Power Supply Control Design Tools – Part 1

In this article, Dr. Ridley discusses the problems in comprehending supposedly-simple power electronics circuits. A free piece of analysis software, the first in a series of six, is provided to readers of this column to help them with their power supply design and analysis.

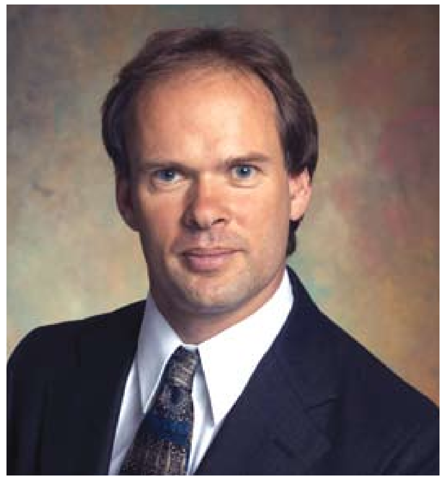
By Dr. Ray Ridley, Ridley Engineering

Modeling Power Supplies

The power electronics field is supposedly simple. We have circuits with maybe half a dozen major components in the power stage, and perhaps 20 more parts in the control loop. We use time-tested techniques like breadboards, oscilloscopes, and frequency response analyzers to measure quantities that we are supposed to be able to predict easily with either simulation or design equations.

Despite the apparent simplicity of the field, it rapidly becomes a bewildering place when you design your first products. There are countless books, references, papers and application notes which try to shed some light on the topics that are involved in design.

In teaching power supply design courses for the last 20 years, I constantly face challenges in presenting material that is up-to-date, accurate, interesting, and most importantly, of immediate practical use in the workplace. Engineers attending the courses are expected to immediately increase their productivity at work, and they do. One of the ways that practicality is kept to the forefront of training is by not getting weighed down with excessive equations.



When presenting courses and showing some of the waveforms or transfer functions of converters, I am constantly asked – do you have the equation for that? Some equations are in the notes I present, but most are in the design software that goes with the courses. Of course I have the design equations, but something has always prevented me from writing them all out. First, they are in the form of software equations, and it is a lot of work to extract them from their native format. But, secondly, I do not believe it would do a service to the engineering community to just provide the equations.

It is a strange fact that for the simple set of the three main converters, the buck, boost, and buck-boost (or flyback) there is no single publication that summarizes all of the design and analysis equations in a single place. It has been on my list of things to do for some time now to generate a complete set of equations and publish them as a poster for the basic converters operating in CCM and DCM, and with voltage-mode and current-mode control.

Texas Instruments once provided a wall chart which made some progress toward this, but it had omissions, out-of-date models, or did not cover all of the functions needed by a designer. It is still quite useful if you can find one.

Too Many Equations

I have come to realize that the ultimate set of equations in a concise printed reference won't happen, and finally last week I was able to put my finger on why. Christophe Basso has just published an update to his Switch-Mode Power Supply book[1] which is highly recommended as a reference to all those involved in power supply design. In this book, he has a thorough review of multiple different modeling techniques over the years, and he presents summaries, including many of the equations needed for design.

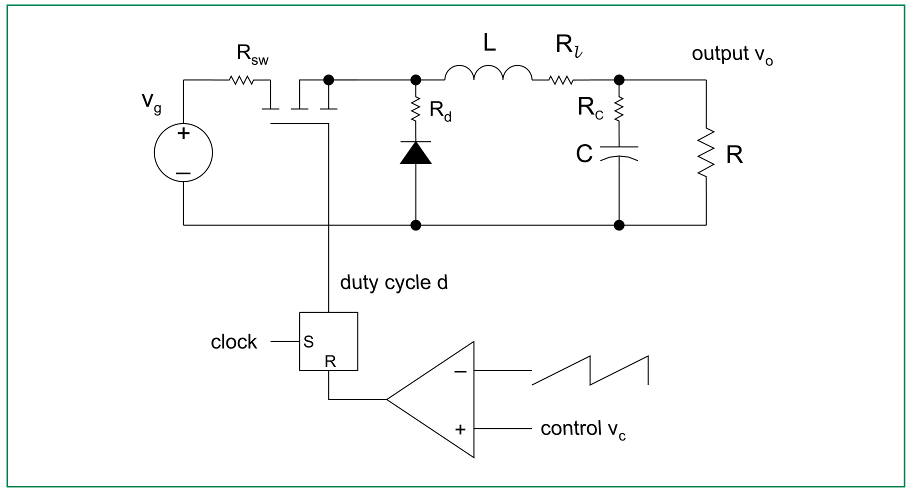


Figure 1: Buck Converter with Parasitic Resistances and Voltage-Mode Control.

There are, in total, over 1300 equations in this book just to cover the basics of operation of our "supposedly simple" converters. Basso's book is very much a practical volume and the equations in there are necessary to understand this field properly. When you have been in this field for a long time, there is a tendency to forget how much background is really needed to be an effective designer. At some point in your career, you need to go through these equations and understand where they come from, and how they impact your circuit. However, at other times, you want to get to the results as quickly as possible because you are subject to an aggressive schedule.

Fortunately, you don't have to digest all the equations before you can design a converter. The big advantage of Basso's book is that it provides Spice models on a CD so you can immediately get to work on your power supply. Understanding the equations can come later.

If you include other advanced aspects of power supplies – resistances that are frequency dependent and nonlinear, multi-output supplies, the intricacies of switching and high frequency snubber networks – the number of equations goes up by an order of magnitude. In fact, it's worse than that, since many of the expressions for quantities become implicit equations that cannot be solved in a closed form.

Different Equations for the Same Problem

There are other problems with collecting a comprehensive set of equations for our converters. If you compare one reference with another, you will find different answers. Rarely will you find two books with exactly the same expression and notation.

In each analysis, approximations are made to arrive at an answer in a useful form. For example, we might assume the ESR of an output capacitor is always much lower than the dc resistance of the output load on the converter. That will lead to one equation. For some high current supplies, however, the ESR of the capacitor may be comparable, or even bigger than the load resistance, and the assumptions this equation will be invalid in predicting responses accurately.

Now let's look at an example circuit and equation. For the simple buck converter of Figure 1, the equation for the control-to-output transfer function is:

$$\frac{\hat{v}_o}{\hat{d}} = V_g \frac{1+sCR_c}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \qquad \text{Eq. 1}$$

Where

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad \text{Eq. 2}$$

The Q of the filter is given in[5] as

$$Q = \frac{1}{\frac{Z_o}{R_l + R} + \frac{R_c + R_l // R}{Z_o}} \qquad \text{Eq. 3}$$

With

$$Z_o = \sqrt{\frac{L}{C}} \qquad \text{Eq. 4}$$

The expression for Q used in the Power 4-5-6 software[2] for the buck converter is:

$$Q = \frac{1}{\frac{Z_o}{R_l} + \frac{DR_{sw} + D'R_d + R_c + R_l}{Z_o}} \qquad \text{Eq. 5}$$

Equations 3 and 5 for the Q of the circuit are clearly not the same. The first one does not include the series resistance of the diode or switch, and the second one does include these resistances. The second equation makes the assumption that the inductor parasitic resistance is considerably smaller than the load resistance, the first does not.

If you refer to the book by Vorpérian[4], even more effects are incorporated into the modeling, including storage-time delay in the power switch.

Clearly, different levels of complexity are possible, and you can spend a tremendous amount of time analyzing this most straightforward of equations. Then, when you build the real circuit, there will still be some discrepancies in the actual measurements versus predictions.

There are no right and wrong answers to this since none of the published models incorporates everything seen in the real circuit. The damping of the filter resonance is affected by switching losses, inductor core losses, and proximity effect losses in the inductor (this one is very complex – the heating effects occur at the switching frequency, yet the damping is at very low frequency). No publication has been written to try and explain the effect of these phenomena on the transfer functions.

More equations, and more variations from one analysis to another, are created when the buck converter operates in discontinuous-conduction mode (DCM). Vorpérian showed quite rightly that the converter remains second order, and the filter of the converter is heavily damped in this region of operation. The equations for DCM are quite complex, and not included here.

Engineering Design Tools

Basso's book[1] is very much aimed at the power designer, and also includes a CD of files to run with Spice. If you like to work with Spice, it is highly recommended that you start with these models for your power supplies.

Sometimes you are so under so much pressure that you do not even have time to get Spice running for your circuit, let alone understand the equation derivations. The intent of this series of articles is to provide the working power supply engineer with some design tools that will speed up the design process. To achieve that, downloadable software is available, and will be provided in 6 parts:

Part 1. Buck converter with voltage mode control in CCM and DCM
Part 2. Buck converter with current-mode control in CCM and DCM
Part 3. Boost converter with voltage mode control in CCM and DCM
Part 4. Boost converter with current-mode control in CCM and DCM
Part 5. Flyback converter with voltage

mode control in CCM and DCM

Part 6. Flyback converter with current-mode control in CCM and DCM

This software can be found at www.ridleyengineering.com. Part 1 is already available, and each successive part will coincide with articles in this magazine.

Part 1 – Buck Converter Analysis with Voltage-Mode Control

The buck converter is the simplest of all, with the equations shown above for the control-to-output transfer function in CCM. After you have downloaded the file for the buck converter, you can enter your converter circuit elements, and immediately plot the control transfer functions. You can also vary input voltage and load resistance to see the effect of different operating conditions on the control transfer function.

In CCM, the control function of the buck converter looks like a simple LC filter, with a gain set by the input voltage to the converter. Damping of the filter is affected by the parasitic component resistances of the circuit and the load resistor. The model provided in the software is sufficiently accurate for all practical converters, and you can use it with confidence.

At light loads, the converter will enter DCM, and the characteristic LC filter characteristic will be replaced by a system with a dominant pole, and a second high frequency pole. The software will automatically detect when this happens for your circuit elements.

Summary

As a power supply designer, you have your hands full trying to get a power supply into production. Unless you have a lot of spare time available, don't get caught up in trying to re-derive equations that vary from paper to paper, or trying to reprogram equations that have already been done before. You will never achieve perfect correlation between modeling and measurements anyway, so make sure your time is well allocated in solving the bigger problems that you may have with making your power supply rugged and reliable.

Make use of the tools that will be available with this column over the coming months, or you Spice simulations of converters that have already been set up by experts in that field[1].

References

1. "Switch-Mode Power Supplies", Christophe P. Basso, published by McGraw-Hill, 2008.

2. Power 4-5-6 Design Software, http://www.ridleyengineering.com/software.htm

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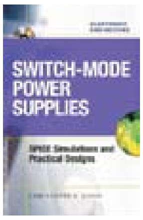

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Power Supply Control Design Tools – Part 2

Buck converter with current-mode control

In this article, Dr. Ridley presents a summary of current-mode control for the buck converter. A free piece of analysis software, the second in a series of six, is provided to readers of this column to aid with the analysis of their current-mode buck converters.

By Dr. Ray Ridley, Ridley Engineering

Modeling Power Supplies with Current-Mode Control

In the last article, (April 2008 Power Systems Design Europe, page 18) the complications of modeling power circuits were discussed in some detail for a buck converter with voltage-mode control. Even for that simple configuration, the analysis can have different levels of complexity. This will depend on how many parasitic components are included in the analysis, and any assumptions made about their relative values.

We don't usually use voltage-mode control for rugged converter design. Current-mode control is the preferred approach, implemented as shown in Figure 1.

A whole new world of mathematical complexity arises when current-mode control is used for a power supply. Fortunately, the full analysis of current-mode control is completed, and you can download the complete book on the topic from www.ridleyengineering.com.

The dynamic analysis of current mode involves advanced techniques, including discrete-time and sampled-data modeling. This is essential to arrive at a model which explains all of the phenomena seen with your converter, and which accurately predicts the measured control-to-output response and loop gain of the current-mode converter.

There are several important points to learn from the full analysis of the current-mode converter:

1. The power stage has a dominant-pole response at low frequencies, determined mainly by the time constant of the output capacitor and load resistor values.

2. The power stage has an additional pair of complex poles at half the switching frequency which, under certain conditions, will create instability in the current feedback loop.

3. The resulting transfer function of the power stage is third-order, even though there are only two state variables in the converter. (This apparent anomaly, for control theorists, is caused by the fact that the switching power converter is a nonlinear, time-varying system.)

4. The second-order double poles at half the switching frequency cannot be ignored, even though they may be well beyond the predicted loop crossover frequency.

5. The capacitor ESR zero is unchanged by the presence of the current loop feedback.

As explained in reference [1], current-mode control has many advantages. These include elimination of the resonant filter frequency, the ability to current share with multiple power stages, simplified compensation design, and inherent peak current limiting.

Designing with Current-Mode Control

While the analysis of current-mode control is quite complex to read and understand, the design process is quite simple. Much simpler, in fact, than voltage-mode control, and this is one of the reasons that current-mode control is so popular today.

Figure 1 shows the current-mode feedback system. The inductor current, or switch current, is sensed and compared to a voltage reference to set the duty cycle of the converter. A sawtooth ramp may also be added to the signal to stabilize the current loop.

Closing the current loop is straightforward. A current transformer, or sense resistor, is used to generate a voltage signal proportional to the actual current in the switch. The only requirement on the design of this network is that the resulting signal should not exceed the

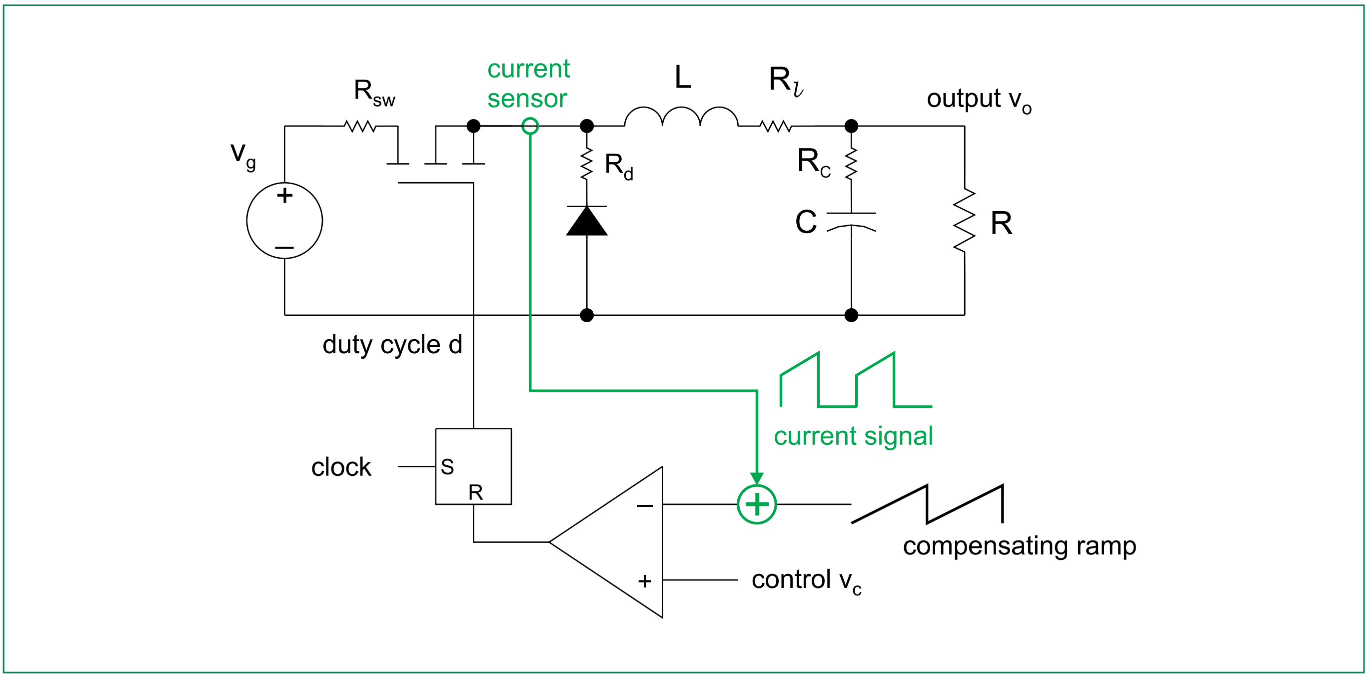

Figure 1: Buck converter with current-mode control. The green components show the current feedback; without these, the control is voltage-mode.

voltage headroom available in the PWM comparator. You do not have to think about the gain of the current loop, or resulting transfer functions at all during this phase of the design.

It is an interesting feature of the current loop that, regardless of how large you make the gain of the current sensing network, the current loop gain remains constant. This is because the PWM modulator gain, which is part of the current loop, is determined by the reciprocal of the slope of the sensed current. The higher the current gain, the lower the gain of the modulator. The two effects exactly cancel each other.

Once the current sense network is selected, you must decide whether you need to add a compensating ramp to the system. This is usually done for converters which will operate at duty cycles above 40%. Further details are given in[1]. Addition of the compensating ramp provides independent control of the PWM modulator gain. This stabilizes the tendency of the current feedback to oscillate at duty cycles approaching 50%.

Buck Converter Current-Mode Software

Software is now available for download that allows you to predict the small-signal response of your buck converter with current-mode control. After entering your power stage values and switching frequency, you can design the current loop parameters of current gain, and compensating ramp value. The software will help you choose the proper values. Once this is done, the transfer function gain and phase of the power stage is plotted for you, and the resulting poles and zeros given.

The software is designed to run under either Excel 2007 or Excel 2003. Make sure when you open the software that the macro features are enabled in order to use the program properly. Please go to www.ridleyengineering.com to download the software.

Summary

As mentioned at the end of the Design Tips in last month's magazine, you have your hands full trying to get a power supply into production. Trying to understand the intricacies of analysis of current-mode control is a useful thing to do, but most engineers simply don't have the time with their aggressive development schedules.

The software tool made available with this article will help you design the current loop properly, and give you the analysis of the converter. Remember, however, the results of any power supply transfer functions should always be verified by measurement. Our power systems are frequently dependent on circuit component parasitics that can be unpredictable, and can also be impacted by noise and improper board layout. Experimental verification[2] is an essential step for a rugged design, and should never be omitted.

References

1. *"A New Small-signal Model for Current-Mode Control", Raymond B. Ridley,1990 PhD dissertation, free download is available at www.ridleyengineering.com/cmode.htm*

2. *"Measuring Frequency Response, Tips and Methods" http://www.ridleyengineering.com/downloads/Spring 2002 feature.pdf*

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Power Supply Control Design Tools – Part 3

Boost converter with voltage-mode control

In this article, Dr. Ridley presents a summary of the boost converter with voltage-mode control. Free analysis software—the third in a series of six—is provided to readers of this column to aid with the analysis of their voltage-mode boost converters.

By Dr. Ray Ridley, Ridley Engineering

Voltage-Mode Boost Converter

The last two articles covered the buck converter in both voltage-mode and current-mode control. The buck is the simplest of all the converters, but as we have seen, the equations can still be very complex when the full range of operation is considered.

The boost converter offers a new set of complications in analysis and characteristics. It can be a challenging converter to stabilize when operating with voltage-mode control as shown in Figure 1.

For the boost converter of Figure 1, the equation for the control-to-output transfer function is:

$$\frac{\hat{v}_o}{\hat{d}} = \frac{V_g}{D'^2}\,\frac{(1+sCR_c)\left(1-s\frac{L_e}{R_L}\right)}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}}$$

Where the resonant frequency is given by

$$\omega_o = \frac{1}{\sqrt{L_e C}}$$

And the equivalent inductance is determined by the duty cycle:

$$L_e = \frac{L}{D'^2}$$

The Q of the filter is a complex combination of the parasitic resistances shown in the circuit, and the load resistance. For this equation, you can refer to either [3] or [5].

Boost Converter Right-Half-Plane Zero

The boost converter adds a new complexity to the control problem – a right-half-plane (RHP) zero. This is caused by the fact that when the boost converter switch is turned on for a longer period of time, the inductor is disconnected from the load for a longer period of time. That means that the output initially drops, even though the control command is trying to make it increase.

Figure 2 shows the effect on the gain

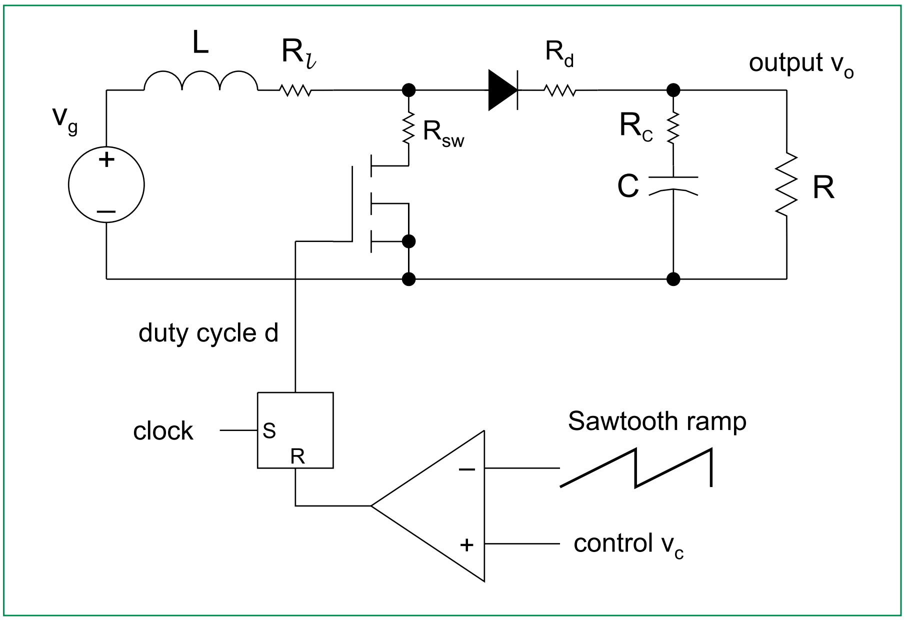

Figure 1: Boost converter with voltage-mode control.

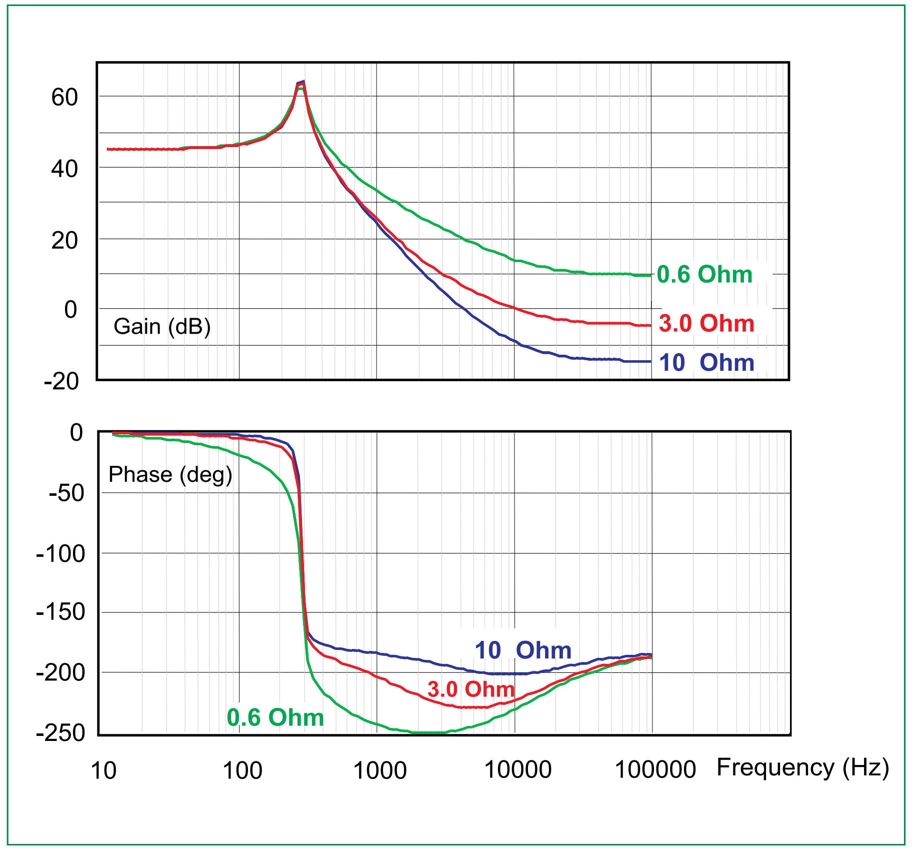

Figure 2: Effect of changing loads on the control characteristic of the boost converter.

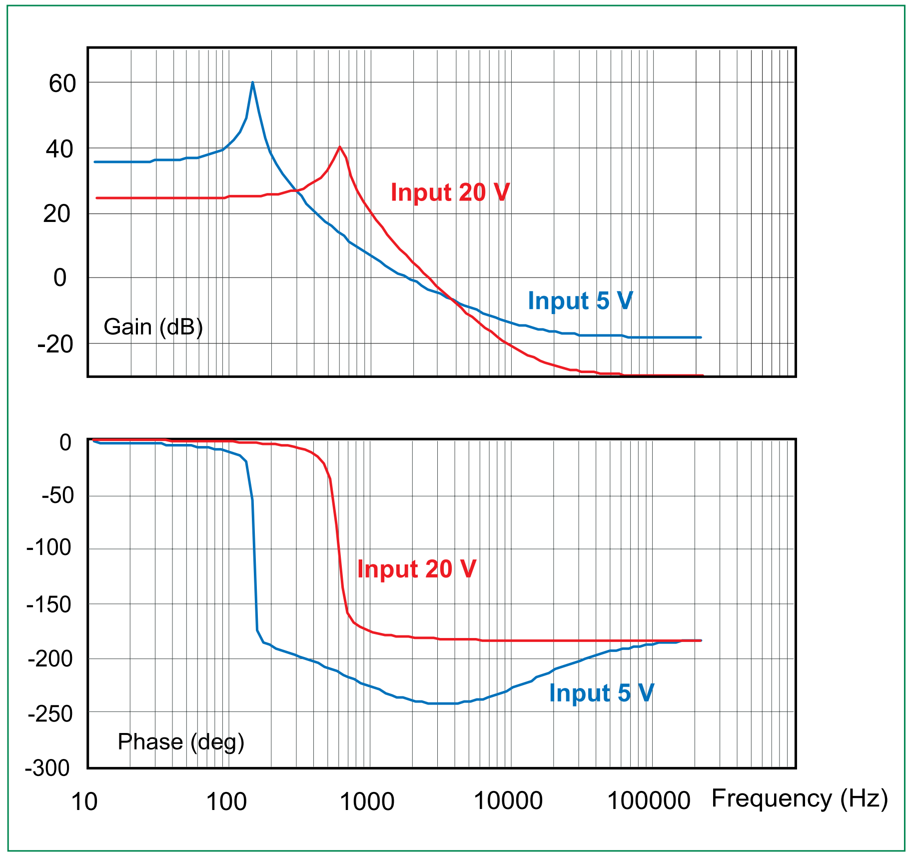

Figure 3: Effect of input line variation on the control characteristic of the boost converter.

and phase of the RHP zero. At heavy loads, the RHP zero frequency is the lowest, and the phase delay is the greatest. At light loads, the RHP zero frequency is higher, and the converter is easier to control.

The operation of the boost converter also causes a shift in the resonant frequency with input voltage, as can be seen from the control equations. Figure 3 shows how the characteristics of the boost converter can vary dramatically with a wide input voltage.

The general rule of thumb for converters with RHP zeros is to design at the lowest input line and the maximum load. This causes the lowest value of RHP zero, and the lowest value of resonant frequency. However, when using voltage-mode control, the moving resonant frequency can create problems at different operating points, and the whole range of operation should be carefully checked with both prediction and measurements.

More equations are created when the boost converter operates in discontinuous-conduction mode (DCM). These are not given in this article, but the free software provided for the boost converter will automatically assess which mode of operation your converter is in, and provide the proper transfer function.

Important Characteristics

There are several important points to remember about the boost converter operating in continuous-conduction mode:

1. There is a double pole at the resonant frequency of the LC filter. The frequency of this double pole will move with the operating point of the converter since it is determined by the equivalent inductance of the circuit, and this is a function of duty cycle. At low line, the resonant frequency has its lowest value.

2. As with all switching power supplies, there is a zero in the control-to-output transfer function corresponding to the ESR of the output filter capacitor.

3. The boost converter has a right-half-plane zero which can make control very difficult. This RHP zero is a function of the inductor (smaller is better) and the load resistance (light load is better than heavy load). The bandwidth of the control feedback loop is restricted to about 1/5th the RHP zero frequency.

In discontinuous conduction mode, the resonant frequency of the filter is eliminated from the control characteristic, as predicted by the switch model in[5]. This simplifies the control loop design, but higher power boost converters are usually designed to operate in CCM for efficiency reasons.

Boost Converter Voltage-Mode Software

Software is available for download that allows you to predict the small-signal response of your boost converter with voltage-mode control. After entering your power stage values and switching frequency, the transfer function gain and phase of the power stage is plotted for you, and the resulting poles and zeros given.

The software is designed to run under either Excel 2007 or Excel 2003. Make sure when you open the software that the macro features are enabled in order to use the program properly. Please go to http://www.ridleyengineering.com/freesoftware.htm to download the software.

Summary

The boost converter is an essential topology for stepping up the input voltage, and is applied in many areas of power conversion. This includes dc-dc converters, lighting applications, power factor correction circuits, battery discharging circuits, and many other applications. It is a good topology, but care and time must be taken to properly design the control loop. The inductor should be chosen carefully for a controllable power stage RHP zero characteristic. As with all converters, measurement [2] is essential to ensure a stable and rugged product.

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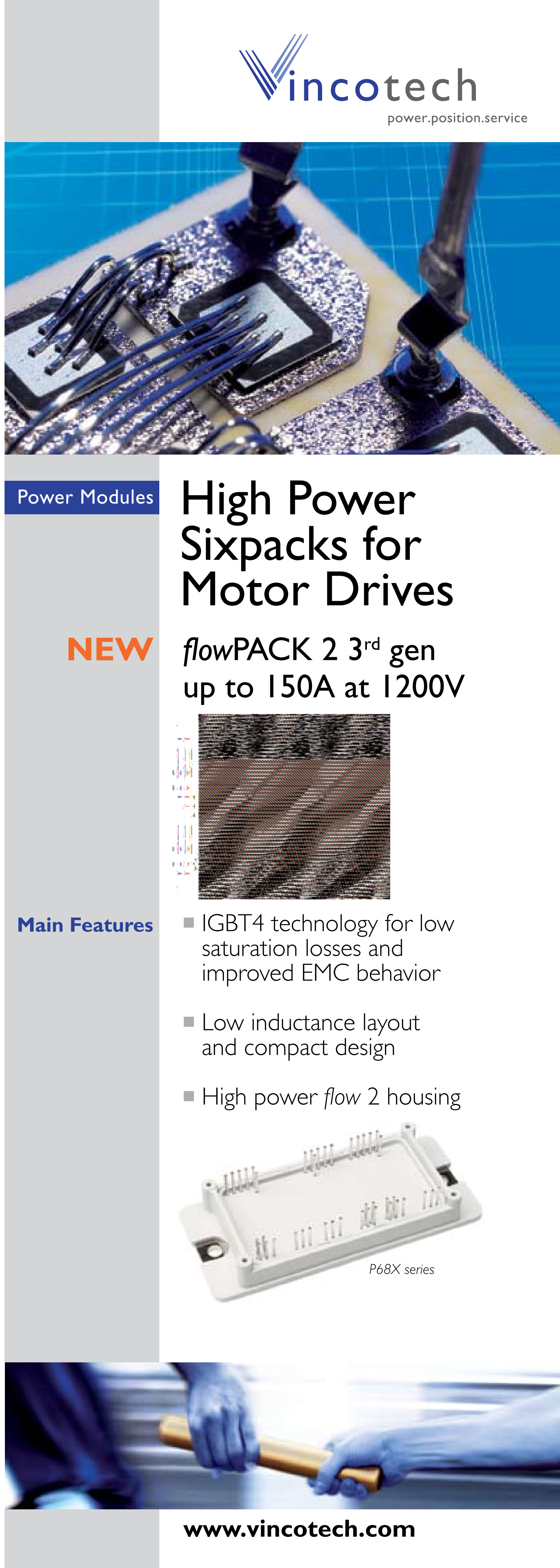

Power Supply Control Design Tools – Part 4

Boost converter with current-mode control

In this article, Dr. Ridley presents a summary of current-mode control for the boost converter. A free piece of analysis software, the fourth in a series of six, is provided to readers of this column to aid with the analysis of their current-mode boost converters.

By Dr. Ray Ridley, Ridley Engineering

Modeling Power Supplies with Current-Mode Control

In the last article, the complications of modeling power circuits were discussed in some detail for a boost converter with voltage-mode control. The boost converter was shown to have the complication of a right-half-plane zero which makes control with voltage-mode very difficult in some cases.

The problem is made much easier with current-mode control. This is always the preferred approach for the boost converter, implemented as shown in Figure 1.

As with the buck converter, a whole new world of mathematical complexity arises when current-mode control is used for a power supply. The full analysis of current-mode control is completed, and you can download the complete book on the topic from www.ridleyengineering.com.

The dynamic analysis of current mode involves advanced techniques, including discrete-time and sampled-data modeling. This is essential to arrive at a model which explains all of the phenomena seen with your converter, and which accurately predicts the measured control-to-output response and loop gain of the current-mode converter.

There are several important points to learn from the full analysis of the current-mode boost converter:

1. The power stage has a dominant-pole response at low frequencies, determined mainly by the time constant of the output capacitor and load resistor values.

2. The power stage has an additional pair of complex poles at half the switching frequency which, under certain conditions, will create instability in the current feedback loop. The damping of these complex poles is controlled by the addition of a compensating ramp.

3. The resulting transfer function of the power stage is third-order, even though there are only two state variables in the converter. (This apparent anomaly, for control theorists, is caused by the fact that the switching power converter is a nonlinear, time-varying system.)

4. The second-order double poles at half the switching frequency cannot be ignored, even though they may be well beyond the predicted loop crossover frequency.

5. The capacitor ESR zero is unchanged by the presence of the current loop feedback.

6. Finally, and most importantly, the current-mode boost converter retains the exact same RHP zero as the voltage-mode converter. However, since the current feedback has eliminated the double poles of the filter resonance, it is not difficult to control this RHP zero effectively.

As explained in reference[1], current-mode control has many advantages. These include elimination of the reso-

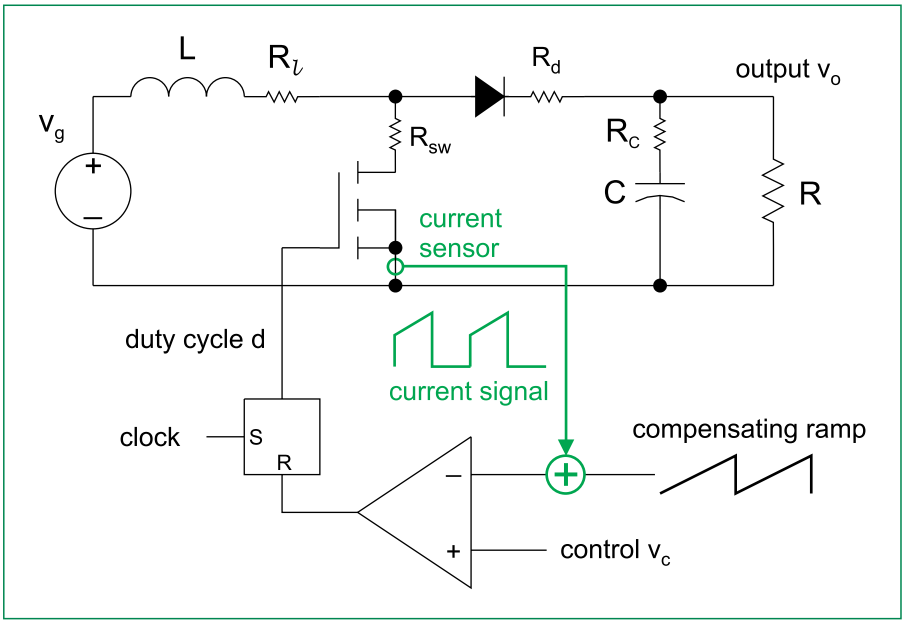

Figure 1: Boost converter with current-mode control. The green components show the current feedback; without these, the control is voltage-mode.

nant filter frequency, the ability to current share with multiple power stages, simplified compensation design, and inherent peak current limiting. The control is also optimum when operating in either continuous- or discontinuous-conduction mode, and there is no problem in operating the converter in both of these regions.

Designing with Current-Mode Control

While the analysis of current-mode control is quite complex to understand, the design process is quite simple. Much simpler, in fact, than voltage-mode control, and this is one of the reasons that current-mode control is so popular today.

Figure 1 shows the current-mode feedback system. The inductor current, or switch current, is sensed and compared to a voltage reference to set the duty cycle of the converter. A sawtooth ramp is added to the signal to stabilize the current loop if duty cycles approaching 50% are used.

Closing the current loop is straightforward. A current transformer, or sense resistor, is used to generate a voltage signal proportional to the actual current in the switch. The only requirement on the design of this network is that the resulting signal should not exceed the voltage headroom available in the PWM comparator. You do not have to think about the gain of the current loop, or resulting transfer functions at all during this phase of the design.

It is an interesting feature of the current loop that, regardless of how large you make the gain of the current sensing network, the current loop gain remains constant. This is because the PWM modulator gain, which is part of the current loop, is determined by the reciprocal of the slope of the sensed current. The higher the current gain, the lower the gain of the modulator. The two effects exactly cancel each other.

Once the current sense network is selected, you must decide whether you need to add a compensating ramp to the system. This is usually done for converters which will operate at duty cycles above 40%. Further details are given in [1]. Addition of the compensating ramp provides independent control of the PWM modulator gain. This stabilizes the tendency of the current feedback to oscillate at duty cycles approaching 50%.

Boost Converter Current-Mode Software

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The software is designed to run under either Excel 2007 or Excel 2003. Make sure when you open the software that the macro features are enabled in order to use the program properly. Please go to www.ridleyengineering.com to download the software.

Summary

If you work with a boost converter, it is advisable to use current-mode control. While the analysis is complex, the software tool made available with this article will help you design the current loop properly and show the transfer functions of the converter. Remember, however, the results of any power supply transfer functions should always be verified by measurement. Power systems are frequently dependent on circuit component parasitics that can be unpredictable, and can also be impacted by noise and improper board layout. Experimental verification[2] is an essential step for a rugged design, and should never be omitted.

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Power Supply Control Design Tools – Part 5

Buck-Boost Converter with Voltage-Mode Control

In this article, Dr. Ridley presents a summary of the buck-boost converter with voltage-mode control. Free analysis software—the fifth in a series of six—is provided to readers of this column to aid with the analysis of their voltage-mode buck-boost converters.

By Dr. Ray Ridley, Ridley Engineering

Voltage-Mode Buck-Boost Converter

In the early days of power electronics, there were three basic topologies: the buck, the boost, and the buck-boost. Variations of these three topologies solved most power conversion problems, and continue to do so today.

In the last 4 articles, the buck and boost converter control characteristics have been presented, using voltage-mode or current-mode control. The final two articles of this series present the buck-boost converter (or, in its isolated version, the flyback converter.) Like the boost converter, the buck-boost can be a challenging converter to stabilize.

Figure 1 shows the standard buck-boost converter operating with voltage-mode control.

For the buck-boost converter of Figure 1, the equation for the control-to-output transfer function is:

$$\frac{\hat{v}_o}{\hat{d}} = \frac{V_g}{D'^2}\,\frac{\left(1+sCR_c\right)\left(1-s\frac{DL_e}{R_L}\right)}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \qquad \text{Eq. 1}$$

Where the resonant frequency is given by

$$\omega_o = \frac{1}{\sqrt{L_e C}} \qquad \text{Eq. 2}$$

And the equivalent inductance is determined by the duty cycle:

$$L_e = \frac{L}{D'^2} \qquad \text{Eq. 3}$$

The Q of the filter is a complex combination of the parasitic resistances shown in the circuit and the load resistance. For this equation, you can refer to either [3] or [5].

Buck-Boost Converter Right-Half-Plane Zero

Like the boost converter, the buck-boost converter has a right-half-plane (RHP) zero, as seen in the transfer function above. As with the boost, when the buck-boost converter switch is turned on for a longer period of time, the inductor is disconnected from the load for a longer period of time. That means that the output initially drops, even though the control command is trying to make it increase. This is the classic characteristic of a RHP zero.

Figure 2 shows the effect on the gain and phase of the RHP zero. At heavy loads, the RHP zero frequency is the lowest, and the phase delay is the greatest. At light loads, the RHP zero frequency is higher, and the converter is easier to control.

The operation of the buck-boost converter also causes a shift in the resonant frequency with input voltage, as can be seen from the control equations. Figure 3 shows how the characteristics of the buck-boost converter varies significantly with a wide input voltage.

For converters with RHP zeros, design is usually done at the lowest input line and the maximum load. This condition has the lowest value of RHP zero, and the lowest value of resonant frequency. The moving resonant frequency can create problems at different operating points, and the whole range of operation should be carefully checked with both

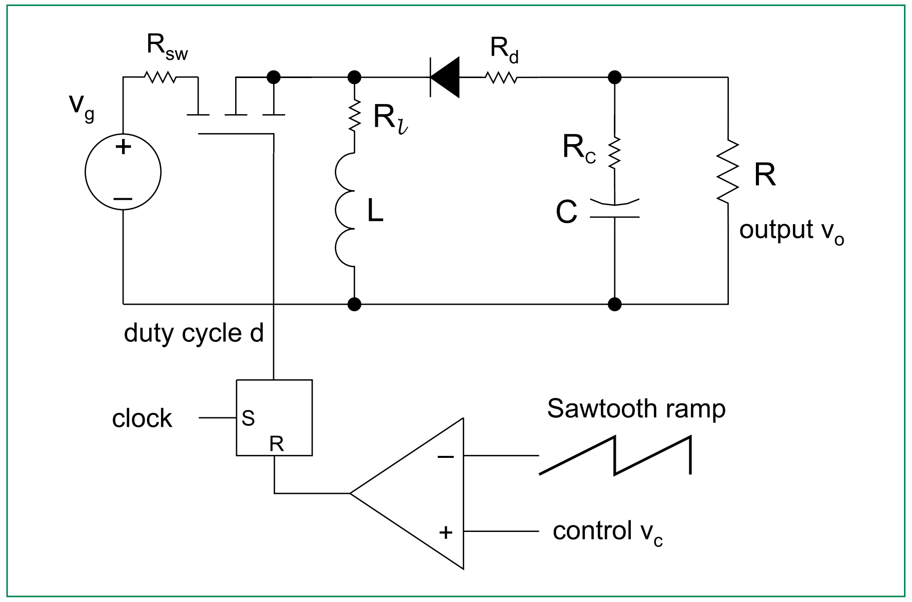

Figure 1: Buck-Boost converter with voltage-mode control.

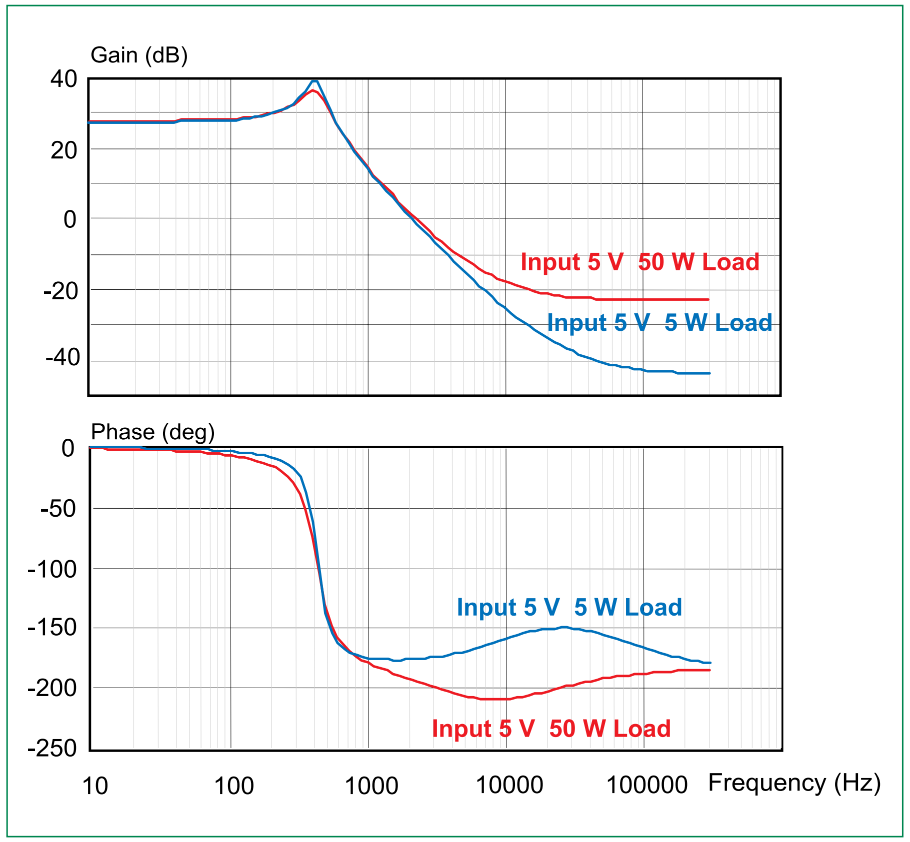

Figure 2: Effect of changing loads on the control characteristic of the buck-boost converter.

prediction and measurements.

More equations are created when the buck-boost converter operates in discontinuous-conduction mode (DCM). The free software provided for the boost converter will automatically assess which mode of operation your converter is in, and provide the proper transfer function.

Important Characteristics

There are several important points to remember about the buck-boost converter operating in continuous-conduction mode:

1. There is a double pole at the resonant frequency of the LC filter. The frequency of this double pole will move with the operating point of the converter since it is determined by the equivalent inductance of the circuit, and this is a function of duty cycle. At low line, the resonant frequency has its lowest value.

2. There is a zero in the control-to-output transfer function corresponding to the ESR of the output filter capacitor.

3. The buck-boost converter has a right-half-plane zero which can make control very difficult. This RHP zero is a function of the inductor (smaller is better) and the load resistance (light load is better than heavy load). The bandwidth of the control feedback loop is restricted to about 1/5th the RHP zero frequency.

In discontinuous conduction mode, the resonant frequency of the filter is eliminated from the control characteristic, as predicted by the switch model in [5]. This simplifies the control loop design, but higher power boost converters are usually designed to operate in CCM for efficiency reasons.

Most buck-boost (or flyback) converters are low power – 10 W or less. And most of them are designed to operate only in discontinuous conduction mode. This makes the control much simpler, but it is often very difficult to avoid going into CCM under all conditions. As power levels rise, trying to keep the converter in only DCM puts excessive stress on the power switch. As we will see with the next article, current-mode control makes the RHP zero issue much more manageable.

Buck-Boost Converter Voltage-Mode Software

Software is available for download that allows you to predict the small-signal response of your buck-boost converter with voltage-mode control. After entering your power stage values and switching frequency, the transfer function gain and phase of the power stage is plotted for you, and the resulting poles and zeros given.

The software is designed to run under either Excel 2007 or Excel 2003. Make sure when you open the software that the macro features are enabled in order to use the program properly. Please go to http://www.ridleyengineering.com/freesoftware.htm to download the software.

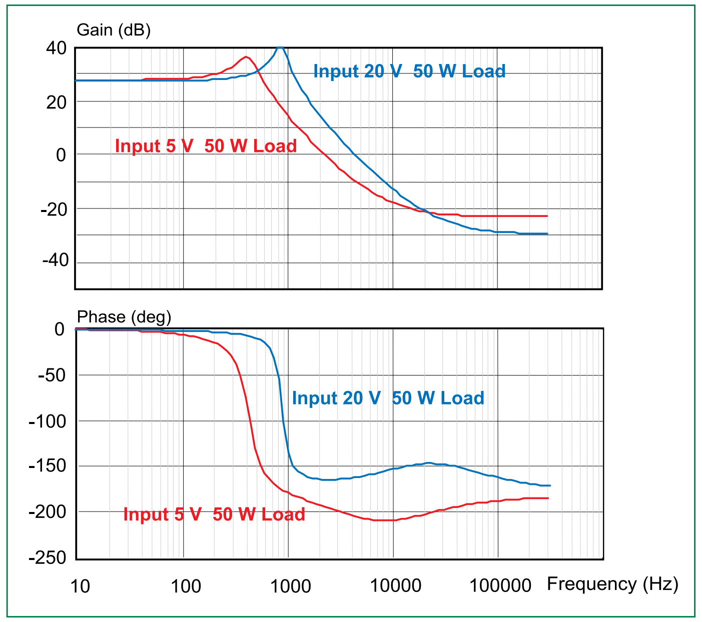

Figure 3: Effect of input line variation on the control characteristic of the buck-boost converter.

Summary

The buck-boost converter is very widely used in industry, in the format of the isolated flyback. It is a very useful topology, but care and time must be taken to properly design the control loop, especially when operating in CCM. The inductor should be chosen carefully for a controllable power stage RHP zero characteristic. It is not necessary to run the converter always in DCM, but this may be the preferred mode of operation for low-power converters. As with all converters, measurement[2] is essential to ensure a stable and rugged product.

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