PROJECT #1 SINE-Δ PWM INVERTER

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1. Problem Description

In this simulation, we will study Sine-Δ Pulse Width Modulation (PWM) technique. We will use the SEMIKRON® IGBT Flexible Power Converter for this purpose. The system configuration is given below:

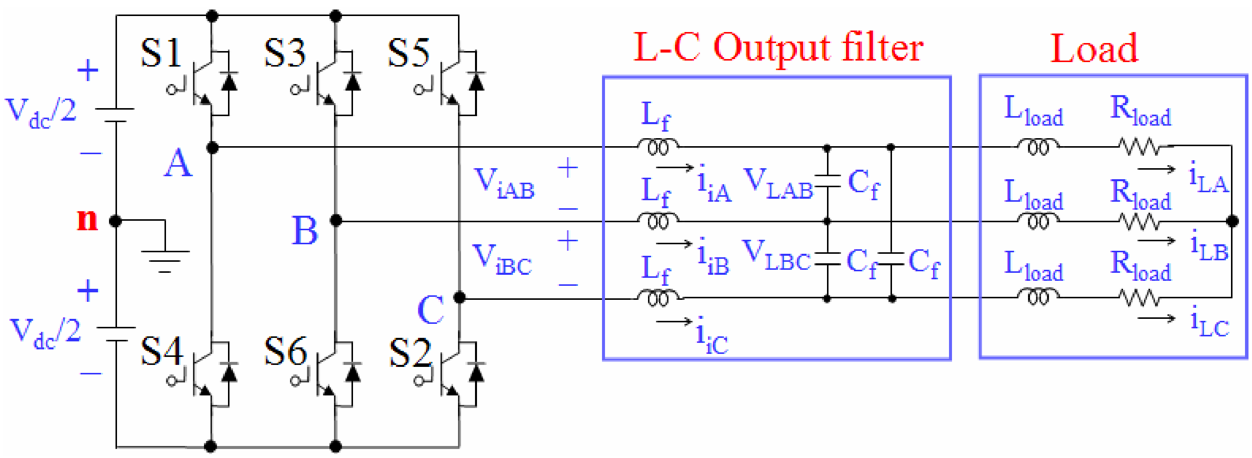


Fig. 1 Circuit model of three-phase PWM inverter with a center-taped grounded DC bus.

The system parameters for this converter are as follows:

- IGBTs: SEMIKRON SKM 50 GB 123D, Max ratings: V_{CES} = 600 V, I_C = 80 A
- DC- link voltage: V_{dc} = 400 V
- Fundamental frequency: f = 60 Hz
- PWM (carrier) frequency: f_z = 3 kHz
- Modulation index: m = 0.8
- Output filter: L_f = 800 μH and C_f = 400 μF
- Load: L_{load} = 2 mH and R_{load} = 5 Ω

Using Matlab/Simulink, simulate the circuit model described in Fig. 1 and plot the waveforms of V_i (= [V_{iAB} V_{iBC} V_{iCA}]), I_i (= [i_{iA} i_{iB} i_{iC}]), V_L (= [V_{LAB} V_{LBC} V_{LCA}]), and I_L (= [i_{LA} i_{LB} i_{LC}]).

2. Sine-Δ PWM

2.1 Principle of Pulse Width Modulation (PWM)

Fig. 2 shows circuit model of a single-phase inverter with a center-taped grounded DC bus, and Fig 3 illustrates principle of pulse width modulation.

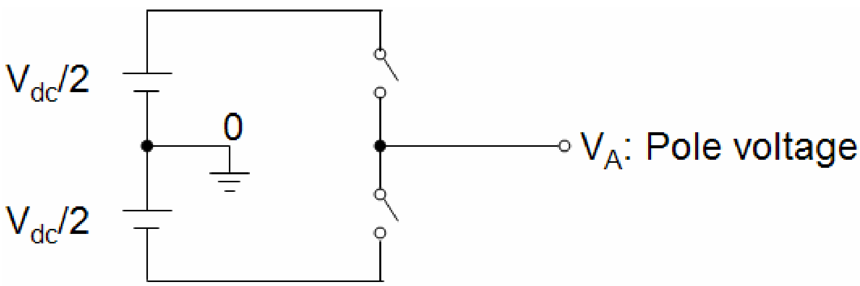


Fig. 2 Circuit model of a single-phase inverter.

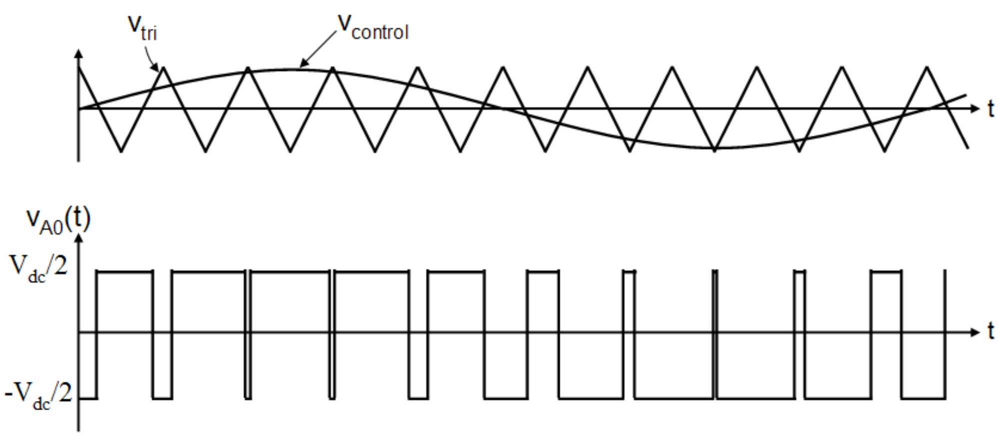


Fig. 3 Pulse width modulation.

As depicted in Fig. 3, the inverter output voltage is determined in the following:

- When $V_{control} > V_{tri}$, $V_{A0} = V_{dc}/2$
- When $V_{control} < V_{tri}$, $V_{A0} = -V_{dc}/2$

Also, the inverter output voltage has the following features:

- PWM frequency is the same as the frequency of V_{tri}
- Amplitude is controlled by the peak value of $V_{control}$
- Fundamental frequency is controlled by the frequency of $V_{control}$

Modulation index (m) is defined as:

$$\therefore m = \frac{v_{control}}{v_{tri}} = \frac{peak \quad of \quad (V_{A0})_1}{V_{dc}/2},$$

where, $(V_{A0})_1$: fundamental frequecny component of V_{A0}

2.2 Three-Phase Sine-Δ PWM Inverter

Fig. 4 shows circuit model of three-phase PWM inverter and Fig. 5 shows waveforms of carrier wave signal (V_{tri}) and control signal ($V_{control}$), inverter output line to neutral voltage (V_{A0}, V_{B0}, V_{C0}), inverter output line to line voltages (V_{AB}, V_{BC}, V_{CA}), respectively.

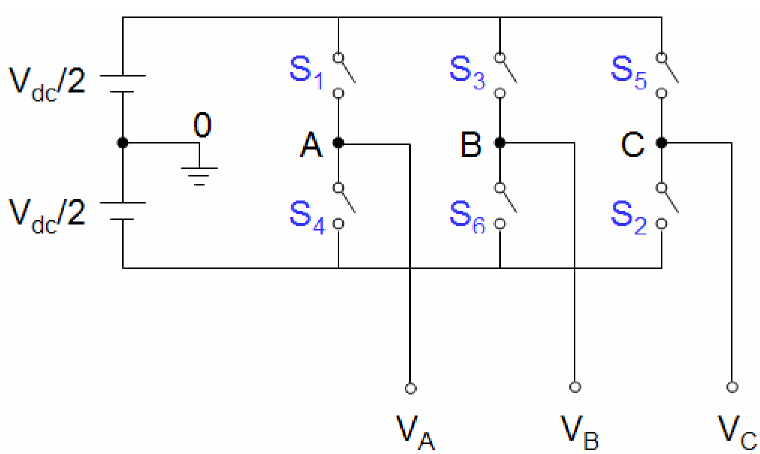


Fig. 4 Three-phase PWM Inverter.

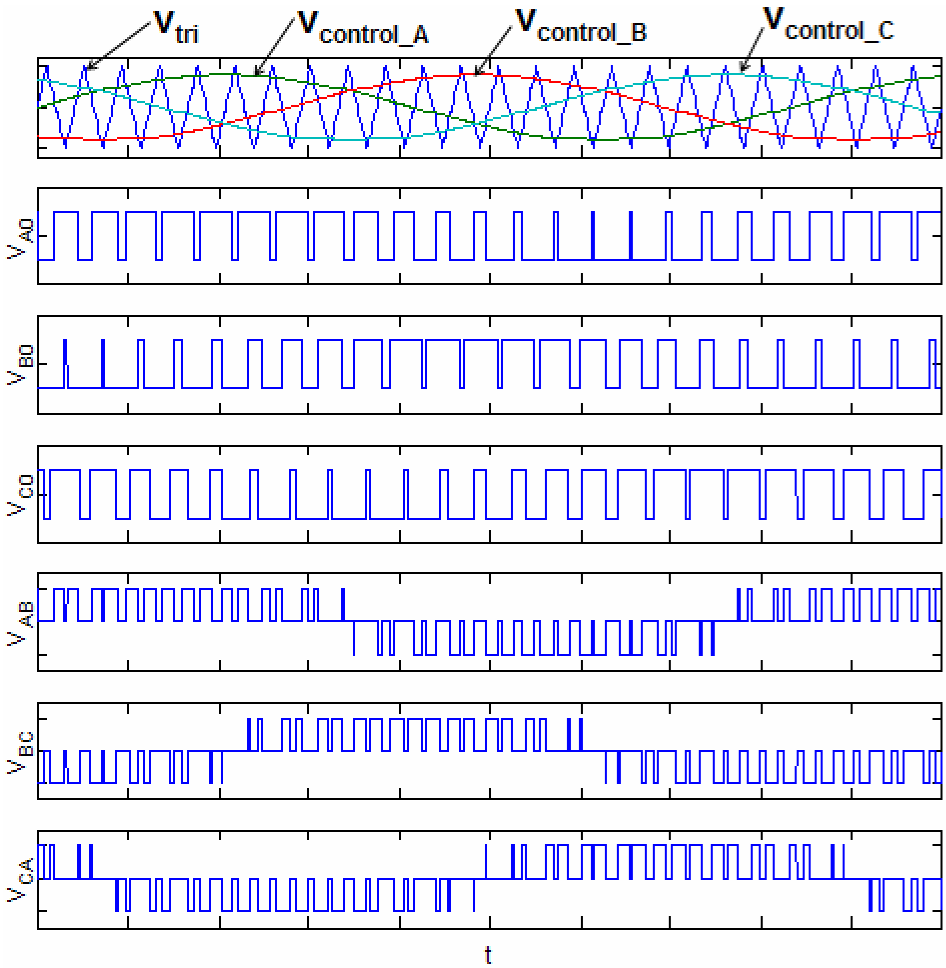


Fig. 5 Waveforms of three-phase sine-Δ PWM inverter.

As described in Fig. 5, the frequency of V_{tri} and $V_{control}$ is:

- Frequency of $V_{tri} = f_s$
- Frequency of $V_{control} = f1$

where, f_s = PWM frequency and f_1 = Fundamental frequency

The inverter output voltages are determined as follows:

- When $V_{control} > V_{tri}$, $V_{A0} = V_{dc}/2$
- When $V_{control} < V_{tri}$, $V_{A0} = -V_{dc}/2$

where, $V_{AB} = V_{A0} - V_{B0}$, $V_{BC} = V_{B0} - V_{C0}$, $V_{CA} = V_{C0} - V_{A0}$

3. State-Space Model

Fig. 6 shows L-C output filter to obtain current and voltage equations.

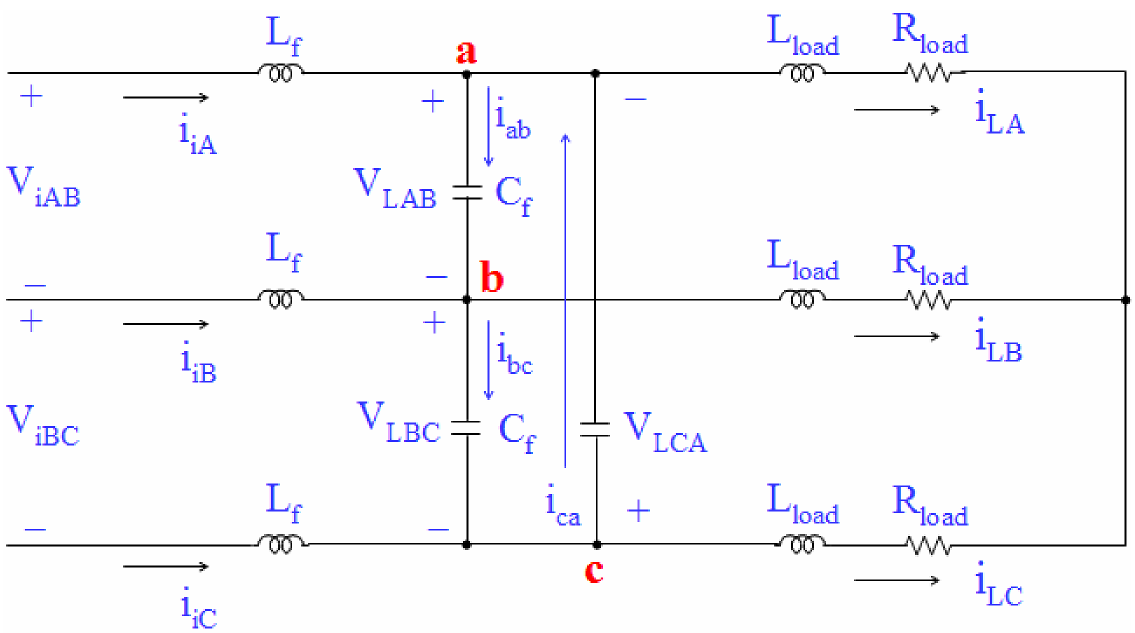


Fig. 6 L-C output filter for current/voltage equations.

By applying Kirchoff's current law to nodes a, b, and c, respectively, the following current equations are derived:

① node "a":

$$i_{iA} + i_{ca} = i_{ab} + i_{LA} \quad \Rightarrow \quad i_{iA} + C_f \frac{dV_{LCA}}{dt} = C_f \frac{dV_{LAB}}{dt} + i_{LA}. \tag{1}$$

② node "b":

$$i_{iB} + i_{ab} = i_{bc} + i_{LB} \quad \Rightarrow \quad i_{iB} + C_f \frac{dV_{LAB}}{dt} = C_f \frac{dV_{LBC}}{dt} + i_{LB}. \tag{2}$$

③ node "c":

$$i_{iC} + i_{bc} = i_{ca} + i_{LC} \quad \Rightarrow \quad i_{iC} + C_f \frac{dV_{LBC}}{dt} = C_f \frac{dV_{LCA}}{dt} + i_{LC} . \tag{3}$$

where, $i_{ab} = C_f \frac{dV_{LAB}}{dt}$, $i_{bc} = C_f \frac{dV_{LBC}}{dt}$, $i_{ca} = C_f \frac{dV_{LCA}}{dt}$.

Also, (1) to (3) can be rewritten as the following equations, respectively:

① subtracting (2) from (1):

$$\begin{aligned} & i_{iA} - i_{iB} + C_f\left(\frac{dV_{LCA}}{dt} - \frac{dV_{LAB}}{dt}\right) = C_f\left(\frac{dV_{LAB}}{dt} - \frac{dV_{LBC}}{dt}\right) + i_{LA} - i_{LB} \\ & \Rightarrow \quad C_f\left(\frac{dV_{LCA}}{dt} + \frac{dV_{LBC}}{dt} - 2\cdot\frac{dV_{LAB}}{dt}\right) = -i_{iA} + i_{iB} + i_{LA} - i_{LB} \end{aligned} . \tag{4}$$

② subtracting (3) from (2):

$$\begin{aligned} & i_{iB} - i_{iC} + C_f\left(\frac{dV_{LAB}}{dt} - \frac{dV_{LBC}}{dt}\right) = C_f\left(\frac{dV_{LBC}}{dt} - \frac{dV_{LCA}}{dt}\right) + i_{LB} - i_{LC} \\ & \Rightarrow \quad C_f\left(\frac{dV_{LAB}}{dt} + \frac{dV_{LCA}}{dt} - 2\cdot\frac{dV_{LBC}}{dt}\right) = -i_{iB} + i_{iC} + i_{LB} - i_{LC} \end{aligned} . \tag{5}$$

③ subtracting (1) from (3):

$$\begin{aligned} & i_{iC} - i_{iA} + C_f\left(\frac{dV_{LBC}}{dt} - \frac{dV_{LCA}}{dt}\right) = C_f\left(\frac{dV_{LCA}}{dt} - \frac{dV_{LAB}}{dt}\right) + i_{LC} - i_{LA} \\ & \Rightarrow \quad C_f\left(\frac{dV_{LAB}}{dt} + \frac{dV_{LBC}}{dt} - 2\cdot\frac{dV_{LCA}}{dt}\right) = -i_{iC} + i_{iA} + i_{LC} - i_{LA} \end{aligned} . \tag{6}$$

To simplify (4) to (6), we use the following relationship that an algebraic sum of line to line load voltages is equal to zero:

$$V_{LAB} + V_{LBC} + V_{LCA} = 0. \quad (7)$$

Based on (7), the (4) to (6) can be modified to a first-order differential equation, respectively:

$$\begin{cases} \dfrac{dV_{LAB}}{dt} = \dfrac{1}{3C_f} i_{iAB} - \dfrac{1}{3C_f}\left(i_{LAB}\right) \\ \dfrac{dV_{LBC}}{dt} = \dfrac{1}{3C_f} i_{iBC} - \dfrac{1}{3C_f}\left(i_{LBC}\right), \\ \dfrac{dV_{LCA}}{dt} = \dfrac{1}{3C_f} i_{iCA} - \dfrac{1}{3C_f}\left(i_{LCA}\right) \end{cases} \quad (8)$$

where, $i_{iAB} = i_{iA} - i_{iB}$, $i_{iBC} = i_{iB} - i_{iC}$, $i_{iCA} = i_{iC} - i_{iA}$ and $i_{LAB} = i_{LA} - i_{LB}$, $i_{LBC} = i_{LB} - i_{LC}$, $i_{LCA} = i_{LC} - i_{LA}$.

By applying Kirchoff's voltage law on the side of inverter output, the following voltage equations can be derived:

$$\begin{cases} \dfrac{di_{iAB}}{dt} = -\dfrac{1}{L_f} V_{LAB} + \dfrac{1}{L_f} V_{iAB} \\ \dfrac{di_{iBC}}{dt} = -\dfrac{1}{L_f} V_{LBC} + \dfrac{1}{L_f} V_{iBC} \\ \dfrac{di_{iCA}}{dt} = -\dfrac{1}{L_f} V_{LCA} + \dfrac{1}{L_f} V_{iCA} \end{cases}. \quad (9)$$

By applying Kirchoff's voltage law on the load side, the following voltage equations can be derived:

$$\begin{cases} V_{LAB} = L_{load} \dfrac{di_{LA}}{dt} + R_{load} i_{LA} - L_{load} \dfrac{di_{LB}}{dt} - R_{load} i_{LB} \\ V_{LBC} = L_{load} \dfrac{di_{LB}}{dt} + R_{load} i_{LB} - L_{load} \dfrac{di_{LC}}{dt} - R_{load} i_{LC} \\ V_{LCA} = L_{load} \dfrac{di_{LC}}{dt} + R_{load} i_{LC} - L_{load} \dfrac{di_{LA}}{dt} - R_{load} i_{LA} \end{cases}. \quad (10)$$

Equation (10) can be rewritten as:

$$\begin{cases} \frac{di_{LAB}}{dt} = -\frac{R_{load}}{L_{load}} i_{LAB} + \frac{1}{L_{load}} V_{LAB} \\ \frac{di_{LBC}}{dt} = -\frac{R_{load}}{L_{load}} i_{LBC} + \frac{1}{L_{load}} V_{LBC} \\ \frac{di_{LCA}}{dt} = -\frac{R_{load}}{L_{load}} i_{LCA} + \frac{1}{L_{load}} V_{LCA} \end{cases} . \tag{11}$$

Therefore, we can rewrite (8), (9) and (11) into a matrix form, respectively:

$$\begin{aligned} \frac{d\mathbf{V}_L}{dt} &= \frac{1}{3C_f}\mathbf{I}_i - \frac{1}{3C_f}\mathbf{I}_L \\ \frac{d\mathbf{I}_i}{dt} &= -\frac{1}{L_f}\mathbf{V}_L + \frac{1}{L_f}\mathbf{V}_i \\ \frac{d\mathbf{I}_L}{dt} &= \frac{1}{L_{load}}\mathbf{V}_L - \frac{R_{load}}{L_{load}}\mathbf{I}_L \end{aligned} , \tag{12}$$

where, $\mathbf{V}_L = [V_{LAB}\ V_{LBC}\ V_{LCA}]^T$, $\mathbf{I}_i = [i_{iAB}\ i_{iBC}\ i_{iCA}]^T = [i_{iA}\text{-}i_{iB}\ i_{iB}\text{-}i_{iC}\ i_{iC}\text{-}i_{iA}]^T$, $\mathbf{V}_i = [V_{iAB}\ V_{iBC}\ V_{iCA}]^T$, $\mathbf{I}_L = = [i_{LAB}\ i_{LBC}\ i_{LCA}]^T = [i_{LA}\text{-}i_{LB}\ i_{LB}\text{-}i_{LC}\ i_{LC}\text{-}i_{LA}]^T$.

Finally, the given plant model (12) can be expressed as the following continuous-time state space equation

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) + \mathbf{Bu}(t), \tag{13}$$

where, $\mathbf{X} = \begin{bmatrix} \mathbf{V}_L \\ \mathbf{I}_i \\ \mathbf{I}_L \end{bmatrix}_{9\times1}$, $\mathbf{A} = \begin{bmatrix} 0_{3\times3} & \frac{1}{3C_f} I_{3\times3} & -\frac{1}{3C_f} I_{3\times3} \\ -\frac{1}{L_f} I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ \frac{1}{L_{load}} I_{3\times3} & 0_{3\times3} & -\frac{R_{load}}{L_{load}} I_{3\times3} \end{bmatrix}_{9\times9}$, $\mathbf{B} = \begin{bmatrix} 0_{3\times3} \\ \frac{1}{L_f} I_{3\times3} \\ 0_{3\times3} \end{bmatrix}_{9\times3}$, $\mathbf{u} = [\mathbf{V}_i]_{3\times1}$.

Note that load line to line voltage $\mathbf{V}_L$**,** inverter output current $\mathbf{I}_i$, and the load current $\mathbf{I}_L$ are the state variables of the system, and the inverter output line-to-line voltage $\mathbf{V}_i$ is the control input (**u**).

4. Simulation Steps

1). Initialize system parameters using Matlab

2). Build Simulink Model

- Generate carrier wave (V_{tri}) and control signal ($V_{control}$) based on modulation index (m)
- Compare V_{tri} to $V_{control}$ to get V_{iAn}, V_{iBn}, V_{iCn}.
- Generate the inverter output voltages (V_{iAB}, V_{iBC}, V_{iCA},) for control input (u)
- Build state-space model
- Send data to Workspace

3). Plot simulation results using Matlab

5. Simulation results

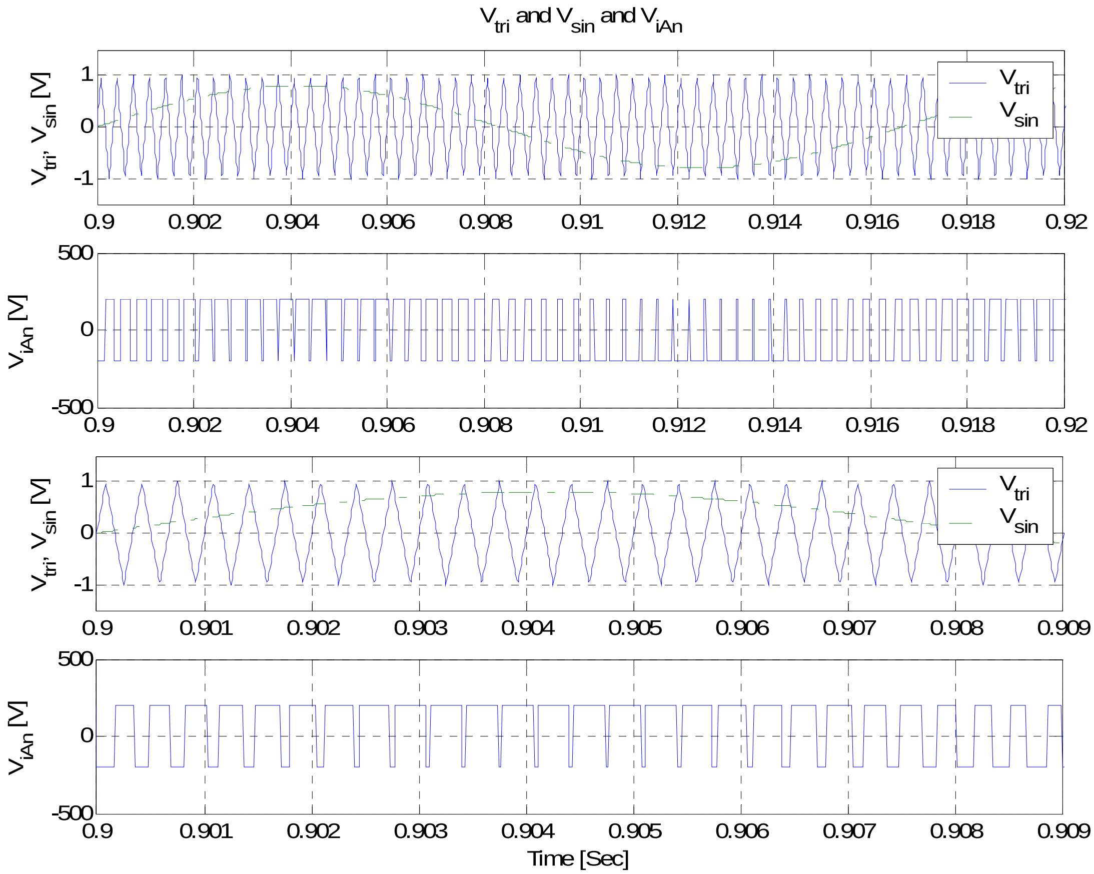


Fig. 7 Waveforms of carrier wave, control signal, and inverter output line to neutral voltage.

(a) Carrier wave (V_{tri}) and control signal (V_{sin})

(b) Inverter output line to neutral voltage (V_{iAn})

(c) Enlarged carrier wave (V_{tri}) and control signal (V_{sin})

(d) Enlarged inverter output line to neutral voltage (V_{iAn})

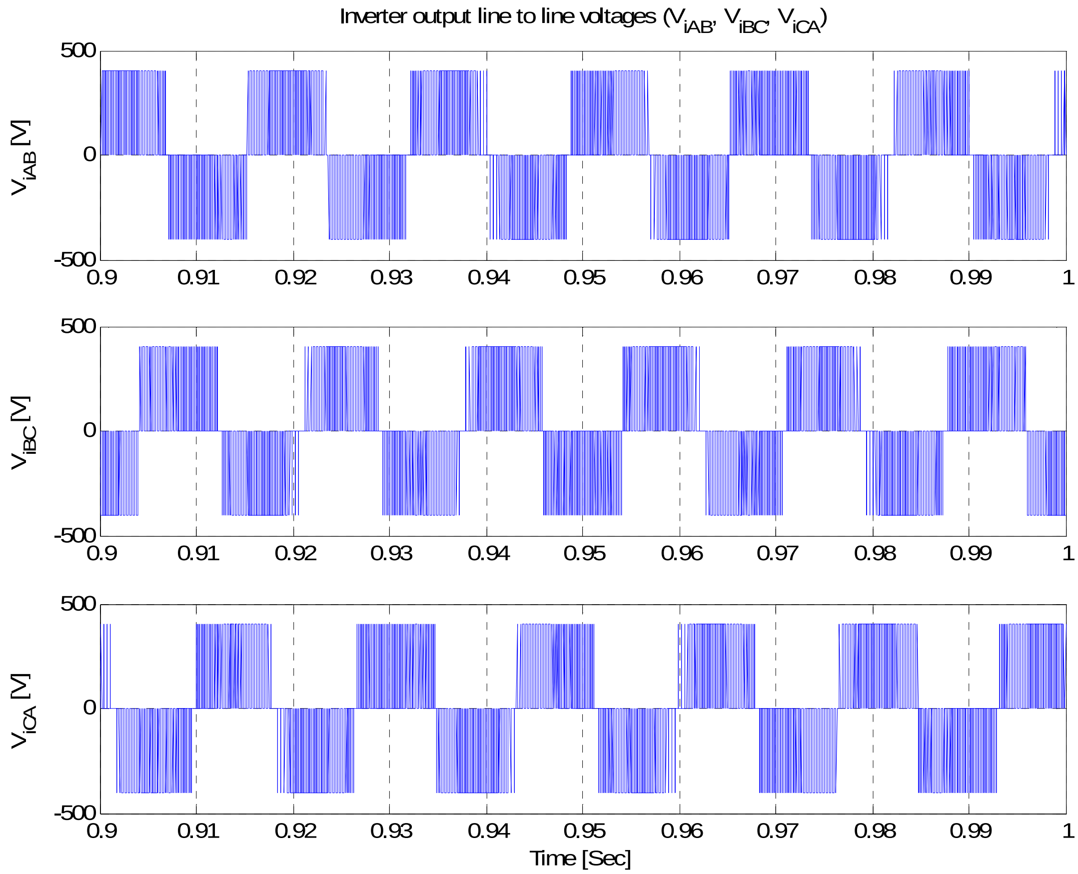


Fig. 8 Simulation results of inverter output line to line voltages (V_{iAB}, V_{iBC}, V_{iCA})

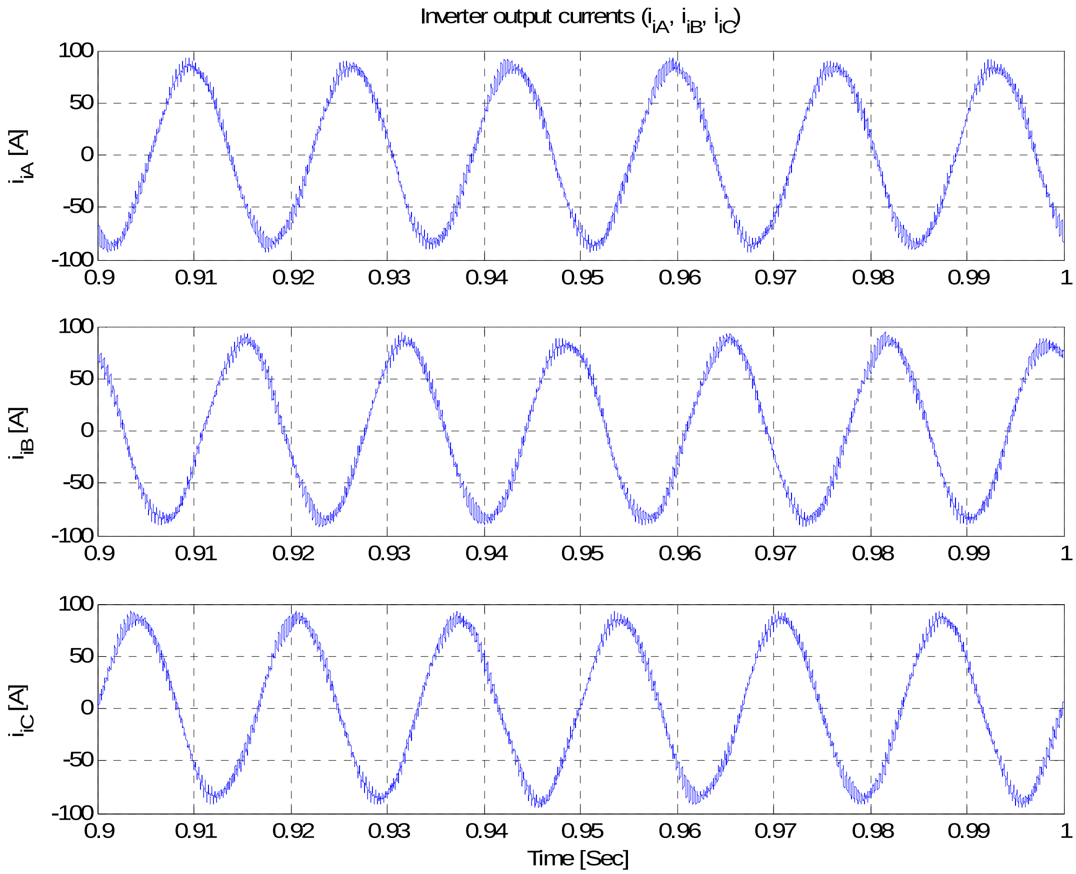


Fig. 9 Simulation results of inverter output currents (i_{iA}, i_{iB}, i_{iC})

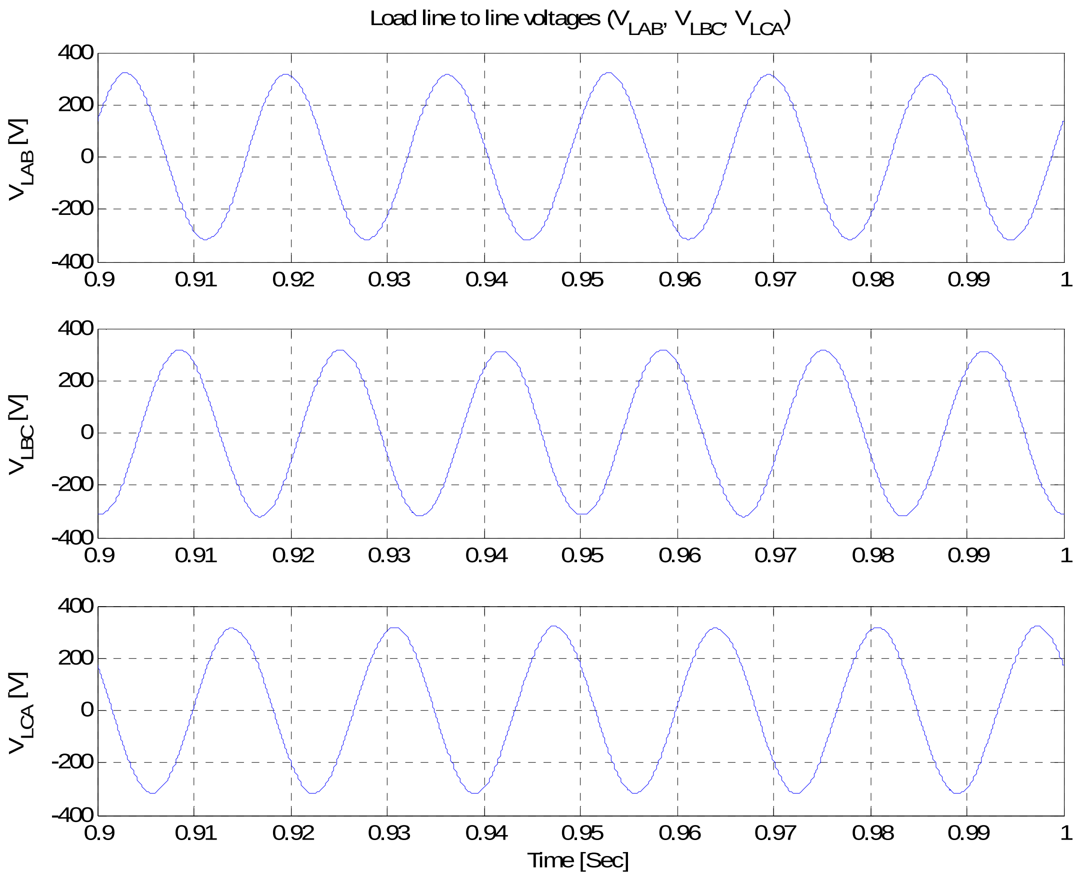


Fig. 10 Simulation results of load line to line voltages (V_{LAB}, V_{LBC}, V_{LCA})

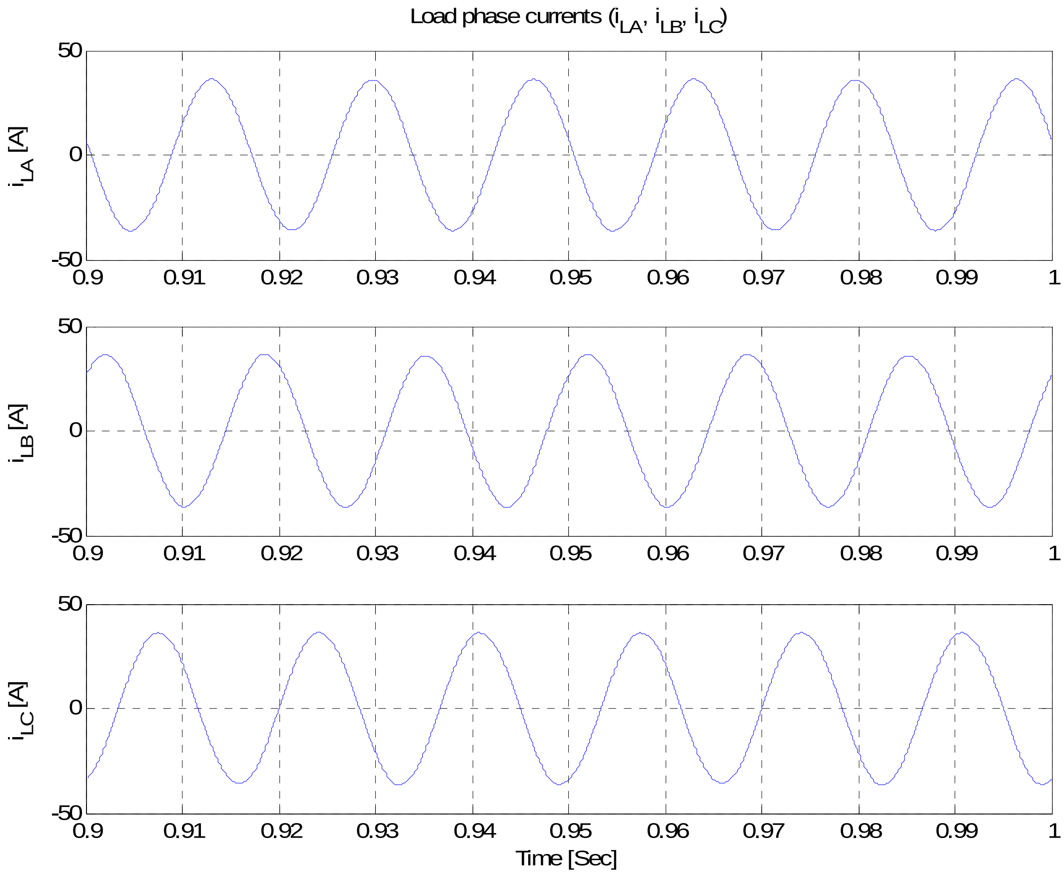


Fig. 11 Simulation results of load phase currents (i_{LA}, i_{LB}, i_{LC})

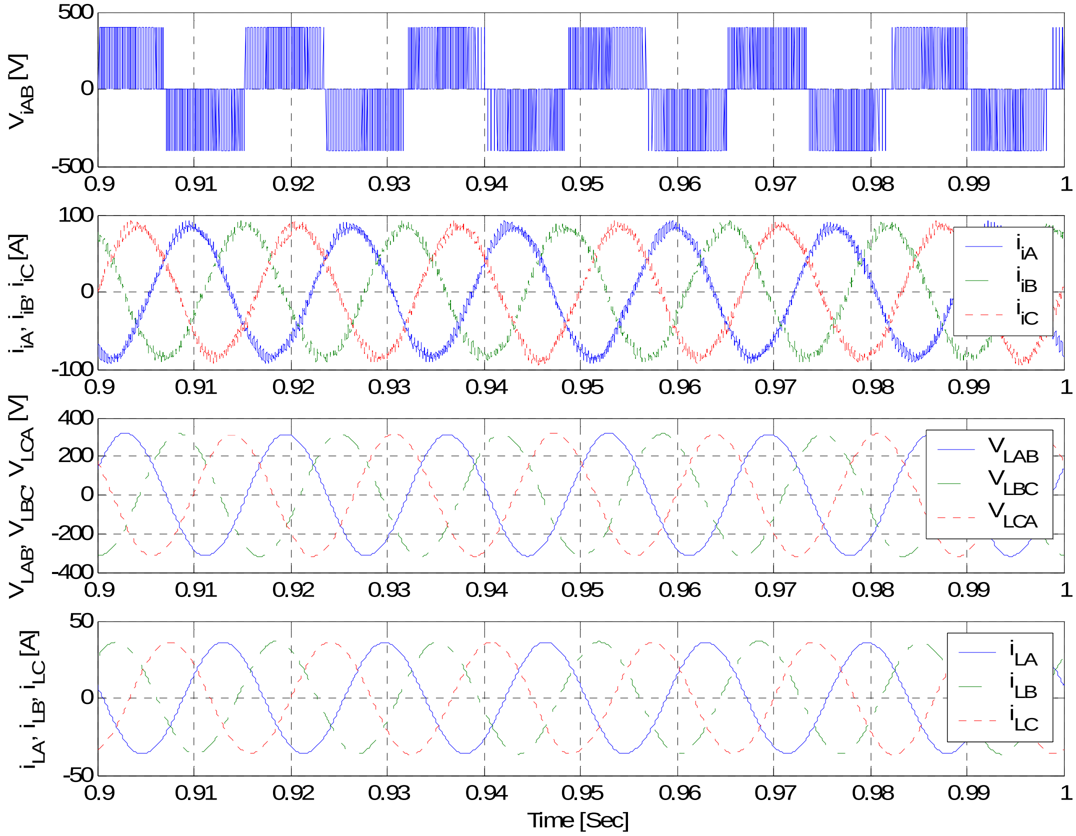


Fig. 12 Simulation waveforms.

(a) Inverter output line to line voltage (V_{iAB})

(b) Inverter output current (i_{iA})

(c) Load line to line voltage (V_{LAB})

(d) Load phase current (i_{LA})

Appendix

Matlab/Simulink Codes

A.1 Matlab Code for System Parameters

```
% Written by Jin Woo Jung, Date: 02/20/05
% ECE743, Simulation Project #1 (Sine PWM Inverter)
% Matlab program for Parameter Initialization

clear all % clear workspace

% Input data
Vdc= 400; % DC-link voltage
Lf= 800e-6;% Inductance for output filter
Cf= 400e-6; % Capacitance for output filter
Lload = 2e-3; %Load inductance
Rload= 5; % Load resistance
f= 60; % Fundamental frequency
fz = 3e3; % Switching frequency
m= 0.8; % Modulation index

% Coefficients for State-Space Model
A=[zeros(3,3)     eye(3)/(3*Cf)    -eye(3)/(3*Cf)
  -eye(3)/Lf      zeros(3,3)        zeros(3,3)
  eye(3,3)/Lload  zeros(3,3)     -eye(3)*Rload/Lload]; % system matrix

B=[zeros(3,3)
   eye(3)/Lf
   zeros(3,3)];  % coefficient for the control variable u

C=[eye(9)];  % coefficient for the output y

D=[zeros(9,3)]; % coefficient for the output y
Ks = 1/3*[-1 0 1; 1 -1 0; 0 1 -1]; % Conversion matrix to transform [iiAB iiBC iiCA] to [iiA iiB
iiC]
```

A.2 Matlab Code for Plotting the Simulation Results

```
% Written by Jin Woo Jung
% Date: 02/20/05
% ECE743, Simulation Project #1 (Sine-PWM)

% Matlab program for plotting Simulation Results
% using Simulink

ViAB = Vi(:,1);
ViBC = Vi(:,2);
ViCA = Vi(:,3);

VLAB= VL(:,1);
VLBC= VL(:,2);
VLCA= VL(:,3);

iiA= IiABC(:,1);
iiB= IiABC(:,2);
iiC= IiABC(:,3);

iLA= ILABC(:,1);
iLB= ILABC(:,2);
iLC= ILABC(:,3);



figure(1)
subplot(3,1,1)
plot(t,ViAB)
axis([0.9 1 -500 500])
ylabel('V_i_A_B [V]')
title('Inverter output line to line voltages (V_i_A_B, V_i_B_C, V_i_C_A)')
```

```
grid

subplot(3,1,2)
plot(t,ViBC)
axis([0.9 1 -500 500])
ylabel('V_i_B_C [V]')
grid

subplot(3,1,3)
plot(t,ViCA)
axis([0.9 1 -500 500])
ylabel('V_i_C_A [V]')
xlabel('Time [Sec]')
grid



figure(2)
subplot(3,1,1)
plot(t,iiA)
axis([0.9 1 -100 100])
ylabel('i_i_A [A]')
title('Inverter output currents (i_i_A, i_i_B, i_i_C)')
grid

subplot(3,1,2)
plot(t,iiB)
axis([0.9 1 -100 100])
ylabel('i_i_B [A]')
grid

subplot(3,1,3)
```

```
plot(t,iiC)
axis([0.9 1 -100 100])
ylabel('i_i_C [A]')
xlabel('Time [Sec]')
grid

figure(3)
subplot(3,1,1)
plot(t,VLAB)
axis([0.9 1 -400 400])
ylabel('V_L_A_B [V]')
title('Load line to line voltages (V_L_A_B, V_L_B_C, V_L_C_A)')
grid

subplot(3,1,2)
plot(t,VLBC)
axis([0.9 1 -400 400])
ylabel('V_L_B_C [V]')
grid

subplot(3,1,3)
plot(t,VLCA)
axis([0.9 1 -400 400])
ylabel('V_L_C_A [V]')
xlabel('Time [Sec]')
grid

figure(4)
subplot(3,1,1)
plot(t,iLA)
axis([0.9 1 -50 50])
```

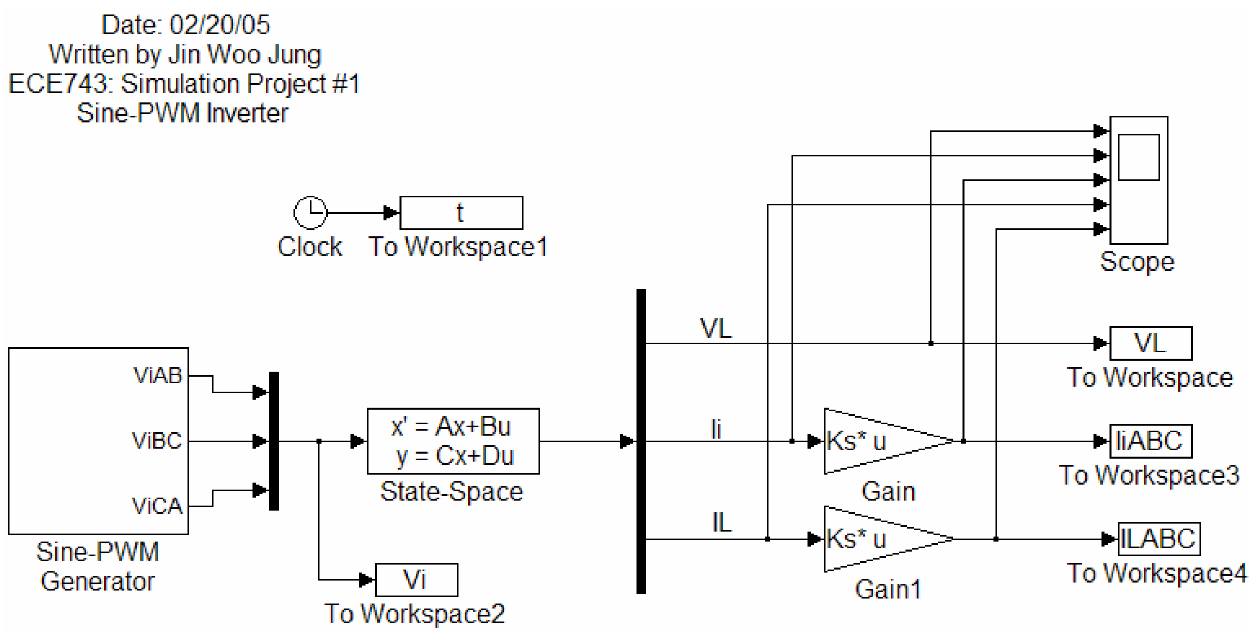
```
ylabel('i_L_A [A]')
title('Load phase currents (i_L_A, i_L_B, i_L_C)')
grid

subplot(3,1,2)
plot(t,iLB)
axis([0.9 1 -50 50])
ylabel('i_L_B [A]')
grid

subplot(3,1,3)
plot(t,iLC)
axis([0.9 1 -50 50])
ylabel('i_L_C [A]')
xlabel('Time [Sec]')
grid

figure(5)
subplot(4,1,1)
plot(t,ViAB)
axis([0.9 1 -500 500])
ylabel('V_i_A_B [V]')
grid

subplot(4,1,2)
plot(t,iiA,'-', t,iiB,'-.',t,iiC,':')
axis([0.9 1 -100 100])
ylabel('i_i_A, i_i_B, i_i_C [A]')
legend('i_i_A', 'i_i_B', 'i_i_C')
grid
```

```
subplot(4,1,3)
plot(t,VLAB,'-', t,VLBC,'-.',t,VLCA,':')
axis([0.9 1 -400 400])
ylabel('V_L_A_B, V_L_B_C, V_L_C_A [V]')
legend('V_L_A_B', 'V_L_B_C', 'V_L_C_A')
grid

subplot(4,1,4)
plot(t,iLA,'-', t,iLB,'-.',t,iLC,':')
axis([0.9 1 -50 50])
ylabel('i_L_A, i_L_B, i_L_C [A]')
legend('i_L_A', 'i_L_B', 'i_L_C')
xlabel('Time [Sec]')
grid


%For only Sine PWM
figure(6)
subplot(4,1,1)
plot(t,Vtri,'-', t,Vsin,'-.')
axis([0.9 0.917 -1.5 1.5])
ylabel('V_t_r_i, V_s_i_n [V]')
legend('V_t_r_i', 'V_s_i_n')
title('V_t_r_i and V_s_i_n')
grid

subplot(4,1,2)
plot(t,ViAn)
axis([0.9 0.917 -500 500])
ylabel('V_i_A_n [V]')
grid
```

```
subplot(4,1,3)
plot(t,Vtri,'-', t,Vsin,'-.')
axis([0.9 0.909 -1.5 1.5])
ylabel('V_t_r_i, V_s_i_n [V]')
legend('V_t_r_i', 'V_s_i_n')
grid

subplot(4,1,4)
plot(t,ViAn)
axis([0.9 0.909 -500 500])
ylabel('V_i_A_n [V]')
xlabel('Time [Sec]')
grid
```

A.3 Simulink Code

Simulink Model for Overall System



Simulink Model for “Sine-PWM Generator”