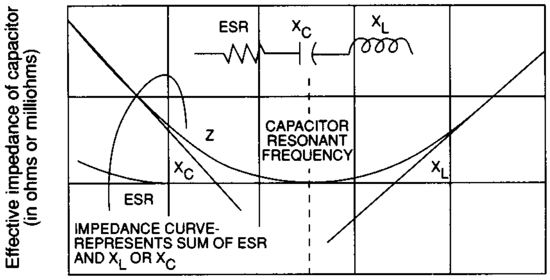


**Figure 1.14** Impedance characteristics of a capacitor.

The capacitance of any capacitor is determined by the surface area of the two conducting plates, the

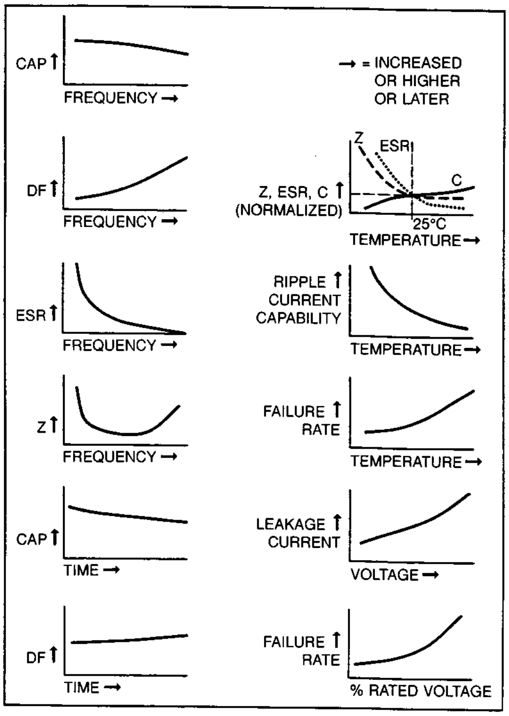
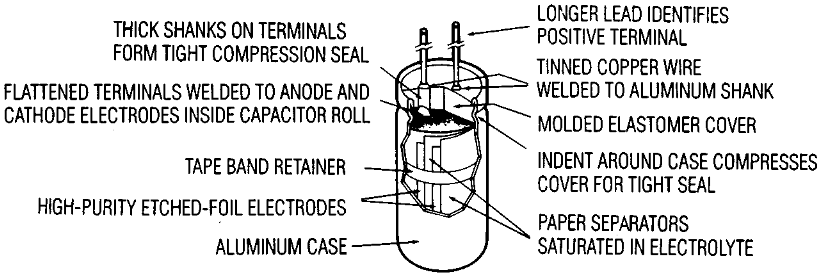


**FIGURE 1.15** Variations in aluminum electrolytic characteristics caused by temperature, frequency, time, and applied voltage. (Courtesy of Sprague Electric Company.)



Voltage Range: 6.3 to 63 WVDC
Capacitance Range: 0.47 to 3300 μF
Size Range: 0.197" dia. x 0.433" long to 0.630" dia. x 1.614" long
Primary Applications: Coupling, decoupling, bypass, and filtering.
Vertical installation on high-density printed wiring boards in transistorized radios, portable TV sets, auto radios, tape recorders, etc.

**(Courtesy**

**FIGURE 1.16** Verti-lytic® miniature single-ended aluminum electrolytic capacitor. (Courtesy of Sprague Electric Company.)

distance between the plates, and the dielectric constant of the insulating material between the plates [see Eq. (1.21)].

In tantalum electrolytics, the distance between the plates is the thickness of the tantalum pentoxide film, and since the dielectric constant of the tantalum pentoxide is high, the capacitance of a tantalum capacitor is high.

Tantalum capacitors contain either liquid or solid electrolytes. The liquid electrolyte in wet-slug and foil capacitors, generally sulfuric acid, forms the cathode (negative) plate. In solid-electrolyte capacitors, a dry material, manganese dioxide, forms the cathode plate.

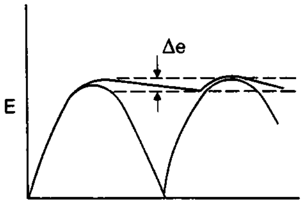
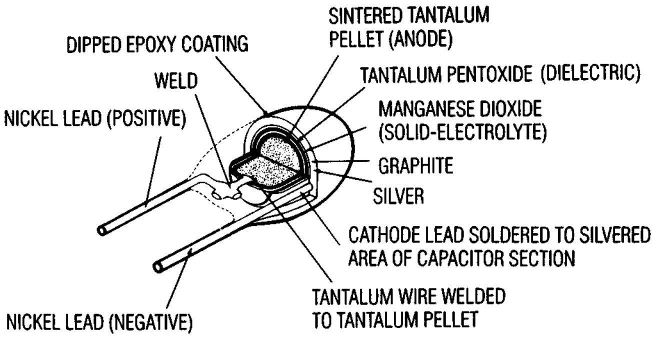


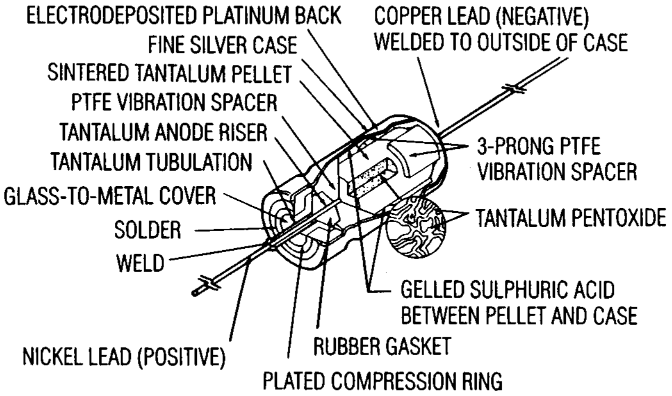
**FIGURE 1.17** Full-wave capacitor charge and discharge.

***Foil Tantalum Capacitors.*** *Foil tantalum capacitors* can be designed to voltage values up to 300 V dc. Of the three types of tantalum electrolytic capacitors, the foil design has the lowest capacitance per unit volume and is best suited for the higher voltages primarily found in older designs of equipment. It is expensive and used only where neither a solid-electrolyte (Fig. 1.18) nor a wet-slug (Fig. 1.19) tantalum capacitor can be employed.



Voltage Range: 3 to 50 WVDC
Capacitance Range: 0.10 to 680 μF
Size Range: 0.175" dia. x 0.280" high to 0.400" dia. x 0.750" high
Primary Applications: For printed wiring boards applications where low cost, small size, high stability, low d-c leakage, and low dissipation factor are important.

**FIGURE 1.18** Tantalex® solid electrolyte tantalum capacitor. (Courtesy of Sprague Electric Company.)



Voltage Range: 6 to 125 WVDC
Capacitance Range: 1.7 to 1200 μF
Size Range: 0.188" dia. x 0.453" long to 0.375" dia. x 1.062" long
Primary Applications: Industrial and military equipment where reliability and premium performance with respect to low d-c leakage current, high inrush current capability, and high volumetric efficiency.

**FIGURE 1.19** Hermetically sealed sintered-anode tantalum capacitor. (Courtesy of Sprague Electric Company.)

Foil tantalum capacitors are generally designed for operation over the temperature range of –55 to +125°C (–67 to +257°F) and are found primarily in industrial and military electronics equipment.

*Solid-electrolyte sintered-anode tantalum capacitors* differ from the wet versions in their electrolyte, which is manganese dioxide.

Another variation of the solid-electrolyte tantalum capacitor encases the element in plastic resins, such as epoxy materials offering excellent reliability and high stability for consumer and commercial electronics with the added feature of low cost.

Still other designs of "solid tantalum" capacitors use plastic film or sleeving as the encasing material, and others use metal shells that are backfilled with an epoxy resin. Finally, there are small tubular and rectangular molded plastic encasements.

*Wet-electrolyte sintered-anode tantalum capacitors,* often called "wet-slug" tantalum capacitors, use a pellet of sintered tantalum powder to which a lead has been attached, as shown in Fig. 1.19. This anode has an enormous surface area for its size.

Wet-slug tantalum capacitors are manufactured in a voltage range to 125 V dc.

***Use Considerations.*** Foil tantalum capacitors are used only where high-voltage constructions are required or where there is substantial reverse voltage applied to a capacitor during circuit operation.

Wet sintered-anode capacitors, or "wet-slug" tantalum capacitors, are used where low dc leakage is required. The conventional "silver can" design will not tolerate reverse voltage. In military or aerospace applications where utmost reliability is desired, tantalum cases are used instead of silver cases. The tantalum-cased wet-slug units withstand up to 3 V reverse voltage and operate under higher ripple currents and at temperatures up to 200°C (392°F).

Solid-electrolyte designs are the least expensive for a given rating and are used where their very small size is important. They will typically withstand a reverse voltage up to 15% of the rated dc working voltage. They also have good low-temperature performance characteristics and freedom from corrosive electrolytes.

## Inductors

**Inductance** is used for the storage of magnetic energy. Magnetic energy is stored as long as current keeps flowing through the inductor. In a perfect **inductor**, the current of a sine wave lags the voltage by 90°.

### Impedance

**Inductive reactance** $X_L$, the impedance of an inductor to an ac signal, is found by the equation

$$X_L = 2\pi f L \qquad (1.34)$$

where $X_L$ = inductive reactance, Ω; $f$ = frequency, Hz; and $L$ = inductance, H.

The type of wire used for its construction does not affect the inductance of a **coil**. $Q$ of the coil will be governed by the resistance of the wire. Therefore coils wound with silver or gold wire have the highest $Q$ for a given design.

To increase inductance, inductors are connected in series. The total inductance will always be greater than the largest inductor.

$$L_T = L_1 + L_2 + \cdots + L_n \qquad (1.35)$$

To reduce inductance, inductors are connected in parallel.

$$L_T = \frac{1}{1/L_1 + 1/L_2 + \cdots + 1/L_n} \qquad (1.36)$$

The total inductance will always be less than the value of the lowest inductor.

### Mutual Inductance

**Mutual inductance** is the property that exists between two conductors carrying current when their magnetic lines of force link together.

The mutual inductance of two coils with fields interacting can be determined by the equation

$$M = \frac{L_A - L_B}{4} \tag{1.37}$$

where $M$ = mutual inductance of $L_A$ and $L_B$, H; $L_A$ = total inductance, H, of coils $L_1$ and $L_2$ with fields aiding; and $L_B$ = total inductance, H, of coils $L_1$ and $L_2$ with fields opposing.

The *coupled inductance* can be determined by the following equations. In parallel with fields aiding,

$$L_T = \frac{1}{\frac{1}{L_1 + M} + \frac{1}{L_2 + M}} \tag{1.38}$$

In parallel with fields opposing,

$$L_T = \frac{1}{\frac{1}{L_1 - M} - \frac{1}{L_2 - M}} \tag{1.39}$$

In series with fields aiding,

$$L_T = L_1 + L_2 + 2M \tag{1.40}$$

In series with fields opposing,

$$L_T = L_1 + L_2 - 2M \tag{1.41}$$

where $L_T$ = total inductance, H; $L_1$ and $L_2$ = inductances of the individual coils, H; and $M$ = mutual inductance, H.

When two coils are inductively coupled to give transformer action, the coupling coefficient is determined by

$$K = \frac{M}{\sqrt{L_1 L_2}} \tag{1.42}$$

where $K$ = coupling coefficient; $M$ = mutual inductance, H; and $L_1$ and $L_2$ = inductances of the two coils, H.

An inductor in a circuit has a reactance equal to $j2\pi fL\ \Omega$. Mutual inductance in a circuit has a reactance equal to $j2\pi fL\ \Omega$. The operator $j$ denotes that the reactance dissipates no energy; however, it does oppose current flow.

The energy stored in an inductor can be determined by the equation

$$W = \frac{LI^2}{2} \tag{1.43}$$

where $W$ = energy, J (W · s); $L$ = inductance, H; and $I$ = current, A.

### Coil Inductance

Inductance is related to the turns in a coil as follows:

1. The inductance is proportional to the square of the turns.
2. The inductance increases as the length of the **winding** is increased.
3. A shorted turn decreases the inductance, affects the frequency response, and increases the insertion loss.
4. The inductance increases as the permeability of the core material increases.
5. The inductance increases with an increase in the cross-sectional area of the core material.
6. Inductance is increased by inserting an iron core into the coil.
7. Introducing an air gap into a choke reduces the inductance.

A conductor moving at any angle to the lines of force cuts a number of lines of force proportional to the sine of the angles. Thus,

$$V = \beta L v \sin\theta \times 10^{-8} \tag{1.44}$$

where $\beta$ = flux density; $L$ = length of the conductor, cm; and $v$ = velocity, cm/s, of conductor moving at an angle $\theta$.

The maximum voltage induced in a conductor moving in a magnetic field is proportional to the number of magnetic lines of force cut by that conductor. When a conductor moves parallel to the lines of force, it cuts no lines of force; therefore, no current is generated in the conductor. A conductor that moves at right angles to the lines of force cuts the maximum number of lines per inch per second, therefore creating a maximum voltage. The right-hand rule determines direction of the induced electromotive force (emf). The emf is in the direction in which the axis of a right-hand screw, when turned with the velocity vector, moves through the smallest angle toward the flux density vector.

The **magnetomotive force** (mmf) in **ampere-turns** produced by a coil is found by multiplying the number of turns of wire in the coil by the current flowing through it.

$$\begin{aligned}\text{Ampere-turns} &= T\left(\frac{V}{R}\right) \\ &= TI\end{aligned} \tag{1.45}$$

where $T$ = number of turns; $V$ = voltage, V; and $R$ = resistance, Ω.

The inductance of a single layer, a spiral, and multilayer coils can be calculated by using either Wheeler's or Nagaoka's equations. The accuracy of the calculation will vary between 1 and 5%. The inductance of a single-layer coil can be calculated using Wheeler's equation:

$$L = \frac{B^2N^2}{9B + 10A} \quad \mu\text{H} \tag{1.46}$$

For the multilayer coil,

$$L = \frac{0.8B^2N^2}{6B + 9A + 10C} \quad \mu\text{H} \tag{1.47}$$

For the spiral coil,

$$L = \frac{B^2N^2}{8B + 11C} \quad \mu\text{H} \tag{1.48}$$

where $B$ = radius of the winding, $N$ = number of turns in the coil, $A$ = length of the winding, and $C$ = thickness of the winding.

## Q

*Q* is the ratio of the inductive reactance to the internal resistance of the coil and is affected by frequency, inductance, dc resistance, inductive reactance, the type of winding, the core losses, the distributed capacity, and the permeability of the core material.

The *Q* for a coil where *R* and *L* are in series is

$$Q = \frac{2\pi f L}{R} \tag{1.49}$$

where $f$ = frequency, Hz; $L$ = inductance, H; and $R$ = resistance, Ω.

The *Q* of the coil can be measured using the circuit of Fig. 1.20 for frequencies up to 1 MHz. The voltage across the inductance (*L*) at resonance equals $Q(V)$ (where *V* is the voltage developed by the oscillator); therefore, it is only necessary to measure the output voltage from the oscillator and the voltage across the inductance.

The oscillator voltage is driven across a low value of resistance, *R*, about 1/100 of the anticipated rf resistance of the *LC* combination, to assure that the measurement will not be in error by more than 1%. For most measurements, *R* will be about 0.10 Ω and should have a voltage of 0.1 V. Most oscillators cannot be operated into this low impedance, so a step-down matching transformer must be employed. Make *C* as large as convenient to minimize the ratio of the impedance looking from the voltmeter to the impedance of the test circuit. The *LC* circuit is then tuned to resonate and the resultant voltage measured. The value of *Q* may then be equated

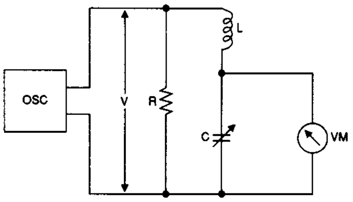


**FIGURE 1.20** Circuit for measuring the *Q* of a coil.

$$Q = \frac{\text{resonant voltage across } C}{\text{voltage across } R} \tag{1.50}$$

The *Q* of any coil may be approximated by the equation

$$\begin{aligned} Q &= \frac{2\pi f L}{R} \\ &= \frac{X_L}{R} \end{aligned} \tag{1.51}$$

where $f$ = the frequency, Hz; $L$ = the inductance, H; $R$ = the dc resistance, Ω (as measured by an ohmmeter); and $X_L$ = the inductive reactance of the coil.

## Time Constant

When a dc voltage is applied to an *RL* circuit, a certain amount of time is required to change the circuit [see text with Eq. (1.31)]. The time constant can be determined with the equation

$$T = \frac{L}{R} \tag{1.52}$$

where $R$ = resistance, Ω; $L$ = inductance, H; and $T$ = time, s.

The *right-hand rule* is used to determine the direction of a magnetic field around a conductor carrying a direct current. Grasp the conductor in the right hand with the thumb extending along the conductor pointing in the direction of the current. With the fingers partly closed, the finger tips will point in the direction of the magnetic field.

Maxwell's rule states, "If the direction of travel of a right-handed corkscrew represents the direction of the current in a straight conductor, the direction of rotation of the corkscrew will represent the direction of the magnetic lines of force."

## Impedance

The total impedance created by resistors, capacitors, and inductors in circuits can be determined with the following equations.

For resistance and capacitance in series,

$$Z = \sqrt{R^2 + X_C^2} \tag{1.53}$$

$$\theta = \arctan \frac{X_C}{R} \tag{1.54}$$

For resistance and inductance in series,

$$Z = \sqrt{R^2 + X_L^2} \tag{1.55}$$

$$\theta = \arctan \frac{X_L}{R} \tag{1.56}$$

For inductance and capacitance in series,

$$Z = \begin{cases} X_L - X_C & \text{when } X_L > X_C \quad (1.57) \\ X_C - X_L & \text{when } X_C > X_L \quad (1.58) \end{cases}$$

For resistance, inductance, and capacitance in series,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{1.59}$$

$$\theta = \arctan \frac{X_L - X_C}{R} \tag{1.60}$$

For capacitance and resistance in parallel,

$$Z = \frac{RX_C}{\sqrt{R^2 + X_C^2}} \tag{1.61}$$

For resistance and inductance in parallel,

$$Z = \frac{RX_L}{\sqrt{R^2 + X_L^2}} \tag{1.62}$$

For capacitance and inductance in parallel,

$$Z = \begin{cases} \dfrac{X_L X_C}{X_L - X_C} & \text{when } X_L > X_C \qquad (1.63) \\ \dfrac{X_C X_L}{X_C - X_L} & \text{when } X_C > X_L \qquad (1.64) \end{cases}$$

For inductance, capacitance, and resistance in parallel,

$$Z = \frac{R X_L X_C}{\sqrt{X_L^2 X_C^2 + R^2 (X_L - X_C)^2}} \tag{1.65}$$

$$\theta = \arctan \frac{R(X_L - X_C)}{X_L X_C} \tag{1.66}$$

For inductance and series resistance in parallel with resistance,

$$Z = R_2 \sqrt{\frac{R_1^2 + X_L^2}{(R_1 + R_2)^2 + X_L^2}} \tag{1.67}$$

$$\theta = \arctan \frac{X_L R_2}{R_1^2 + X_L^2 + R_1 R_2} \tag{1.68}$$

For inductance and series resistance in parallel with capacitance,

$$Z = X_C \sqrt{\frac{R^2 + X_L^2}{R^2 + (X_L - X_C)^2}} \tag{1.69}$$

$$\theta = \arctan \frac{X_L (X_C - X_L) - R^2}{R X_C} \tag{1.70}$$

For capacitance and series resistance in parallel with inductance and series resistance,

$$Z = \sqrt{\frac{(R_1^2 + X_L^2)(R_2^2 + X_C^2)}{(R_1 + R_2)^2 + (X_L - X_C)^2}} \tag{1.71}$$

$$\theta = \arctan \frac{X_L (R_2^2 + X_C^2) - X_C (R_1^2 + X_L^2)}{R_1 (R_2^2 + X_C^2) + R_2 (R_1^2 + X_L^2)} \tag{1.72}$$

where $Z$ = impedance, Ω; $R$ = resistance, Ω; $L$ = inductance, H; $X_L$ = inductive reactance, Ω; $X_C$ = capacitive reactance, Ω; and $\theta$ = **phase** angle, degrees, by which current leads voltage in a capacitive circuit or lags voltage in an inductive circuit (0° indicates an in-phase condition).

### Resonant Frequency

When an inductor and capacitor are connected in series or parallel, they form a resonant circuit. The **resonant frequency** can be determined from the equation

$$\begin{aligned} f &= \frac{1}{2\pi\sqrt{LC}} \\ &= \frac{1}{2\pi C X_C} \\ &= \frac{X_L}{2\pi L} \end{aligned} \tag{1.73}$$

where $f$ = frequency, Hz; $L$ = inductance, H; $C$ = capacitance, F; and $X_L$, $X_C$ = impedance, Ω.

The resonant frequency can also be determined through the use of a reactance chart developed by the Bell Telephone Laboratories (Fig. 1.21). This chart can be used for solving problems of inductance, capacitance, frequency, and impedance. If two of the values are known, the third and fourth values may be found with its use.

## Defining Terms

**Air capacitor:** A fixed or variable capacitor in which air is the dielectric material between the capacitor's plates.

**Ambient temperature:** The temperature of the air or liquid surrounding any electrical part or device. Usually refers to the effect of such temperature in aiding or retarding removal of heat by radiation and convection from the part or device in question.

**Ampere-turns:** The magnetomotive force produced by a coil, derived by multiplying the number of turns of wire in a coil by the current (A) flowing through it.

**Anode:** The positive electrode of a capacitor.

**Capacitive reactance:** The opposition offered to the flow of an alternating or pulsating current by capacitance measured in ohms.

**Capacitor:** An electrical device capable of storing electrical energy and releasing it at some predetermined rate at some predetermined time. It consists essentially of two conducting surfaces (electrodes) separated by an insulating material or dielectric. A capacitor stores electrical energy, blocks the flow of direct current, and permits the flow of alternating current to a degree dependent essentially upon capacitance and frequency. The amount of energy stored, $E = 0.5\ CV^2$.

**Cathode:** The capacitor's negative electrode.

**Coil:** Anumber of turns of wire in the form of a spiral. The spiral may be wrapped around an iron core or an insulating form, or it may be self-supporting. A coil offers considerable opposition to ac current but very little to dc current.

**Conductor:** Abare or insulated wire or combination of wires not insulated from one another, suitable for carrying an electric current.

**Dielectric:** The insulating (nonconducting) medium between the two electrodes (plates) of a capacitor.

**Dielectric constant:** The ratio of the capacitance of a capacitor with a given dielectric to that of the same capacitor having a vacuum dielectric.

**Disk capacitor:** A small single-layer ceramic capacitor with a dielectric insulator consisting of conductively silvered opposing surfaces.

**Dissipation factor (DF):** The ratio of the effective series resistance of a capacitor to its reactance at a specified frequency measured in percent.

**Electrolyte:** Current-conducting solution between two electrodes or plates of a capacitor, at least one of which is covered by a dielectric.

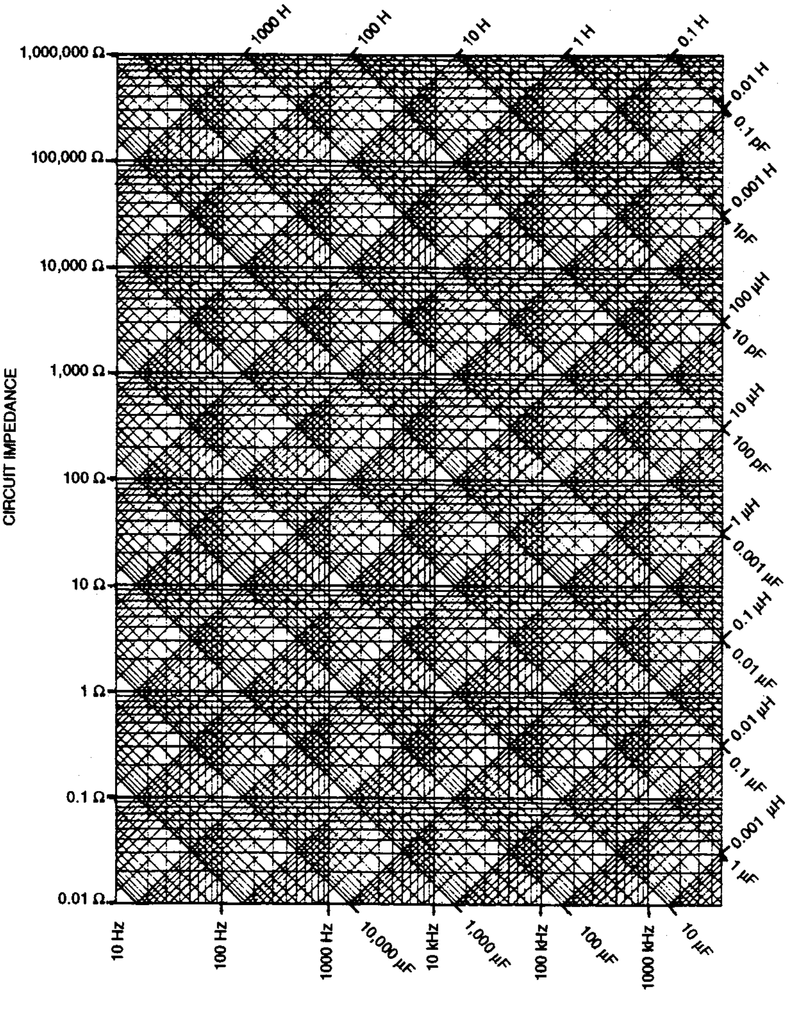


**FIGURE 1.21** Reactance chart. (Courtesy AT&T Bell Laboratories.)

**Electrolytic capacitor:** A capacitor solution between two electrodes or plates of a capacitor, at least one of which is covered by a dielectric.

**Equivalent series resistance (ESR):** All internal series resistance of a capacitor concentrated or "lumped" at one point and treated as one resistance of a capacitor regardless of source, i.e., lead resistance, termination losses, or dissipation in the dielectric material.

**Farad:** The basic unit of measure in capacitors. Acapacitor charged to 1 volt with a charge of 1 coulomb (1 ampere flowing for 1 second) has a capacitance of 1 farad.

**Field:** Ageneral term referring to the region under the influence of a physical agency such as electricity, magnetism, or a combination produced by an electrical charged object.

**Impedance (*Z*):** Total opposition offered to the flow of an alternating or pulsating current measured in ohms. (Impedance is the vector sum of the resistance and the capacitive and inductive reactance, i.e., the ratio of voltage to current.)

**Inductance:** The property which opposes any change in the existing current. Inductance is present only when the current is changing.

**Inductive reactance ($X_L$):** The opposition to the flow of alternating or pulsating current by the inductance of a circuit.

**Inductor:** Aconductor used to introduce inductance into a circuit.

**Leakage current:** Stray direct current of relatively small value which flows through a capacitor when voltage is impressed across it.

**Magnetomotive force:** The force by which the magnetic field is produced, either by a current flowing through a coil of wire or by the proximity of a magnetized body. The amount of magnetism produced in the first method is proportional to the current through the coil and the number of turns in it.

**Mutual inductance:** The property that exists between two current-carrying conductors when the magnetic lines of force from one link with those from another.

**Negative-positive-zero (NPO):** An ultrastable temperature coefficient (±30 ppm/°C from –55 to 125°C) temperature-compensating capacitor.

**Phase:** The angular relationship between current and voltage in an ac circuit. The fraction of the period which has elapsed in a periodic function or wave measured from some fixed origin. If the time for one period is represented as 360° along a time axis, the phase position is called phase angle.

**Polarized capacitor:** An electrolytic capacitor in which the dielectric film is formed on only one metal electrode. The impedance to the flow of current is then greater in one direction than in the other. Reversed polarity can damage the part if excessive current flow occurs.

**Power factor (PF):** The ratio of effective series resistance to impedance of a capacitor, expressed as a percentage.

**Quality factor (*Q*):** The ratio of the reactance to its equivalent series resistance.

**Reactance (*X*):** Opposition to the flow of alternating current. Capacitive reactance ($X_c$) is the opposition offered by capacitors at a specified frequency and is measured in ohms.

**Resonant frequency:** The frequency at which a given system or object will respond with maximum amplitude when driven by an external sinusoidal force of constant amplitude.

**Reverse leakage current:** Anondestructive current flowing through a capacitor subjected to a voltage of polarity opposite to that normally specified.

**Ripple current:** The total amount of alternating and direct current that may be applied to an electrolytic capacitor under stated conditions.

**Temperature coefficient (TC):** A capacitor's change in capacitance per degree change in temperature. May be positive, negative, or zero and is usually expressed in parts per million per degree Celsius (ppm/°C) if the characteristics are linear. For nonlinear types, TC is expressed as a percentage of room temperature (25°C)capacitance.

**Time constant:** In a capacitor-resistor circuit, the number of seconds required for the capacitor to reach 63.2% of its full charge after a voltage is applied. The time constant of a capacitor with a capacitance ($C$) in farads in series with a resistance ($R$) in ohms is equal to $R \times C$ seconds.

**Winding:** Aconductive path, usually wire, inductively coupled to a magnetic core or cell.

## Related Topic

55.5 Dielectric Materials

## References

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# 1.3 Transformers

## *C. Sankaran*

The electrical transformer was invented by an American electrical engineer, William Stanley, in 1885 and was used in the first ac lighting installation at Great Barrington, Massachusetts. The first transformer was used to step up the power from 500 to 3000 V and transmitted for a distance of 1219 m (4000 ft). At the receiving end the voltage was stepped down to 500 V to power street and office lighting. By comparison, present transformers are designed to transmit hundreds of megawatts of power at voltages of 700 kV and beyond for distances of several hundred miles.

Transformation of power from one voltage level to another is a vital operation in any transmission, distribution, and utilization network. Normally, power is generated at a voltage that takes into consideration the cost of generators in relation to their operating voltage. Generated power is transmitted by overhead lines many miles and undergoes several voltage transformations before it is made available to the actual user. Figure 1.22 shows a typical power flow line diagram.

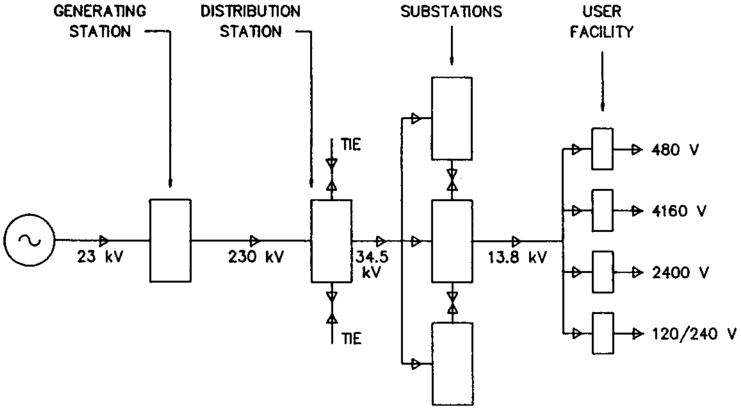

**FIGURE 1.22** Power flow line diagram.

## Types of Transformers

Transformers are broadly grouped into two main categories: dry-type and liquid-filled transformers. Dry-type transformers are cooled by natural or forced circulation of air or inert gas through or around the transformer enclosure. Dry-type transformers are further subdivided into ventilated, sealed, or encapsulated types depending upon the construction of the transformer. Dry transformers are extensively used in industrial power distribution for rating up to 5000 kVA and 34.5 kV.

Liquid-filled transformers are cooled by natural or forced circulation of a liquid coolant through the windings of the transformer. This liquid also serves as a **dielectric** to provide superior voltage-withstand characteristics. The most commonly used liquid in a transformer is a mineral oil known as transformer oil that has a continuous operating temperature rating of 105°C, a flash point of 150°C, and a fire point of 180°C. A good grade transformer oil has a **breakdown strength** of 86.6 kV/cm (220 kV/in.) that is far higher than the breakdown strength of air, which is 9.84 kV/cm (25 kV/in.) at atmospheric pressure.

Silicone fluid is used as an alternative to mineral oil. The breakdown strength of silicone liquid is over 118 kV/cm (300 kV/in.) and it has a flash point of 300°C and a fire point of 360°C. Silicone-fluid-filled transformers are classified as less flammable. The high dielectric strengths and superior thermal conductivities of liquid coolants make them ideally suited for large high-voltage power transformers that are used in modern power generation and distribution.

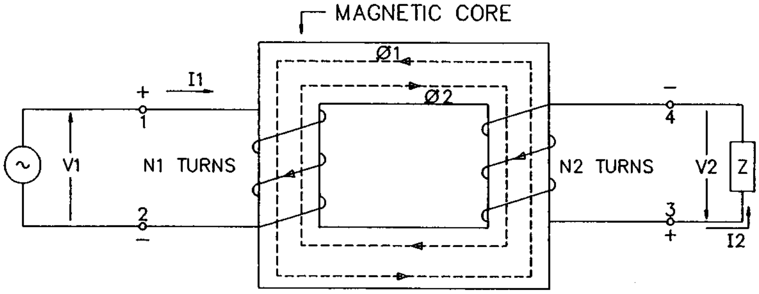

**FIGURE 1.23** Electrical power transfer.

## Principle of Transformation

The actual process of transfer of electrical power from a voltage of $V_1$ to a voltage of $V_2$ is explained with the aid of the simplified transformer representation shown in Fig. 1.23. Application of voltage across the primary winding of the transformer results in a **magnetic field** of $\phi_1$ Wb in the magnetic core, which in turn induces a voltage of $V_2$ at the secondary terminals. $V_1$ and $V_2$ are related by the expression $V_1/V_2 = N_1/N_2$, where $N_1$ and $N_2$ are the number of turns in the primary and secondary windings, respectively. If a load current of $I_2$ A is drawn from the secondary terminals, the load current establishes a magnetic field of $\phi_2$ Wb in the core and in the direction shown. Since the effect of load current is to reduce the amount of primary magnetic field, the reduction in $\phi_1$ results in an increase in the primary current $I_1$ so that the net magnetic field is almost restored to the initial value and the slight reduction in the field is due to leakage **magnetic flux**. The currents in the two windings are related by the expression $I_1/I_2 = N_2/N_1$. Since $V_1/V_2 = N_1/N_2 = I_2/I_1$, we have the expression $V_1 \cdot I_1 = V_2 \cdot I_2$. Therefore, the voltamperes in the two windings are equal in theory. In reality, there is a slight loss of power during transformation that is due to the energy necessary to set up the magnetic field and to overcome the losses in the transformer core and windings. Transformers are static power conversion devices and are therefore highly efficient. Transformer efficiencies are about 95% for small units (15 kVA and less), and the efficiency can be higher than 99% for units rated above 5 MVA.

## Electromagnetic Equation

Figure 1.24 shows a magnetic core with the area of cross section $A = W \cdot D$ m$^2$. The transformer primary winding that consists of $N$ turns is excited by a sinusoidal voltage $v = V\sin(\omega t)$, where $\omega$ is the angular frequency given by the expression $\omega = 2\pi f$ and $f$ is the frequency of the applied voltage waveform. $\phi$ is magnetic field in the core due to the excitation current $i$:

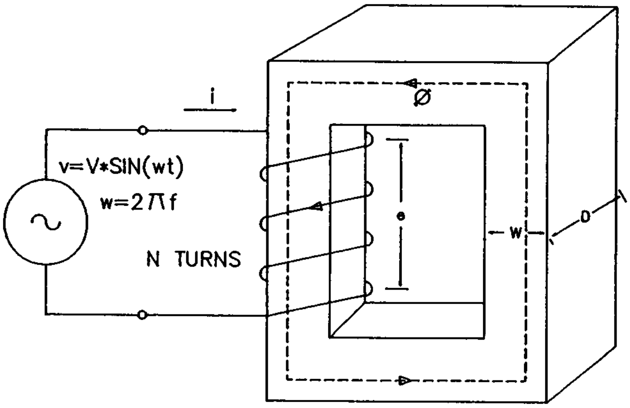

**FIGURE 1.24** Electromagnetic relation.

$$\phi = \Phi \sin\left(\omega t - \frac{\pi}{2}\right) = -\Phi \cos(\omega t)$$

Induced voltage in the winding

$$e = -N\frac{d\phi}{dt} = N\frac{d[\Phi \cos(\omega t)]}{dt} = -N\omega\Phi \sin(\omega t)$$

Maximum value of the induced voltage

$$E = N\omega\Phi$$

The root-mean-square value

$$E_{\text{rms}} = \frac{E}{\sqrt{2}} = \frac{2\pi f N\Phi}{\sqrt{2}} = 4.44 f \text{ NBA}$$

where flux $\Phi$ (webers) is replaced by the product of the flux density $B$ (teslas) and the area of cross section of the core.

This fundamental design equation determines the size of the transformer for any given voltage and frequency. Power transformers are normally operated at flux density levels of 1.5 T.

## Transformer Core

The transformer core is the medium that enables the transfer of power from the primary to the secondary to occur in a transformer. In order that the transformation of power may occur with the least amount of loss, the magnetic core is made up of laminations which have the highest permeability, permeability being a measure of the ease with which the magnetic field is set up in the core.

The magnetic field reverses direction every one half cycle of the applied voltage and energy is expended in the core to accomplish the cyclic reversals of the field. This loss component is known as the hysteresis loss $P_h$:

$$P_h = 150.7 V_e f B^{1.6} \quad \text{W}$$

where $V_e$ is the volume of the core in cubic meters, $f$ is the frequency, and $B$ is the maximum flux density in teslas.

As the magnetic field reverses direction and cuts across the core structure, it induces a voltage in the laminations known as eddy voltages. This phenomenon causes eddy currents to circulate in the laminations. The loss due to eddy currents is called the eddy current loss $P_e$:

$$P_e = 1.65 V_e B^2 f^2 t^2 / r$$

where $V_e$ is the volume of the core in cubic meters, $f$ is the frequency, $B$ is the maximum flux density in teslas, $t$ is thickness of the laminations in meters, and $r$ is the resistivity of the core material in ohm-meters.

Hysteresis losses are reduced by operating the core at low flux densities and using core material of high permeability. Eddy current losses are minimized by low flux levels, reduction in thickness of the laminations, and high resistivity core material.

Cold-rolled, grain-oriented silicon steel laminations are exclusively used in large power transformers to reduce core losses. A typical silicon steel used in transformers contains 95% iron, 3% silicon, 1% manganese, 0.2% phosphor, 0.06% carbon, 0.025% sulphur, and traces of other impurities.

## Transformer Losses

The heat developed in a transformer is a function of the losses that occur during transformation. Therefore, the transformer losses must be minimized and the heat due to the losses must be efficiently conducted away from the core, the windings, and the cooling medium. The losses in a transformer are grouped into two categories: (1) no-load losses and (2) load losses. The no-load losses are the losses in the core due to excitation and are mostly composed of hysteresis and eddy current losses. The load losses are grouped into three categories: (1) winding $I^2R$ losses, (2) winding eddy current losses, and (3) other stray losses. The winding $I^2R$ losses are the result of the flow of load current through the resistance of the primary and secondary windings. The winding eddy current losses are caused by the magnetic field set up by the winding current, due to formation of eddy voltages in the conductors. The winding eddy losses are proportional to the square of the rms value of the current and to the square of the frequency of the current. When transformers are required to supply loads that are rich in **harmonic frequency** components, the eddy loss factor must be given extra consideration. The other stray loss component is the result of induced currents in the buswork, core clamps, and tank walls by the magnetic field set up by the load current.

### Transformer Connections

A single-phase transformer has one input (primary) winding and one output (secondary) winding. A conventional three-phase transformer has three input and three output windings. The three windings can be connected in one of several different configurations to obtain three-phase connections that are distinct. Each form of connection has its own merits and demerits.

### Y Connection (Fig. 1.25)

In the Y connection, one end of each of the three windings is connected together to form a Y, or a neutral point. This point is normally grounded, which limits the maximum potential to ground in the transformer to the line to neutral voltage of the power system. The grounded neutral also limits transient overvoltages in the transformer when subjected to lightning or switching surges. Availability of the neutral point allows the transformer to supply line to neutral single-phase loads in addition to normal three-phase loads. Each phase of the Y-connected winding must be designed to carry the full line current, whereas the phase voltages are only 57.7% of the line voltages.

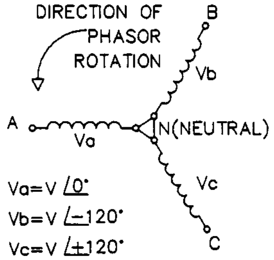

**FIGURE 1.25** Y connection.

### Delta Connection (Fig. 1.26)

In the delta connection, the finish point of each winding is connected to the start point of the adjacent winding to form a closed triangle, or delta. A delta winding in the transformer tends to balance out unbalanced loads that are present on the system. Each phase of the delta winding only carries 57.7% of the line current, whereas the phase voltages are equal to the line voltages.

Large power transformers are designed so that the high-voltage side is connected in Y and the low-voltage side is connected in delta. Distribution transformers that are required to supply single-phase loads are designed in the opposite configuration so that the neutral point is available at the low-voltage end.

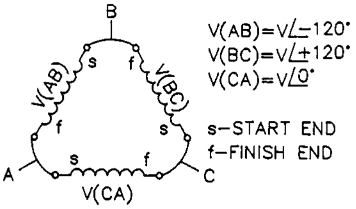

**FIGURE 1.26** Delta connection.

### Open-Delta Connection (Fig. 1.27)

An open-delta connection is used to deliver three-phase power if one phase of a three-phase bank of transformers fails in service. When the failed unit is removed from service, the remaining units can still supply three-phase power but at a reduced rating. An open-delta connection is also used as an economical means to deliver three-phase power using only two single-phase transformers. If *P*

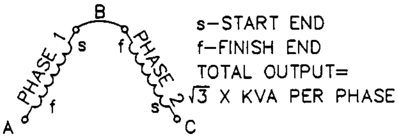

**FIGURE 1.27** Open-delta connection.

is the total three-phase kVA, then each transformer of the open-delta bank must have a rating of $P/\sqrt{3}$ kVA. The disadvantage of the open-delta connection is the unequal **regulation** of the three phases of the transformer.

### T Connection (Fig. 1.28)

The T connection is used for three-phase power transformation when two separate single-phase transformers with special configurations are available. If a voltage transformation from $V_1$ to $V_2$ volts is required, one of the units (main transformer) must have a voltage ratio of $V_1/V_2$ with the midpoint of each winding brought out. The other unit must have a ratio of $0.866V_1/0.866V_2$ with the neutral point brought out, if needed.

The Scott connection is a special type of T connection used to transform three-phase power to two-phase power for operation of electric furnaces and two-phase motors. It is shown in Fig. 1.29.

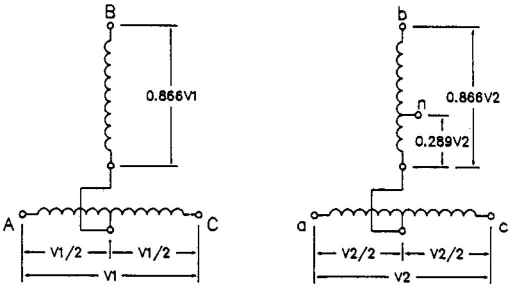

**FIGURE 1.28** T connection.

### Zigzag Connection (Fig. 1.30)

This connection is also called the interconnected star connection where the winding of each phase is divided into two halves and interconnected to form a zigzag configuration. The zigzag connection is mostly used to derive a neutral point for grounding purposes in three-phase, three-wire systems. The neutral point can be used to (1) supply single-phase loads, (2) provide a safety ground, and (3) sense and limit ground fault currents.

## Transformer Impedance

Impedance is an inherent property in a transformer that results in a voltage drop as power is transferred from the primary to the secondary side of the power system. The impedance of a transformer consists of two parts: resistance ($R$) and reactance ($X$). The resistance component is due to the resistance of the material of the winding and the percentage value of the voltage drop due to resistance becomes less as the rating of the transformer increases. The reactive component, which is also known as leakage reactance, is the result of incomplete linkage of the magnetic field set up by the secondary winding with the turns of the primary winding, and vice versa. The net impedance of the transformer is given by $Z = \sqrt{R^2 + X^2}$. The impedance value marked on the transformer is the percentage voltage drop due to this impedance under full-load operating conditions:

$$\%\ \text{impedance}\ z = IZ\left(\frac{100}{V}\right)$$

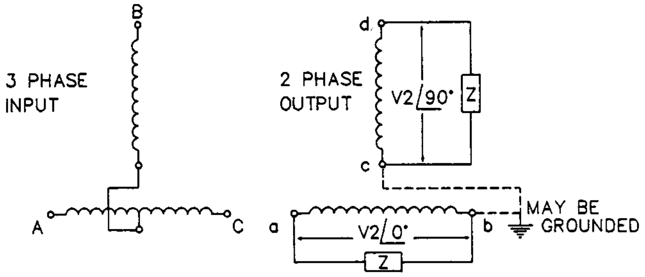

**FIGURE 1.29** Three-phase–two-phase transformation.

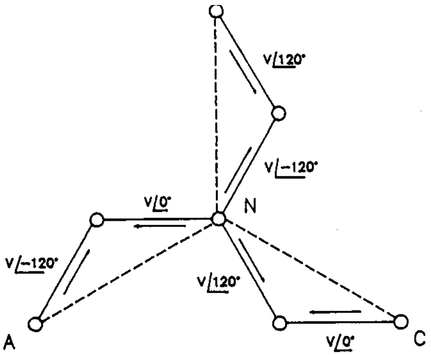

**FIGURE 1.30** Zigzag connection.

where $I$ is the full-load current of the transformer, $Z$ is the impedance in ohms of the transformer, and $V$ is the voltage rating of the transformer winding. It should be noted that the values of $I$ and $Z$ must be referred to the same side of the transformer as the voltage $V$.

Transformers are also major contributors of impedance to limit the fault currents in electrical power systems.

## Defining Terms

**Breakdown strength:** Voltage gradient at which the molecules of medium break down to allow passage of damaging levels of electric current.

**Dielectric:** Solid, liquid, or gaseous substance that acts as an insulation to the flow of electric current.

**Harmonic frequency:** Integral multiples of fundamental frequency. For example, for a 60-Hz supply the harmonic frequencies are 120, 180, 240, 300, . . .

**Magnetic field:** Magnetic force field where lines of magnetism exist.

**Magnetic flux:** Term for lines of magnetism.

**Regulation:** The change in voltage from no-load to full-load expressed as a percentage of full-load voltage.

## Related Topics

9.3 Wye ⇔ Delta Transformations • 36.1 Magnetism • 61.6 Protection • 64.1 Transformer Construction

## References and Further Information

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# 1.4 Electrical Fuses

*Nick Angelopoulos*

The fuse is a simple and reliable safety device. It is second to none in its ease of application and its ability to protect people and equipment.

The fuse is a current-sensitive device. It has a conductor with a reduced cross section (element) normally surrounded by an arc-quenching and heat-conducting material (filler). The entire unit is enclosed in a body fitted with end contacts. A basic fuse element design is illustrated in Fig. 1.32.

## Ratings

Most fuses have three electrical ratings: ampere rating, voltage rating, and **interrupting rating**. The ampere rating indicates the current the fuse can carry without melting or exceeding specific temperature rise limits. The voltage rating, ac or dc, usually indicates the maximum system voltage that can be applied to the fuse. The interrupting rating (I.R.) defines the maximum short-circuit current that a fuse can safely interrupt. If a fault current higher than the interrupting rating causes the fuse to operate, the high internal pressure may cause the fuse to rupture. It is imperative, therefore, to install a fuse, or any other type of protective device, that has an interrupting rating not less than the available short-circuit current. A violent explosion may occur if the interrupting rating of any protective device is inadequate.

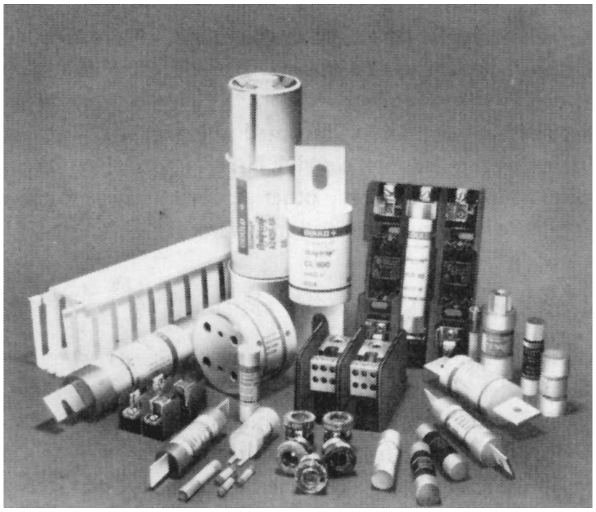


**FIGURE 1.31** A variety of plug, cartridge, and blade type fuses.

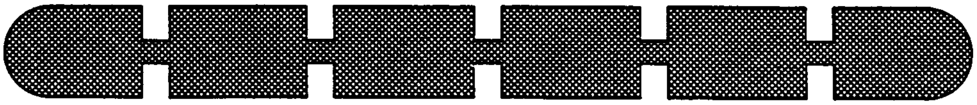


**FIGURE 1.32** Basic fuse element.

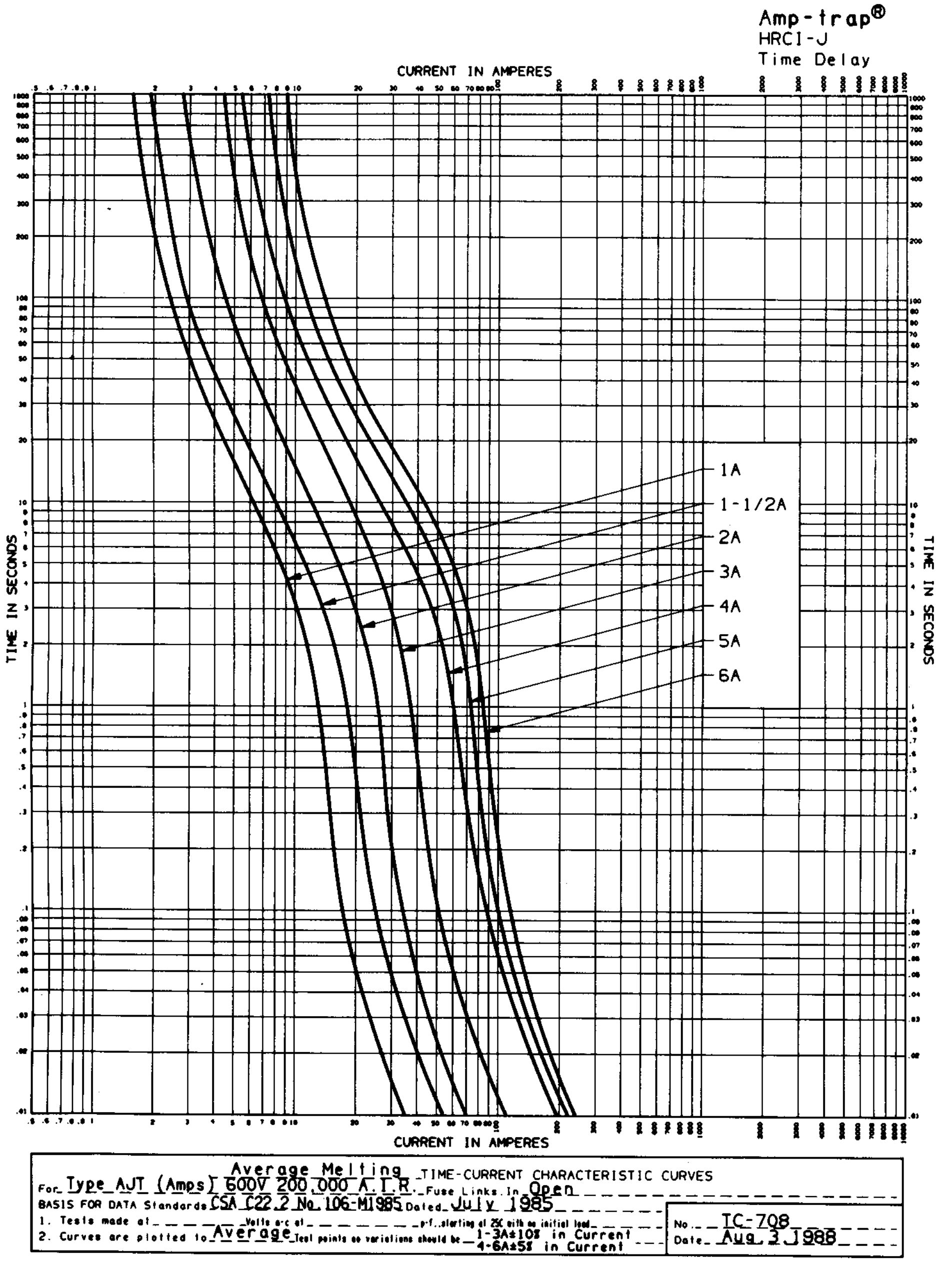
A fuse must perform two functions. The first, the "passive" function, is one that tends to be taken for granted. In fact, if the fuse performs the passive function well, we tend to forget that the fuse exists at all. The passive function simply entails that the fuse can carry up to its normal load current without aging or overheating. Once the current level exceeds predetermined limits, the "active" function comes into play and the fuse operates. It is when the fuse is performing its active function that we become aware of its existence.

In most cases, the fuse will perform its active function in response to two types of circuit conditions. The first is an overload condition, for instance, when a hair dryer, teakettle, toaster, and radio are plugged into the same circuit. This overload condition will eventually cause the element to melt. The second condition is the overcurrent condition, commonly called the short circuit or the fault condition. This can produce a drastic, almost instantaneous, rise in current, causing the element to melt usually in less than a quarter of a cycle. Factors that can lead to a fault condition include rodents in the electrical system, loose connections, dirt and moisture, breakdown of insulation, foreign contaminants, and personal mistakes. Preventive maintenance and care can reduce these causes. Unfortunately, none of us are perfect and faults can occur in virtually every electrical system—we must protect against them.

## Fuse Performance

Fuse performance characteristics under overload conditions are published in the form of *average melting time–current characteristic curves,* or simply *time-current curves.* Fuses are tested with a variety of currents, and the melting times are recorded. The result is a graph of time versus current coordinates that are plotted on log-log scale, as illustrated in Fig. 1.33.

Under short-circuit conditions the fuse operates and fully opens the circuit in less than 0.01 s. At 50 or 60 Hz, this represents operation within the first half cycle. The current waveform let-through by the fuse is the shaded, almost triangular, portion shown in Fig. 1.34(a). This depicts a fraction of the current that would have been let through into the circuit had a fuse not been installed.

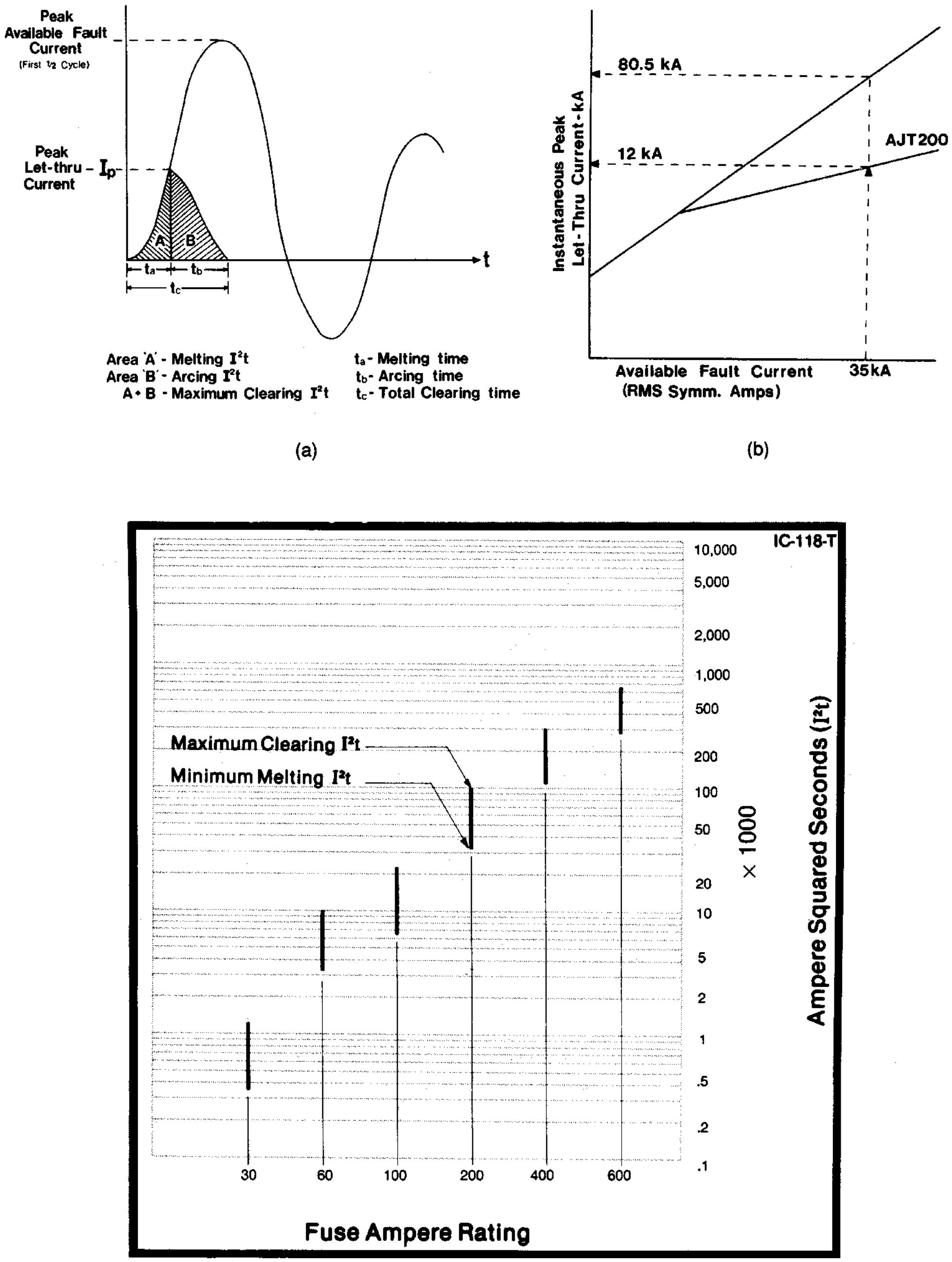


Average Melting TIME-CURRENT CHARACTERISTIC CURVES
For Type AJT (Amps) 600V 200,000 A.I.R. Fuse Links In Open
BASIS FOR DATA Standards CSA C22.2 No. 106-M1985 Dated July 1985
1. Tests made at ___ Volts a-c at ___ p-f., starting at 25C with no initial load ___
2. Curves are plotted to Average Test points no variations should be 1-3A±10% in Current 4-6A±5% in Current
No. TC-708
Date Aug. 3 1988

**FIGURE 1.33** Time-current characteristic curves.

Fuse short-circuit performance characteristics are published in the form of peak let-through ($I_p$) graphs and $I^2t$ graphs. $I_p$ (peak current) is simply the peak of the shaded triangular waveform, which increases as the fault current increases, as shown in Fig. 1.34(b). The electromagnetic forces, which can cause mechanical damage to equipment, are proportional to $I_p^2$.

$I^2t$ represents heat energy measured in units of $A^2$ s (ampere squared seconds) and is documented on $I^2t$ graphs. These $I^2t$ graphs, as illustrated in Fig. 1.34(c), provide three values of $I^2t$: minimum melting $I^2t$, arcing $I^2t$, and total clearing $I^2t$. $I^2t$ and $I_p$ short-circuit performance characteristics can be used to coordinate fuses and other equipment. In particular, $I^2t$ values are often used to selectively coordinate fuses in a distribution system.



(a) (b)

(c)

**FIGURE 1.34** (a) Fuse short-circuit operation. (b) Variation of fuse peak let-through current $I_p$. (c) $I^2t$ graph.

## Selective Coordination

In any power distribution system, selective coordination exists when the fuse immediately upstream from a fault operates, leaving all other fuses further upstream unaffected. This increases system reliability by isolating the faulted branch while maintaining power to all other branches. Selective coordination is easily assessed by comparing the $I^2t$ characteristics for feeder and branch circuit fuses. The branch fuse should have a total clearing $I^2t$ value that is less than the melting $I^2t$ value of the feeder or upstream fuse. This ensures that the branch fuse will melt, arc, and clear the fault before the feeder fuse begins to melt.

## Standards

Overload and short-circuit characteristics are well documented by fuse manufacturers. These characteristics are standardized by product standards written in most cases by safety organizations such as CSA (Canadian Standards Association) and UL (Underwriters Laboratories). CSA standards and UL specify product designations, dimensions, performance characteristics, and temperature rise limits. These standards are used in conjunction with national code regulations such as CEC (Canadian Electrical Code) and NEC (National Electrical Code) that specify how the product is applied.

IEC (International Electrotechnical Commission—Geneva, Switzerland) was founded to harmonize electrical standards to increase international trade in electrical products. Any country can become a member and participate in the standards-writing activities of IEC. Unlike CSA and UL, IEC is not a certifying body that certifies or approves products. IEC publishes consensus standards for national standards authorities such as CSA (Canada), UL (USA), BSI (UK) and DIN (Germany) to adopt as their own national standards.

## Products

North American low-voltage distribution fuses can be classified under two types: Standard or Class H, as referred to in the United States, and **HRC (high rupturing capacity)** or current-limiting fuses, as referred to in Canada. It is the interrupting rating that essentially differentiates one type from the other.

Most Standard or Class H fuses have an interrupting rating of 10,000 A. They are not classified as HRC or current-limiting fuses, which usually have an interrupting rating of 200,000 A. Selection is often based on the calculated available short-circuit current.

In general, short-circuit currents in excess of 10,000 A do not exist in residential applications. In commercial and industrial installations, short-circuit currents in excess of 10,000 A are very common. Use of HRC fuses usually means that a fault current assessment is not required.

## Standard—Class H

In North America, Standard or Class H fuses are available in 250- and 600-V ratings with ampere ratings up to 600 A. There are primarily three types: one-time, time-delay, and renewable. Rating for rating, they are all constructed to the same dimensions and are physically interchangeable in standard-type fusible switches and fuse blocks.

One-time fuses are not reusable once blown. They are used for general-purpose resistive loads such as lighting, feeders, and cables.

Time-delay fuses have a specified delay in their overload characteristics and are designed for motor circuits. When started, motors typically draw six times their full load current for approximately 3 to 4 seconds. This surge then decreases to a level within the motor full-load current rating. Time-delay fuse overload characteristics are designed to allow for motor starting conditions.

Renewable fuses are constructed with replaceable links or elements. This feature minimizes the cost of replacing fuses. However, the concept of replacing fuse elements in the field is not acceptable to most users today because of the potential risk of improper replacement.

## HRC

HRC or current-limiting fuses have an interrupting rating of 200 kA and are recognized by a letter designation system common to North American fuses. In the United States they are known as Class J, Class L, Class R, etc., and in Canada they are known as HRCI-J, HRC-L, HRCI-R, and so forth. HRC fuses are available in ratings up to 600 V and 6000 A. The main differences among the various types are their dimensions and their short-circuit performance ($I_p$ and $I^2t$) characteristics.

One type of HRC fuse found in Canada, but not in the United States, is the HRCII-C or Class C fuse. This fuse was developed originally in England and is constructed with bolt-on-type blade contacts. It is available in a voltage rating of 600 V with ampere ratings from 2 to 600 A. Some higher ampere ratings are also available but are not as common. HRCII-C fuses are primarily regarded as providing short-circuit protection only. Therefore, they should be used in conjunction with an overload device.

HRCI-R or Class R fuses were developed in the United States. Originally constructed to Standard or Class H fuse dimensions, they were classified as Class K and are available in the United States with two levels of short-circuit performance characteristics: Class K1 and Class K5. However, they are not recognized in Canadian Standards. Under fault conditions, Class K1 fuses limit the $I_p$ and $I^2t$ to lower levels than do Class K5 fuses. Since both Class K1 and K5 are constructed to Standard or Class H fuse dimensions, problems with interchangeability occur. As a result, a second generation of these K fuses was therefore introduced with a rejection feature incorporated in the end caps and blade contacts. This rejection feature, when used in conjunction with rejection-style fuse clips, prevents replacement of these fuses with Standard or Class H 10-kA I.R. fuses. These rejection style fuses are known as Class RK1 and Class RK5. They are available with time-delay or non-time-delay characteristics and with voltage ratings of 250 or 600 V and ampere ratings up to 600 A. In Canada, CSA has only one classification for these fuses, HRCI-R, which have the same maximum $I_p$ and $I^2t$ current-limiting levels as specified by UL for Class RK5 fuses.

HRCI-J or Class J fuses are a more recent development. In Canada, they have become the most popular HRC fuse specified for new installations. Both time-delay and non-time-delay characteristics are available in ratings of 600 V with ampere ratings up to 600 A. They are constructed with dimensions much smaller than HRCI-R or Class R fuses and have end caps or blade contacts which fit into 600-V Standard or Class H-type fuse clips.

However, the fuse clips must be mounted closer together to accommodate the shorter fuse length. Its shorter length, therefore, becomes an inherent rejection feature that does not allow insertion of Standard or HRCI-R fuses. The blade contacts are also drilled to allow bolt-on mounting if required. CSA and UL specify these fuses to have maximum short-circuit current-limiting $I_p$ and $I^2t$ limits lower than those specified for HRCI-R and HRCII-C fuses. HRCI-J fuses may be used for a wide variety of applications. The time-delay type is commonly used in motor circuits sized at approximately 125 to 150% of motor full-load current.

HRC-L or Class L fuses are unique in dimension but may be considered as an extension of the HRCI-J fuses for ampere ratings above 600 A. They are rated at 600 V with ampere ratings from 601 to 6000 A. They are physically larger and are constructed with bolt-on-type blade contacts. These fuses are generally used in low-voltage distribution systems where supply transformers are capable of delivering more than 600 A.

In addition to Standard and HRC fuses, there are many other types designed for specific applications. For example, there are medium- or high-voltage fuses to protect power distribution transformers and medium-voltage motors. There are fuses used to protect sensitive semiconductor devices such as diodes, SCRs, and triacs. These fuses are designed to be extremely fast under short-circuit conditions. There is also a wide variety of dedicated fuses designed for protection of specific equipment requirements such as electric welders, capacitors, and circuit breakers, to name a few.

## Trends

Ultimately, it is the electrical equipment being protected that dictates the type of fuse needed for proper protection. This equipment is forever changing and tends to get smaller as new technology becomes available. Present trends indicate that fuses also must become smaller and faster under fault conditions, particularly as available short-circuit fault currents are tending to increase.

With free trade and the globalization of industry, a greater need for harmonizing product standards exists. The North American fuse industry is taking big steps toward harmonizing CSA and UL fuse standards, and at the same time is participating in the IEC standards process. Standardization will help the electrical industry to identify and select the best fuse for the job—anywhere in the world.

## Defining Terms

**HRC (high rupturing capacity):** A term used to denote fuses having a high interrupting rating. Most low-voltage HRC-type fuses have an interrupting rating of 200 kA rms symmetrical.

**$I^2t$ (ampere squared seconds):** A convenient way of indicating the heating effect or thermal energy which is produced during a fault condition before the circuit protective device has opened the circuit. As a protective device, the HRC or current-limiting fuse lets through far less damaging $I^2t$ than other protective devices.

**Interrupting rating (I.R.):** The maximum value of short-circuit current that a fuse can safely interrupt.

## Related Topic

1.1 Resistors

## References

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## Further Information

For greater detail the “Shawmut Advisor” (Gould, Inc., 374 Merrimac Street, Newburyport MA 01950) or the “Fuse Technology Course Notes” (Gould Shawmut Company, 88 Horner Avenue, Toronto, Canada M8Z-5Y3) may be referred to for fuse performance and application.