

電磁學

- 動電生磁
- 動磁生電

磁通量改變產生感應電場及電動勢
法拉第定律(Faraday's Law)

法拉第感應定律(Faraday's Law of Induction)

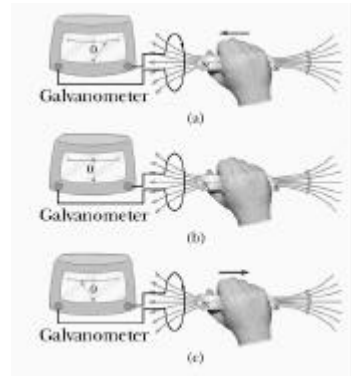
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$

\mathcal{E} : 感應電動勢(induced emf)

Φ_B : 磁通量(magnetic flux)

“-”: 楞次定律(Lenz's Law)

The polarity of the induced emf is such that it tends to produce a current that creates a magnetic flux to oppose the change in magnetic flux through the area enclosed by the current loop.

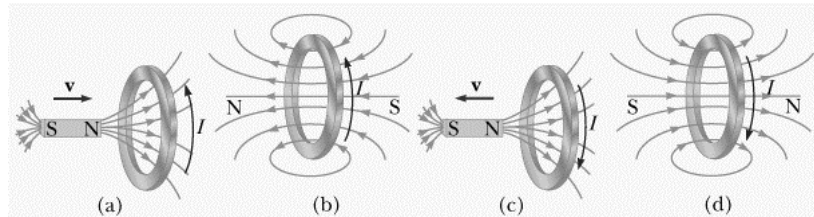


中興大學物理系 孫允武

電磁感應二-1

電磁學

楞次定律

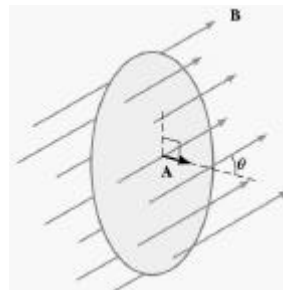


帶電流圈在均勻磁場中的情形

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} (BA \cos \theta)$$

產生感應電動勢的方法

- B 隨時間改變
- 面積隨時間改變
- 角度 θ 隨時間改變
- 上面各項之組合



中興大學物理系 孫允武

電磁感應二-2

電磁學

Motional emf : 導體在磁場中運動產生之電動勢

磁力的觀點 $qE = q\mathbf{u}B$ $E = \mathbf{u}B$
 $\Delta V = E\ell = B\ell\mathbf{u}$

磁通變化的觀點

$$\Phi_B = B\ell x$$

$$E = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell \frac{dx}{dt}$$

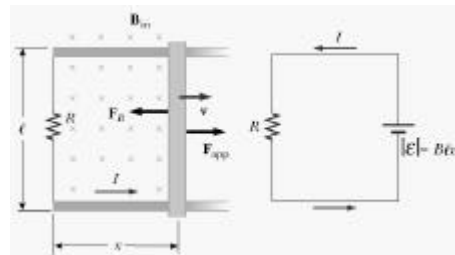
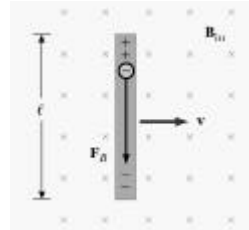
$$E = -B\ell\mathbf{u}$$

加上負載 (loading) $I = \frac{\Delta V}{R} = \frac{B\ell\mathbf{u}}{R}$

若維持等速運動

$$F_{\text{app}} = I\ell B$$

$$P = F_{\text{app}}\mathbf{u} = I\ell B\mathbf{u} = \frac{B^2\ell^2\mathbf{u}^2}{R} = \frac{\Delta V^2}{R}$$



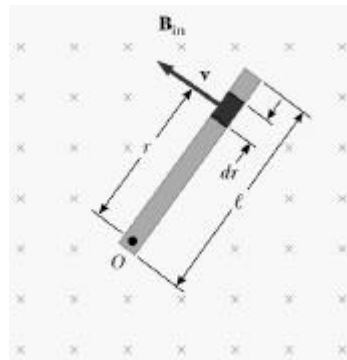
中興大學物理系 孫允武

電磁感應二-3

電磁學

例題

A conducting bar of length ℓ rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field \mathbf{B} is directed perpendicular to the plane of rotation, as shown in the figure. Find the motional emf induced between the ends of the bar.



由磁通量的觀點：

$$d\Phi_B = B dA = B\left(\frac{1}{2}\ell ds\right) = \frac{1}{2}B\ell^2 dq$$

$$E = -\frac{d\Phi_B}{dt} = -\frac{1}{2}B\ell^2 \frac{dq}{dt} = -\frac{1}{2}B\ell^2\omega$$

也可以由每小段 dr 產生之 emf 加起來求得。

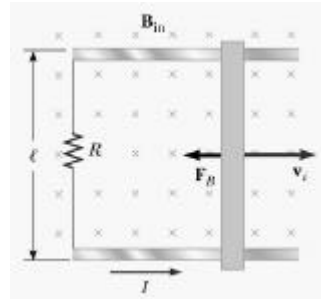
中興大學物理系 孫允武

電磁感應二-4

電磁學

例題

The conducting bar illustrated in the right figure, of mass m and length ℓ , moves on two frictionless parallel rails in the presence of a uniform magnetic field directed in the page. The bar is given an initial velocity \mathbf{v}_i to the right and is released at $t=0$. Find the velocity of the bar as a function of time.



$$F_x = ma = m \frac{d\mathbf{u}}{dt} = -I\ell B = -\frac{B^2 \ell^2}{R} \mathbf{u}$$

$$\frac{d\mathbf{u}}{\mathbf{u}} = -\frac{B^2 \ell^2}{mR} dt \quad \int_{u_i}^u \frac{d\mathbf{u}}{\mathbf{u}} = -\frac{B^2 \ell^2}{mR} \int_0^t dt$$

$$\ln\left(\frac{\mathbf{u}}{\mathbf{u}_i}\right) = -\left(\frac{B^2 \ell^2}{mR}\right)t = -\frac{t}{\tau} \quad \tau = \frac{mR}{B^2 \ell^2}$$

$$\mathbf{u} = \mathbf{u}_i e^{-t/\tau}$$

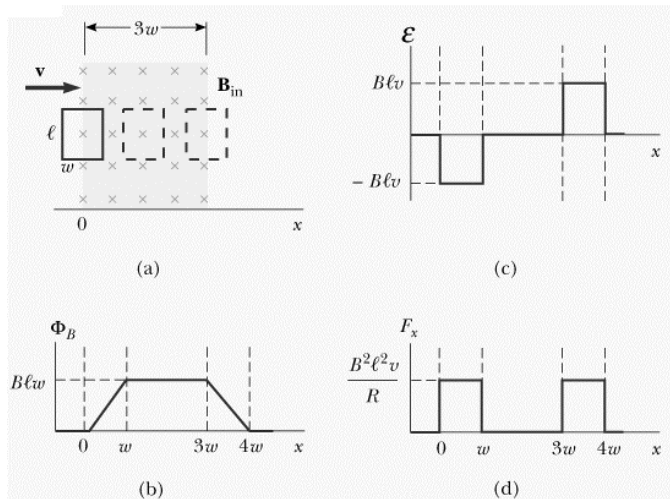
中興大學物理系 孫允武

電磁感應二-5

電磁學

例題

A Loop Moving Through a Magnetic Field



中興大學物理系 孫允武

電磁感應二-6

電磁學

感應電場(induced electric field)

我們再仔細探討感應電動勢的成因。

右圖中 S_1 為在變化磁場中的導線， S_2 則為變化磁場內之任一封閉曲線， S_3 是無磁場區之一封閉曲線。

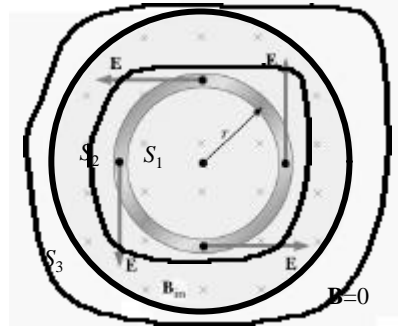
沿 S_1 有感應電場及電流，沿 S_2 及 S_3 呢？

他們都有感應電場，感應電流則必須有導體形成迴路才有。

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$



$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$



不能定義電位能

感應電場是非保守性的，隨時間改變的。和靜電場不同。

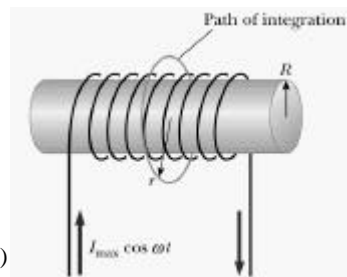
中興大學物理系 孫允武

電磁感應二-7

電磁學

例題

A long solenoid of radius R has n turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{\max} \cos \omega t$, where I_{\max} is the maximum current and ω is the angular frequency of the alternating current source. (a) Determine the magnitude of the induced electric field outside the solenoid, a distance $r > R$ from its long central axis. (b) What is the magnitude of the induced electric field inside the solenoid, a distance r from its axis?



$$(a) \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} (B \pi R^2) = -\pi R^2 \frac{dB}{dt} = -\pi R^2 \frac{d(\mu_0 n I)}{dt} = -\pi R^2 \mu_0 n I_{\max} \frac{d}{dt} \cos \omega t = E(2\pi r)$$

$$E = \frac{\omega R^2 \mu_0 n I_{\max}}{2r} \sin \omega t$$

$$(b) -\frac{d}{dt} (B \pi r^2) = -\pi r^2 \mu_0 n I_{\max} \frac{d}{dt} \cos \omega t = E(2\pi r) \quad E = \frac{\omega \mu_0 n I_{\max}}{2} r \sin \omega t$$

中興大學物理系 孫允武

電磁感應二-8

電磁學

發電機(Generators) 及馬達(Motors)

發電機

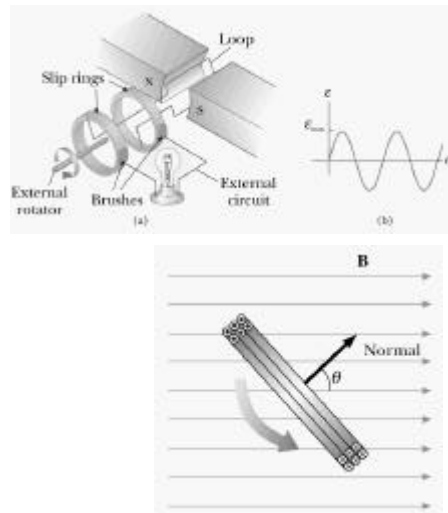
交流發電機

$$\Phi_B = BA \cos \theta = BA \cos \omega t$$

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = -NAB \frac{d}{dt} \cos \omega t$$

$$= NAB \omega \sin \omega t$$

$$\mathcal{E}_{\max} = NAB \omega$$



中興大學物理系 孫允武

電磁感應二-9

電磁學

馬達（電動機）：將發電機逆向操作，及加入電流線圈受磁力而運動旋轉。

馬達啟動時，所需之電流最大。當旋轉至高速時，會產生一反相電動勢（back emf），他會減小線圈中之電流。

當機械負載大時，馬達會變慢，back emf 減小，電流增加。

直流發電機



例題

Assume that a motor in which the coils have a total resistance of 10Ω is supplied by a voltage of 120V . When the motor is running at its maximum speed, the back emf is 70V . Find the current in the coils (a) when the motor is turned on and (b) when it has reached maximum speed.

(a)

$$I = \frac{\mathcal{E}}{R} = \frac{120\text{V}}{10\Omega} = 12\text{A}$$

(b)

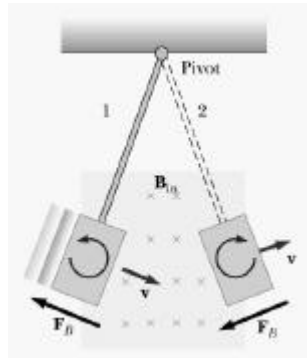
$$I = \frac{\mathcal{E} - \mathcal{E}_{\text{back}}}{R} = \frac{120\text{V} - 70\text{V}}{10\Omega} = 5.0\text{A}$$

中興大學物理系 孫允武

電磁感應二-10

電磁學

渦電流(Eddy Currents)



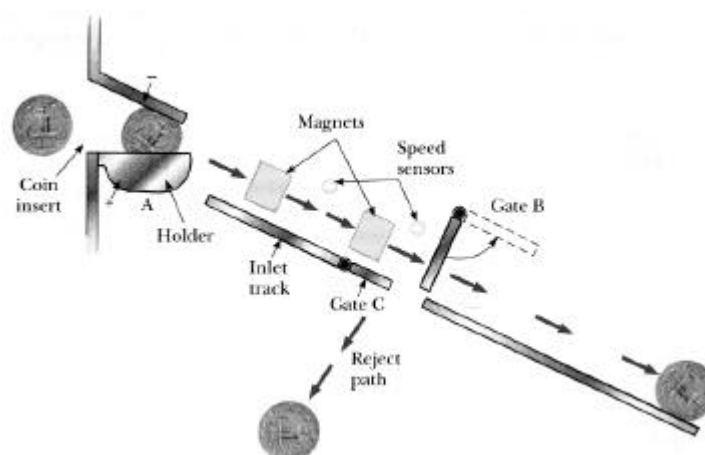
電磁爐也是利用渦電流加熱

中興大學物理系 孫允武

電磁感應二-11

電磁學

販賣機(vending machine)的檢查機制



中興大學物理系 孫允武

電磁感應二-12

電磁學

馬克斯威爾方程式(Maxwell's Equations)---積分形式

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

Gauss's law

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

Gauss's law in magnetism

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Faraday's law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell law

Lorentz force law

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$