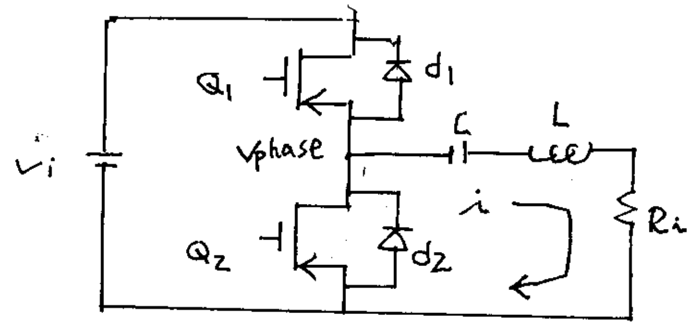
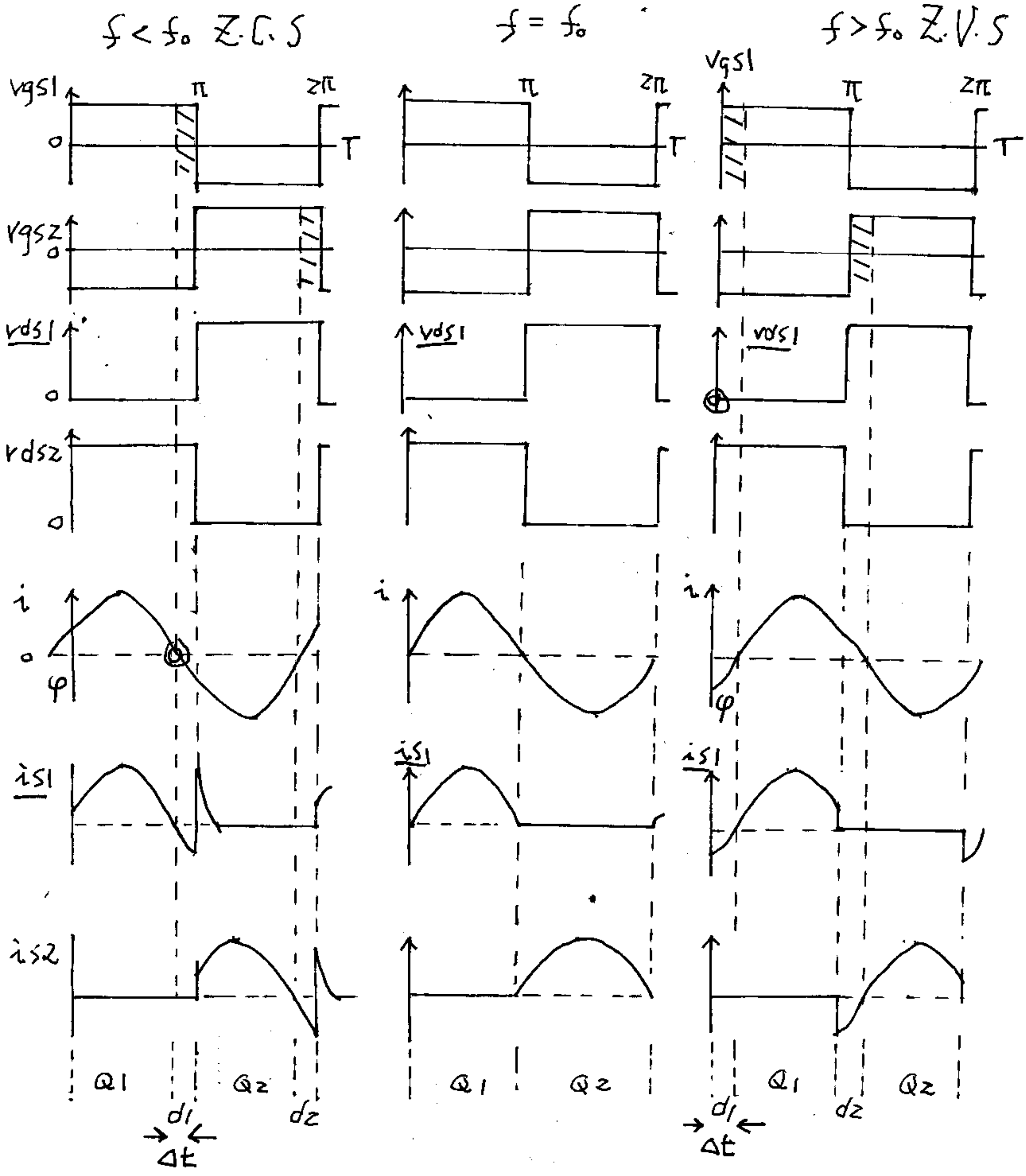
Boundary condition of ZCS & ZVS



#1.



#2.

We define $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Z_0 = \sqrt{\frac{L}{C}} = \omega_0 L = \frac{1}{\omega_0 C}$$

- Loaded quality

$$Q_L = \frac{Z_0}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \sqrt{\frac{L}{C}}/R$$

- unloaded quality

$$Q_0 = \frac{Z_0}{r} = \frac{\omega_0 L}{L} = \frac{1}{\omega_0 C r}$$

where $r = r_{ds} + r_L + r_C$

$R = R_i + r$

說明

1. 當 $f = f_0$ 时，Vphase 看出去的 Load 为 Resistive Load

i in-phase V_{ds1} ∴ 沒有相位移.

2. 當 $f < f_0$ 时，Vphase 看出去 Capacitive Load，i Lead V_{ds1}，$\varphi < 0$，在 $T = \pi - \Delta t$ 之前的 Δt 时，此时不論 V_{gs1} 的 Level 如何，$i = 0$，這就是 Z.C.S。由 Next stage 为 Q2 turn on ∴ d_1 的 recovery time 會造成很大的 current-spike. [Vds 尚未 turn-off，Ids 已經为 0]

3. 當 $f > f_0$ 时，Vphase 看出去为 inductive load，i Leg V_{ds1}，$\varphi > 0$，在 $T = 0 \sim \Delta t$ 时，不論 V_{gs1} 的 level 如何，V_{ds1} 被 d_1 by pass 而为 0，這是 Z.V.S [Ids 尚未 turn-on，Vds 已經为 0]

CLL Resonant — converter

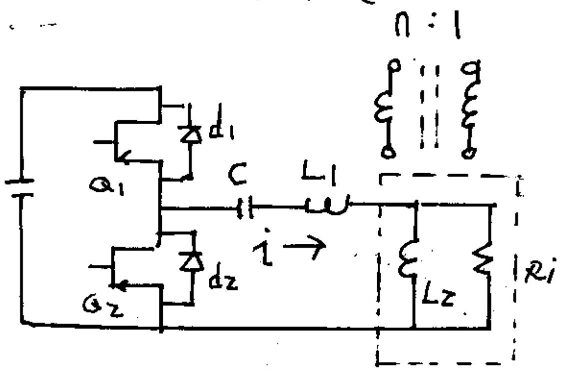


FIG 1.

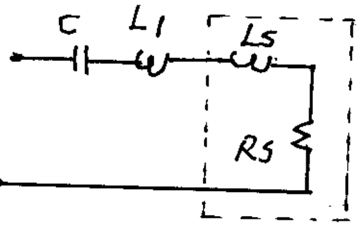


FIG 2

#3. Boundary condition Between Capacitive Load & Inductive Load

⊙ We define :

$$A = \frac{L_1}{L_2} \sim ①$$

$$L = L_1 + L_2 = L_2(1+A) = L_1(1+\frac{1}{A}) \sim ②$$

corner frequency [undamped natural frequency]

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(L_1+L_2)C}} \sim ③$$

$$Z_0 = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}} \sim ④$$

Loaded quality at ω_0

$$Q_L = \frac{R_i}{Z_0} = \frac{R_i}{\omega_0 L} = \omega_0 C R_i \sim ⑤$$

由②式得 $L_2 = \frac{L}{1+A}$ ⑤式得 $L = \frac{R_i}{\omega_0 L}$

合併為 $L_2 = \frac{R_i}{(1+A)\,\omega_0 Q_L} \sim ⑥$

⊙ Fig 2 為 fig 1 的串联等效电路

$$L_{eq} = L_1 + L_s \sim ⑦$$

Resonant frequency

$$\omega r = \frac{1}{\sqrt{Leq.C}} = \frac{1}{\sqrt{(L_1+L_S)C}} \sim ⑧$$

Loaded quality factor at ωr

$$Qr = \frac{\omega r(L_1+L_S)}{RS} = \frac{1}{\omega r C Rs} \sim ⑨$$

Note：由串联共振可知道 Capacitive load 或 inductive load 其 boundary 條件由 ωr 决定之。

⊙ 求 input impedance

由 Fig 1 知道

$$Z = j\omega A L_2 + \frac{1}{j\omega C} + \frac{R_i\, j\omega L_2}{R_i + j\omega L_2}$$

$$= \frac{j\omega A L_2[j\omega C \times (R_i + j\omega L_2)] + [R_i + j\omega L_2] + R_i\, j\omega L_2\, j\omega C}{j\omega C\,[R_i + j\omega L_2]}$$

$$= \frac{[-\omega^2 A L_2 C R_i - \omega^2 L_2 C R_i + R_i] + [j\omega L_2 - j\omega^3 A L_2^2 C]}{j\omega C R_i - \omega^2 L_2 C}$$

分子及分母除以 $-\omega^2 L_2 C$

$$= \frac{\left[A R_i + R_i - \frac{R_i}{\omega^2 L_2 C}\right]^{\triangle 1} + \left[j\omega A L_2 - j\frac{1}{\omega C}\right]^{\triangle 2}}{1 - j\frac{R_i}{\omega L_2}\ \triangle 3} \sim ⑩$$

Recall $L_2 = \frac{R_i}{(1+A)\omega_0 Q_L}$; $C = \frac{Q_L}{\omega_0 R_i}$; $L_2 = \frac{L}{1+A}$

$\omega_0 = \frac{1}{\sqrt{LC}}$ 代入⑩式中的

$\triangle 1$, $\triangle 2$ $\triangle 3$ 式之中

由 ① 式

$$R_i(1+A)-\frac{R_i(1+A)}{\omega^2 LC}=R_i(1+A)\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]$$

由 ② 式

$$j\,R_i\frac{\omega}{\omega_0}\frac{A}{(1+A)}\frac{1}{Q_L}-j\,\frac{\omega_0}{\omega}\frac{R_i}{Q_L}=j\frac{R_i}{Q_L}\left[\frac{\omega}{\omega_0}\frac{A}{(1+A)}-\frac{\omega_0}{\omega}\right]$$

由 ③ 式

$$1-j\frac{R_i}{\omega}\frac{(1+A)\omega_0 Q_L}{R_i}=1-j(1+A)\frac{\omega_0}{\omega}Q_L$$

合併 ① ② ③ 式可得

$$Z=\frac{R_i\left\{(1+A)\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]+j\frac{1}{Q_L}\left[\frac{\omega}{\omega_0}\frac{A}{1+A}-\frac{\omega_0}{\omega}\right]\right\}}{1-jQ_L(1+A)\frac{\omega_0}{\omega}} \sim (11)$$

$$=Ze^{j\varphi}=R_s+jX_s$$

Recall $R_i=Q_L Z_0$

$$\frac{Z}{Z_0}=Q_L\sqrt{\frac{[1+A]^2\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]^2+\frac{1}{Q_L^2}\left[\frac{\omega}{\omega_0}\frac{A}{(1+A)}-\frac{\omega_0}{\omega}\right]^2}{1+\left[Q_L(1+A)\frac{\omega_0}{\omega}\right]^2}} \sim (12)$$

在 ⑪式 分子大括号之中 乘以 $\left[1+jQ_L(1+A)\frac{\omega_0}{\omega}\right]$。分母为实数。
分母

$$(1+A)\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]+j\frac{1}{Q_L}\left[\frac{\omega}{\omega_0}\frac{A}{1+A}-\frac{\omega_0}{\omega}\right] \quad ①$$

$$+\left[jQ_L(1+A)\frac{\omega_0}{\omega}\right](1+A)\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]-\left[(1+A)\frac{\omega_0}{\omega}\right]\left[\frac{\omega}{\omega_0}\frac{A}{1+A}-\frac{\omega_0}{\omega}\right] \quad ②$$

其中②式又可化为

$$jQ\frac{\omega_0}{\omega}(1+A)^2\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]-A+\left(\frac{\omega_0}{\omega}\right)^2(1+A) \quad ③$$

將 ①式 与 ③式合併

Imagine part:

$$j\frac{1}{Q_L}\left[\frac{\omega}{\omega_0}\frac{A}{1+A}-\frac{\omega_0}{\omega}\right]+jQ_L\frac{\omega_0}{\omega}[1+A]^2\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]$$

Real part: 1

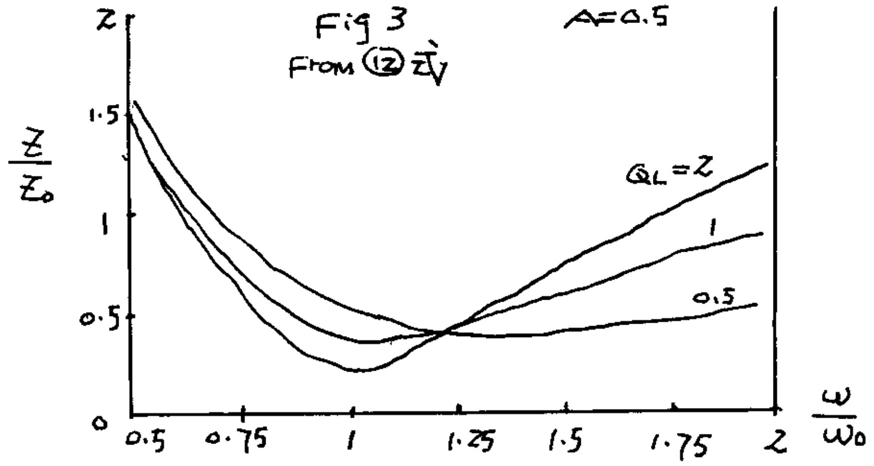
所以 $\varphi=\tan^{-1}\left\{\frac{1}{Q_L}\left(\frac{\omega}{\omega_0}\frac{A}{1+A}-\frac{\omega_0}{\omega}\right)+Q_L\left(\frac{\omega_0}{\omega}\right)(1+A)^2\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]\right\}$ ~ 13

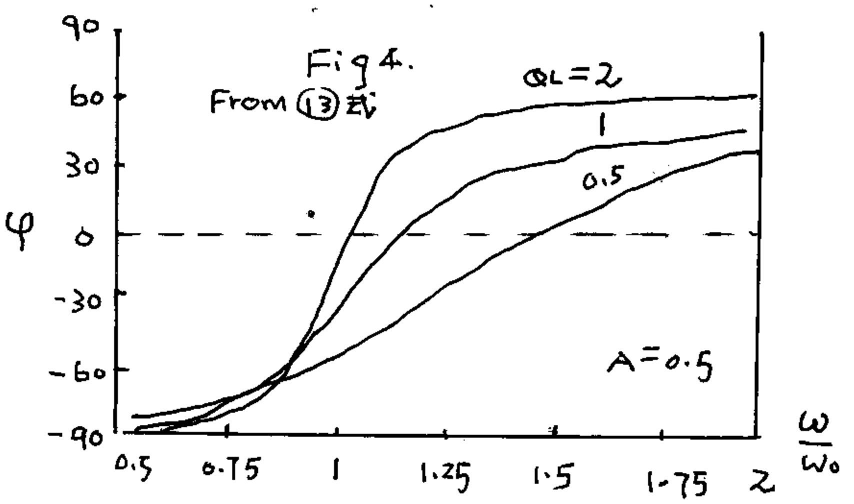
$\therefore R_S=Z\cos\varphi$; $X_S=Z\sin\varphi$.

假如 $\omega=\omega_0$ 时

$$\varphi=\tan^{-1}\frac{1}{Q_L}\left(\frac{A}{1+A}-1\right)=-\tan^{-1}\frac{1}{Q_L}\left(\frac{1}{1+A}\right)<0 \quad \sim 14$$

此公式说明了 $\omega=\omega_0$ 或 $\omega<\omega_0$ 时 Resonant converter 成现了 Capacitive load 的特性.





⊙ 求 Boundary Condition 的 ω_r.

由 Fig4 可知道 LLC Resonant converter 在不同的 Q_L 值，它的 $\frac{\omega_r}{\omega_o}$ 是不一样的，並不像 LC Resonant converter 不同的 Q_L 值只有 $\frac{\omega_r}{\omega_o}$ 值，我们必须求出不同的 $\frac{\omega_r}{\omega_o}$ 值.

由⑬式

令 $\varphi=0$, $\omega\to\omega_r$, $\frac{\omega_r}{\omega_o}=K$

則 $$\frac{1}{Q_L}\left(K\frac{A}{1+A}-\frac{1}{K}\right)+\frac{Q_L}{K}(1+A)^2\left(1-\frac{1}{K^2}\right)=0$$

$$\frac{KA}{1+A}-\frac{1}{K}+\frac{Q_L^2}{K}(1+A)^2\left(1-\frac{1}{K^2}\right)=0$$

$$\frac{K^2A}{1+A}-1+Q_L^2(1+A)^2-\frac{Q_L^2(1+A)^2}{K^2}=0$$

$$\frac{K^4A}{1+A}+K^2\left[Q^2(1+A)^2-1\right]-Q_L^2(1+A)^2=0$$

$$\therefore\ K^2=\frac{1-Q_L^2(1+A)^2\pm\sqrt{\left[Q_L^2(1+A)^2-1\right]^2-4AQ_L^2(1+A)}}{2A(1+A)}$$

$$\therefore\ \frac{\omega_r}{\omega_o}=\sqrt{\frac{(1+A)\left\{1-Q_L^2(1+A)^2\pm\sqrt{\left[Q_L^2(1+A)^2-1\right]^2-4A(1+A)Q_L^2}\right.}{2A}}$$

～⑮式

由⑮式可知，當 $Q_L\to 0$ 時 $\frac{\omega_r}{\omega_o}\to\sqrt{\frac{1+A}{A}}$

除了由⑮式可知道 $\frac{\omega_r}{\omega_o}$，我们也可以應用 Fig2 等效电路求出 ω_r.

$$\omega_r=\frac{1}{\sqrt{L_{eq}\cdot C}}\ ,\ \text{where } L_{eq}=L_1+L_s$$

also, we define $$q_r=\frac{\omega_r L_s}{R_s}=\frac{R_i}{\omega_r L_z}=Q_L(1+A)\left(\frac{\omega_o}{\omega_r}\right)$$ ～⑯式

where $$L_z=\frac{R_i}{(1+A)\omega_o Q_L}$$

③求出 L_s, R_s

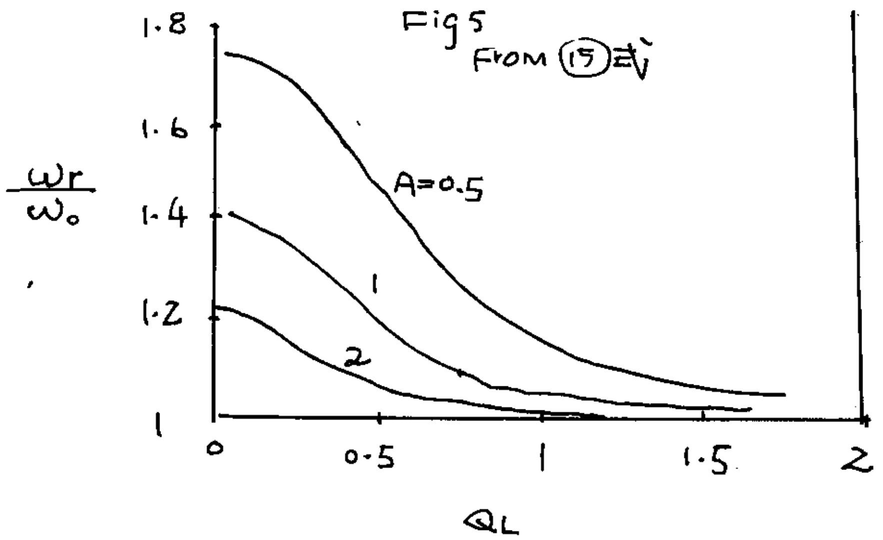
$$R_i \parallel X_{L_2} = \frac{R_i\, j\omega_r L_2}{R_i + j\omega_r L_2} = \frac{R_i(\omega_r L_2)^2 + j\omega_r L_2 R_i^2}{R_i^2 + (\omega_r L_2)^2}$$

$$= \frac{R_i}{1+\left(\frac{R_i}{\omega_r L_2}\right)^2} + j\,\frac{\omega_r L_2}{1+\left(\frac{\omega_r L_2}{R_i}\right)^2}$$

$$R_s = Re\left[R_i \parallel X_{L_2}\right] = \frac{R_i}{1+\left(\frac{R_i}{\omega_r L_2}\right)^2} = \frac{1}{1+q_r^2} \quad \sim (17)$$

$$X_{L_s} = Im\left[R_i \parallel X_{L_2}\right] = \frac{X_{L_2}}{1+\left(\frac{\omega_r L_2}{R_i}\right)^2} = \frac{X_{L_2}}{1+\frac{1}{q_r^2}} \quad \sim (18)$$

當 $q_r^2 \gg 1$ 時 $L_s \approx L_2$, $L \approx L_{eq}$ ∴ $\omega_r \approx \omega_0$



#.4 Voltage Transfer function.

$$V_{ds2} = \begin{cases} V_I & \text{for } 0 < \omega t \le \pi \\ 0 & \text{for } \pi < \omega t \le 2\pi \end{cases}$$

where, V_I fundamental component 為

$v_{i1} = V_m \sin \omega t$, $V_{m(peak)} = \frac{2}{\pi} V_I$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{\sqrt{2}}{\pi} V_I$$

we define $M_{VS} = \frac{V_{rms}}{V_I} = \frac{\sqrt{2}}{\pi}$ ~(19)

From Fig 1, We define ac to ac voltage transfer function

$$M_{VR} = \frac{V_{Ri(rms)}}{V_{rms}} = \frac{R_i // X_{L2}}{Z} \sim (20)$$

已知 $R_i // X_{L2} = \frac{R_i \, j\omega L_2}{R_i + j\omega L_2} = \frac{R_i}{1 + \frac{R_i}{j\omega L_2}}$

將 $L_2 = \frac{R_i}{(1+A)\omega_0 Q_L}$ 代入上式

可得 $R_i // X_{L2} = \frac{R_i}{1 - j Q_L \frac{\omega_0}{\omega}(1+A)}$ ~(21)

Recall (11)式 與 (21)式 代入 ~(20)式

$$M_{VR} = \frac{V_{Ri(rms)}}{V_{rms}} = \frac{1}{(1+A)\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right] + j\frac{1}{Q_L}\left(\frac{\omega}{\omega_0}\frac{A}{1+A} - \frac{\omega_0}{\omega}\right)} = M_{Vr} e^{j\varphi} \sim (22)$$

where M_{Vr} 為 phasor.

$$M_{Vr} = \frac{1}{\sqrt{(1+A)^2\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]^2 + \frac{1}{Q_L^2}\left(\frac{\omega}{\omega_0}\frac{A}{1+A} - \frac{\omega_0}{\omega}\right)^2}} \sim (23)$$

$$\varphi = -\tan^{-1}\left\{\frac{\frac{1}{Q_L}\left(\frac{\omega}{\omega_0}\frac{A}{1+A} - \frac{\omega_0}{\omega}\right)}{(1+A)\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]}\right\} \sim (24)$$

Fig 6 From ㉓式

Note

⊙ A 愈小

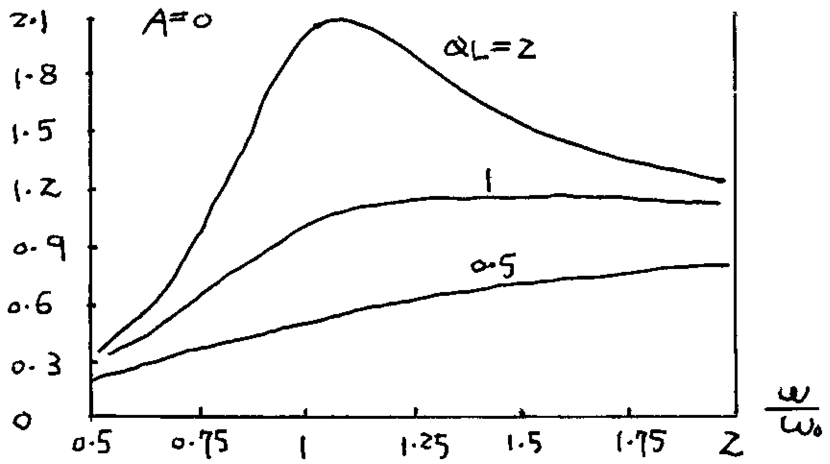
1. MVr 愈不易受 ω 的影响
2. 以相同的 QL 为條件 MVr 較低
3. 以 ZVS/ZCS Boundary $\frac{\omega}{\omega_0}$ 較高

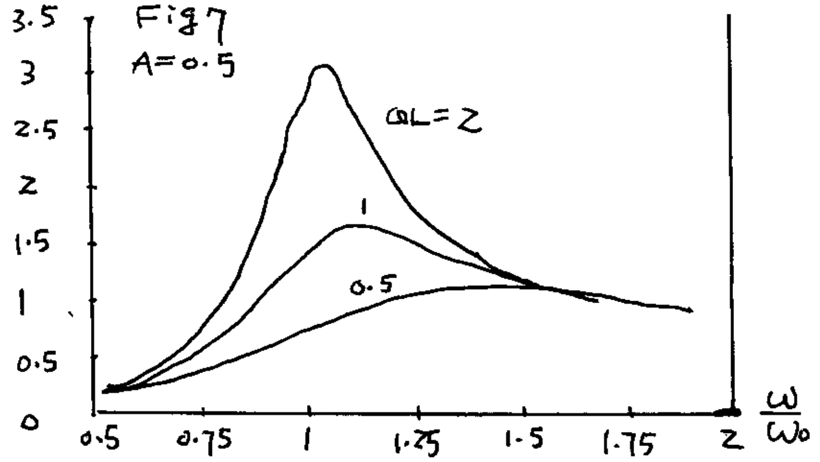
⊙ QL 愈高 MVr 愈高

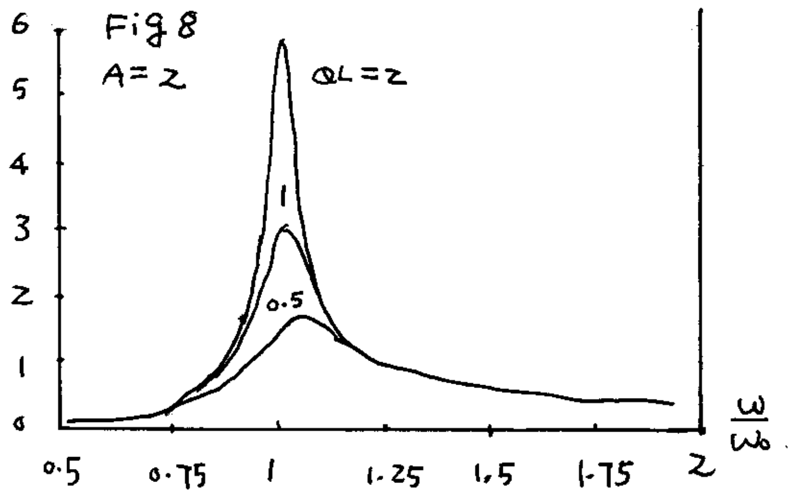
⊙ ω 愈高, MVR 愈低

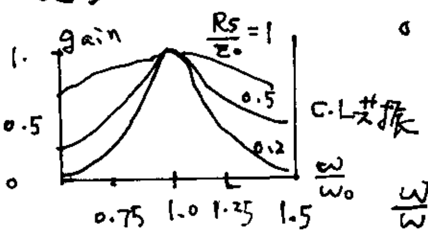
⊙ 以 Fig7 为例 $\frac{\omega}{\omega_0}=1.5$ 時 MVr 不受 Q值影响 为定数。

這是 CLL 共振 与 CL 共振 非常不一樣的地方









$\frac{\omega}{\omega_0}>1.0$ 才有 Z.V.S, 亦为定数

由㉒式知道，如果 MVR=1，其必要條件為

Image part=0

Real part=1 ，$\omega \to \omega_{rs}$

$$\therefore 1-\left(\frac{\omega_0}{\omega_{rs}}\right)^2=\frac{1}{1+A},\ \left(\frac{\omega_0}{\omega_{rs}}\right)^2=\frac{A}{1+A}\ \therefore \frac{\omega_{rs}}{\omega_0}=\sqrt{1+\frac{1}{A}} \quad \sim (25)$$

from ㉕式

$$\omega_{rs}=\omega_0\sqrt{1+\frac{1}{A}}=\sqrt{\frac{1}{LC}}\times\sqrt{\frac{L}{L_1}}=\frac{1}{\sqrt{L_1C}} \quad \sim (26)$$

由㉖式可知在共振時 $XL_1+XC=0$

MVr 永遠 =1.

如果要求 dc to AC voltage transfer function :

$$M_{VI}=\frac{V_{Ri(rms)}}{V_I}=\frac{V_{Ri(rms)}}{V_{rms}}\times\frac{V_{rms}}{V_I}=M_{Vr}\cdot M_{Vs}$$

⑲式與㉓式合併即可

$$M_{VI}=\frac{\sqrt{2}}{\pi\sqrt{(1+A)^2\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]^2+\left[\frac{1}{Q_L}\left(\frac{\omega}{\omega_0}\frac{A}{A+1}-\frac{\omega_0}{\omega}\right)\right]^2}} \quad \sim (27)$$

#5. Energy Parameters

流经 resonant capacitor C 的电流，定義為

$$i = I_m \sin(\omega t - \varphi)$$

⊙ $I_m = \frac{V_m}{Z} = \frac{2V_I}{\pi Z}$

將 (12)式代入上式可得

$$I_m = \frac{2V_I}{\pi Z_o Q_L} \times \sqrt{\frac{1 + [Q_L(\frac{\omega_o}{\omega})(1+A)]^2}{(1+A)^2[1-(\frac{\omega_o}{\omega})^2]^2 + \frac{1}{Q_L^2}(\frac{\omega}{\omega_o}\frac{A}{A+1} - \frac{\omega_o}{\omega})^2}}$$

觀察分母為 MVr^2,

$$\therefore I_m = \frac{2V_I\, MVr}{\pi Z_o Q_L}\sqrt{1 + [Q_L(\frac{\omega_o}{\omega})(1+A)]^2} \sim (28)$$

⊙ output current 的计算

$$I_{om(peak)} = \frac{\sqrt{2}\, V_{Ri}}{R_i} = \frac{\sqrt{2}\cdot MV_I \cdot V_I}{V_I}$$

將 (27)式代入上式

$$I_{om(peak)} = \frac{2V_I}{\pi Z_o Q_L \sqrt{(1+A)^2[1-(\frac{\omega_o}{\omega})^2]^2 + [\frac{1}{Q_L}(\frac{\omega}{\omega_o}\frac{A}{A+1} - \frac{\omega_o}{\omega})]^2}}$$

where $Z_o Q_L = R_i$

$\sim (29)$

⊙ output power

$$P_{Ri} = \frac{V_{Ri}^2}{R_i} = \frac{MV_I^2 V_I^2}{R_i}$$ (27)式代入

$$= \frac{2V_I^2}{\pi^2 Z_o Q_L \left\{(1+A)^2[1-(\frac{\omega_o}{\omega})^2]^2 + [\frac{1}{Q_L}(\frac{\omega}{\omega_o}\frac{A}{A+1} - \frac{\omega_o}{\omega})]^2\right.}$$

$\sim (30)$

⊙ conduction loss

$$P_r = \frac{r I_M^2}{2}$$

将 (28) 式代入上式

$$P_r = \frac{2 r V_I^2 M V_r^2 \left\{ 1 + \left[Q_L \left(\frac{\omega_0}{\omega}\right)(1+A) \right]^2 \right\}}{\pi^2 Z_0^2 Q_L^2} \sim (31)$$

where $r = r_{ds} + r_{Cr} + r_{L1} + r_{Ls}$

$$= r_{ds} + r_{Cr} + r_{L1} + \frac{r_{L2}}{1 + \left(\frac{\omega L_2}{R_i}\right)^2}$$

⊙ efficiency

$$\eta_I = \frac{P_{Ri}}{P_{Ri} + P_r} = \frac{1}{1 + \frac{P_r}{P_{Ri}}} \sim (32)$$

where $$\frac{P_r}{P_{Ri}} = \frac{2 r V_I^2 M V_r^2 \left\{ 1 + \left[Q_L \left(\frac{\omega_0}{\omega}\right)(1+A) \right]^2 \right\}}{\pi^2 Z_0^2 Q_L^2} \times \frac{R_i}{M V_I^2 \cdot V_I^2}$$

where $R_i = Z_0 Q_L$

$$\left(\frac{M V_r}{M V_I}\right)^2 = \frac{1}{M V_s^2} = \frac{\pi^2}{2}$$ 将本式代入上式

再合併 (32) 式

$$\eta_I = \frac{1}{1 + \frac{r}{R_i} \left\{ 1 + \left[Q_L \left(\frac{\omega_0}{\omega}\right)(1+A) \right]^2 \right\}} \sim (33) 式$$

from ㉝式

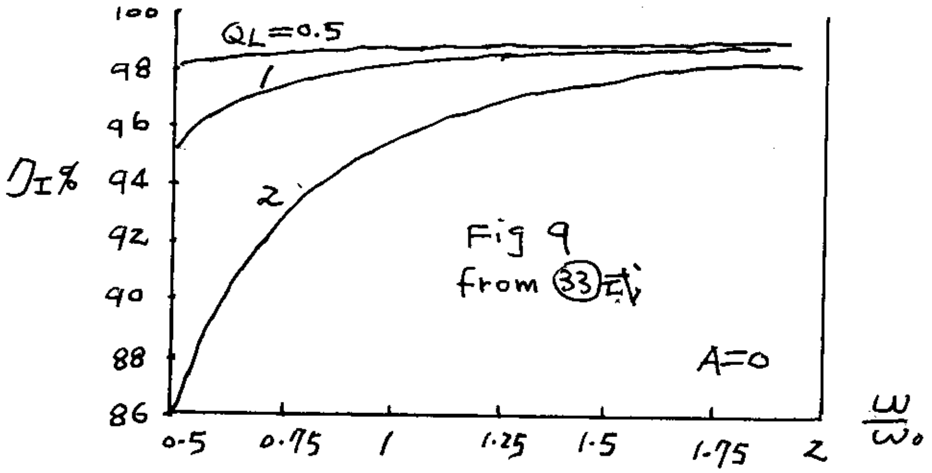
令 $\frac{\omega_0}{\omega} = \frac{1}{K}$ 代入㉝式

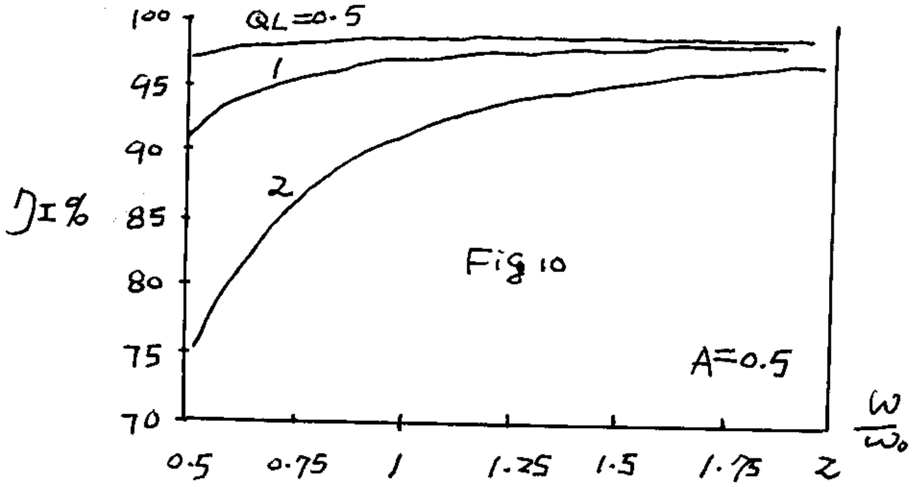
$$\eta_I = \frac{1}{1+\frac{r}{Q_L Z_o}\left[1+\frac{Q_L}{K}(1+A)\right]^2} = \frac{1}{1+\frac{r}{Q_L Z_o}\left[1+\left(\frac{Q_L}{K}\right)^2(1+A)^2+\frac{2Q_L}{K}(1+A)\right]}$$

$$= \frac{1}{1+\frac{r}{Z_o}\left[\frac{1}{Q_L}+\frac{Q}{K^2}(1+A)^2+\frac{2}{K}(1+A)\right]}$$

令 $\frac{d\eta_I}{dQ_L}=0 \quad \therefore \frac{1}{Q_L^2}-\left(\frac{1+A}{K}\right)^2=0$

∴ 在最高效率时 $Q_L = \frac{K}{1+A} = \frac{\frac{\omega}{\omega_0}}{1+A}$ ～ ㉞





$r_{ds}=0.5\Omega$, $r_{cr}=0.08\Omega$
$r_{L1}=r_{L2}=0.8\Omega$
$Z_o=212\Omega$

說明：

1. 由Fig 9, Fig 10 效率圖說明了，A愈小，Q_L愈小，會有比較高的效率。
而且由Fig 11看來，Q_L比較小，I_m會比較小。
但是由Fig 12, Fig 13看來Q_L比較小，輸出功率以及流經負載电流會小得很多。
不利於Heavy power設計.

2. 將$\frac{\omega}{\omega_0}$的值拉高會有比較好的效率，
而且效率不易受Q_L的影响
但由(34)式觀察 $Q_L = \dfrac{\frac{\omega}{\omega_0}}{1+A}$ [最高效率原则]
$\frac{\omega}{\omega_0}$要大，A要小，而且要使Q_L小，似乎是背道而馳，只能做optmized設计.

3. 由Fig 6、7、8觀察A愈小，Q_L愈低
表现出来的是对frequency response是很慢的，
也表示了对於增益反應是很慢的，
不利transient以及variation of V_I。

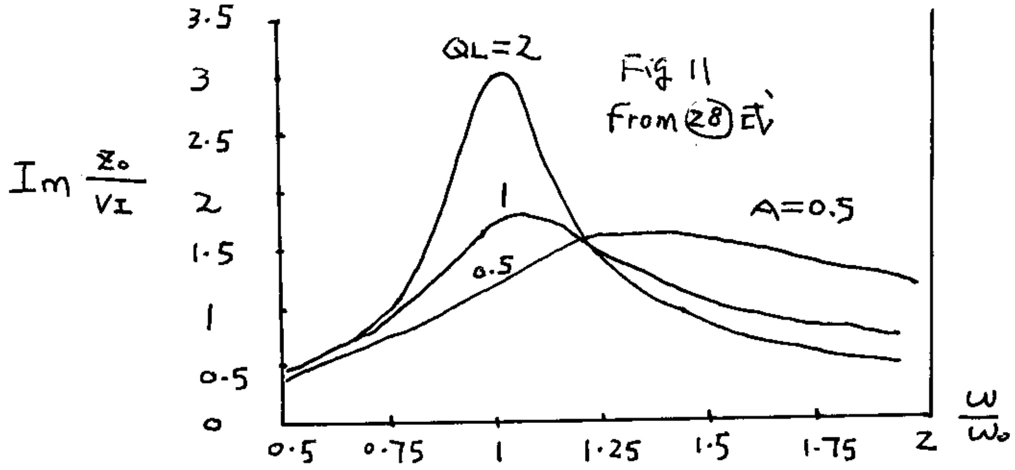


Fig 11
From (28)式
Im $\frac{Z_0}{V_I}$
QL=2
1
0.5
A=0.5
3.5
3
2.5
2
1.5
1
0.5
0
0.5
0.75
1
1.25
1.5
1.75
2
$\frac{\omega}{\omega_0}$

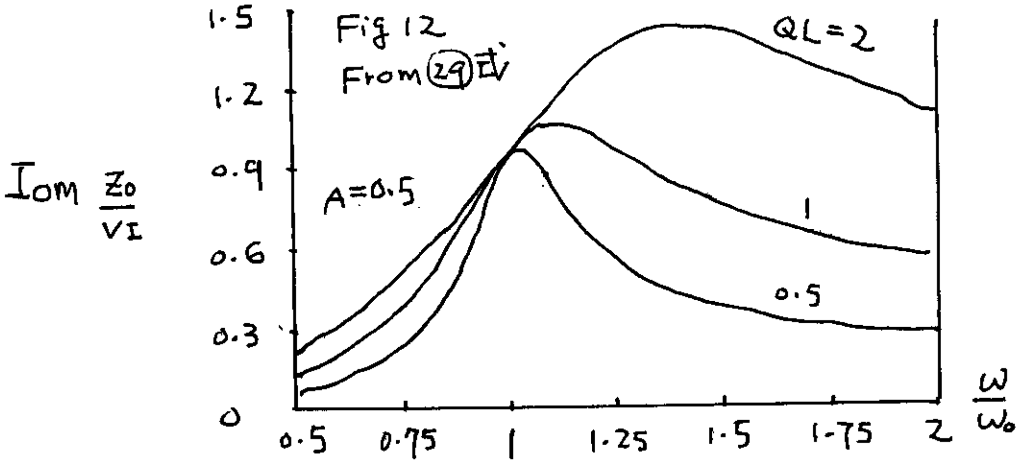


Fig 12
From (29)式
Iom $\frac{Z_0}{V_I}$
QL=2
1
0.5
A=0.5
1.5
1.2
0.9
0.6
0.3
0
0.5
0.75
1
1.25
1.5
1.75
2
$\frac{\omega}{\omega_0}$

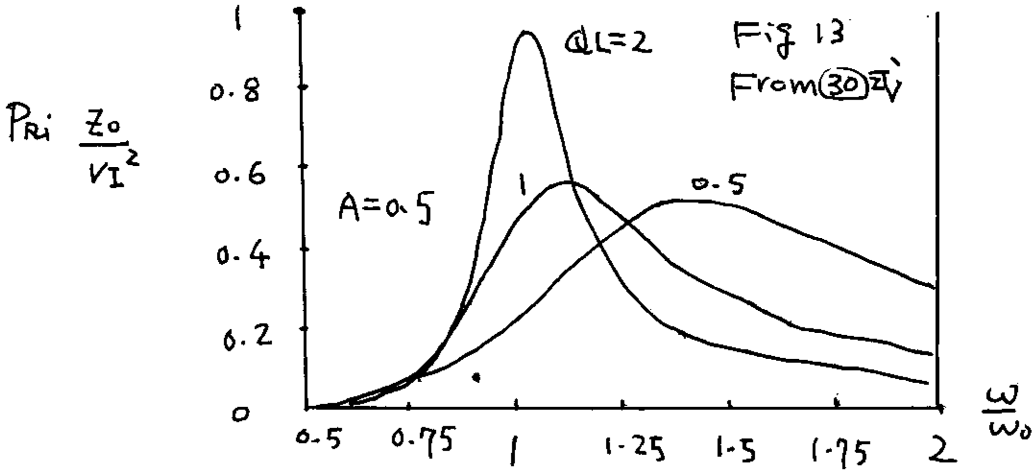


Fig 13
From (30)式
PRi $\frac{Z_0}{V_I^2}$
QL=2
1
0.5
A=0.5
1
0.8
0.6
0.4
0.2
0
0.5
0.75
1
1.25
1.5
1.75
2
$\frac{\omega}{\omega_0}$

#6. Voltage Stress.

⊙ $V_{L1m} = X_{L1} \cdot I_m$

已知 $X_{L1} = \omega \frac{A}{1+A} L = \omega \frac{A}{1+A} \frac{Z_o}{\omega_o}$

將本式與(28)式合併

可得

$$V_{L1m} = \frac{\omega}{\omega_o} \frac{A}{1+A} \frac{2V_I M_{Vr}}{\pi Q_L} \sqrt{1+\left[Q_L \frac{\omega_o}{\omega}(1+A)\right]^2} \sim (35)$$

$$V_{L1m} = (\omega L_1) \frac{V_m}{Z} = \left(\frac{\omega}{\omega_o}\right)(\omega_o L_1) \frac{2V_I}{\pi Z} \sim (36)$$

⊙ $V_{L2m} = \sqrt{2} V_{Ri} = \sqrt{2}\, M_{Vr} \cdot M_{Vs} \cdot V_I = \sqrt{2}\, M_{VI} \cdot V_I$

將本式與(27)式合併

$$V_{L2m} = \frac{2V_I}{\pi \sqrt{(1+A)^2\left[1-\left(\frac{\omega_o}{\omega}\right)^2\right]^2 + \left[\frac{1}{Q_L}\left(\frac{\omega}{\omega_o}\frac{A}{A+1} - \frac{\omega_o}{\omega}\right)\right]^2}} \sim (37)$$

⊙ $V_{Cm} = X_C \cdot I_m$

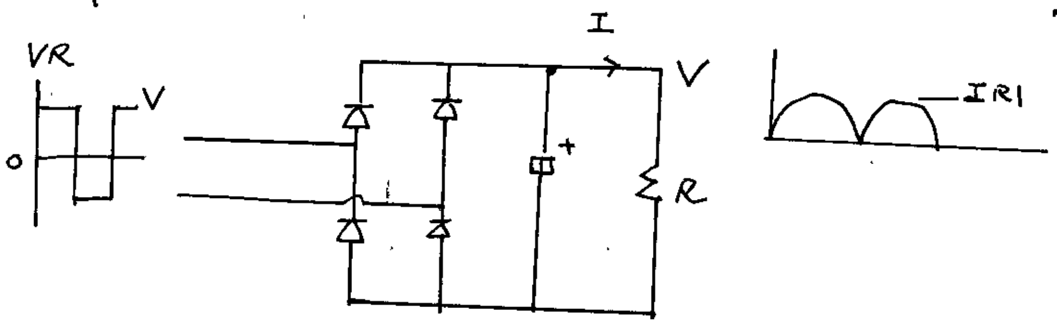
已知 $X_C = \frac{1}{\omega C} = \frac{\omega_o Z_o}{\omega}$

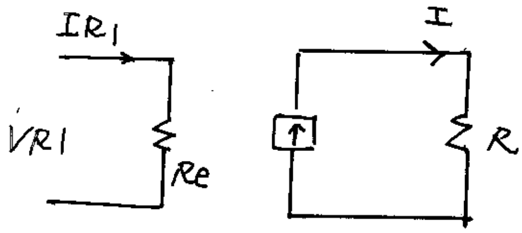
將本式與(28)式合併

$$V_{Cm} = \frac{\omega_o}{\omega}\left(\frac{2V_I M_{Vr}}{\pi Q_L}\right)\sqrt{1+\left[Q_L \frac{\omega_o}{\omega}(1+A)\right]^2} \sim (38)$$

$$V_{Cm} = \frac{1}{\omega C} \frac{V_m}{Z} = \left(\frac{\omega_o}{\omega}\right)\left(\frac{1}{\omega_o C}\right) \frac{2V_I}{\pi Z} \sim (39)$$

#7. Emulated Resistor





由於串聯共振 Secondary 為 Current driving

可以知道輸入電源為 V.

VR fundamental component

$$v_{R1} = \frac{4}{\pi} V \sin(\omega t - \varphi) = V_{R1} \sin(\omega t - \varphi)$$

I 為正弦波為 fundment component，並與 v_{R1} in-phase.

$$\therefore\ I = \frac{2}{T_s}\int_0^{\frac{T_s}{2}} I_{R1}\left|\sin(\omega t - \varphi)\right| dt$$

$$= \frac{2}{\pi} I_{R1}$$

$$R_e = \frac{V_{R1}}{I_{R1}} = \frac{V_{R1}}{V} \times \frac{I}{I_{R1}} \times \frac{V}{I}$$

$$= \frac{4}{\pi} \times \frac{2}{\pi} \times R$$

$$\therefore\ R_e = \frac{8}{\pi^2} \times R \quad \sim (40)$$

實際設計的應用

規格 $V_o = 24V$, $I_o = 10A$, Rectifier $V_f = 0.7V$

$V_{I(min)} = 380V$, $V_{I(max)} = 420V$

$f_s = 100KHz$.

◎ 共振的計算非常複雜，只能作 compromise 的設計，先決定 $A = \frac{L_K}{L_m}$ 的比值.

ST's 建議 $A = \frac{1}{3.5} \sim \frac{1}{7.0}$

這裏我們定義 $A = \frac{1}{5} = 0.2$

$$n = \frac{n_p}{n_s} \geq \frac{V_{I(max)} \times d_1}{V_o + V_f} = \frac{420 \times 0.5}{24 + 0.7} = 8.5$$

We choose $n = 9$

from ㊵ 式

$$R_i = \frac{8}{\pi^2} \times n^2 \times \frac{V_o + V_f}{I_o} = \frac{8}{\pi^2} \times 81 \times \frac{24 + 0.7}{10} = 162.17\Omega$$

from ㉕ 式

$M_{VR} = 1$. 其必要條件 $\frac{f_s}{f_o} = \sqrt{1 + \frac{1}{A}} = \sqrt{6} = 2.45$

$$\therefore f_o = \frac{100k}{2.45} = 40.82k$$

from ㉞ 式，如果以最大效率為設計的條件

$$Q_L = \frac{f_s / f_o}{1 + A} = \frac{2.45}{1.2} = 2.04.$$

from ⑤ 式

$$C = \frac{Q_L}{\omega_o R_i} = \frac{2.04}{2\pi \times 40.82K \times 162.17\Omega} = 0.049\,uF$$

$$L = (L_K + L_M) = \frac{R_i}{\omega_o Q_L} = \frac{162.17\Omega}{2\pi \times 40.82K \times 2.04} = 309.6\,uH$$

$$L_K = \frac{1}{6} \times L = 51.6\,uH$$

$$L_m = \frac{5}{6} \times L = 258\,uH$$

from page 10

我們可以觀察得 Z0，QL 愈高，Mvr 愈高，Voltage-stress 也會愈高，並且以小信号而言，phase-margin 也會較小。

假設 Z0，定義 $Q_L = 1.0$ 為設計條件.

則
$$C = \frac{Q_L}{\omega_0 \cdot R_i} = \frac{1}{2\pi \times 40.82K \times 162.17\Omega} = 0.024uF$$

$$L = \frac{R_i}{\omega_0 Q_L} = \frac{162.17\Omega}{2\pi \times 40.82K} = 631.7uH$$

$$L_K = \frac{1}{6} \times L = 105.3uH$$

$$L_m = \frac{5}{6} \times L = 526.4uH$$

$$f_r = \frac{1}{2\pi\sqrt{L_K \times C}} = \frac{1}{2\pi\sqrt{105.3uH \times 0.024uF}} = 100.02k = f_s.$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{R_i}{Q_L} = 162.17\Omega$$

⊙ from (28)式 以 fourier 第一次諧波計算流經 MOS 的 peak current.

$$I_M = \frac{V_M}{Z} = \frac{2V_I}{\pi Z}$$

$$= \frac{2V_I \cdot M_{vr}}{\pi Z_0 Q_L}\sqrt{1+\left[Q_L \times \frac{\omega_0}{\omega} \times (1+A)\right]^2}$$

$$= \frac{2 \times 400 \times 1}{\pi \times 162.17 \times 1} \times \sqrt{1+\left(\frac{1}{2.04} \times 1.2\right)^2} = 1.82A.$$

from (35) 式

$$V_{L_{km}} = \omega L_K \cdot I_m$$

$$= 2\pi \times 100K \times 105.3\,uH \times 1.82A = 120.4v$$

$$V_{Cm} = \frac{1}{\omega C} \cdot I_m = \frac{1}{2\pi \times 100K \times 0.024\,uF} \times 1.82A = 120.57v$$

from (37) 式

$$V_{Lm\text{-}m} = \frac{2V_I}{\pi\sqrt{(1+A)^2\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]^2 + \left[\frac{1}{Q_L}\left(\frac{\omega}{\omega_0}\frac{A}{1+A} - \frac{\omega_0}{\omega}\right)\right]^2}}$$

$$= \frac{2 \times 400}{\pi\sqrt{0.8536}} = 275.6V$$