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A Novel RC Model of Capacitive-Loaded Parallel and Series-Parallel Resonant DC-DC Converters

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Abstract - A novel analytical methodology is proposed and applied to investigate the steady state processes in voltage-fed parallel and series-parallel resonant DC-DC converters with capacitive output filter. In this methodology the rectifier, output capacitor and load are replaced by an equivalent circuit which includes a capacitor and resistor connected in parallel. Excellent agreement was obtained when comparing numerical values calculated by the proposed model to cycle-by-cycle SPICE simulation and to the numerical results of earlier studies.

INTRODUCTION

Exact analysis of parallel and series-parallel DC-DC resonant converters with capacitive output filter [1, 2, 3] is rather complex due to the fact that the equations include unknown time instances at which the output rectifier begins and ceases to conduct. The simplified analysis proposed by Steigerwald [4] is based on the method of the first harmonic. This approach is applicable to parallel and series-parallel converters with an LC output filter, but does not cover the case of a capacitor filter, in which the input current of the rectifier flows only a part of the switching period.

The objectives of the present study were as follows:

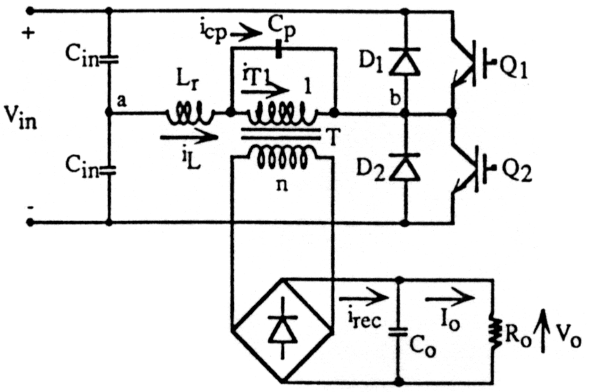
1. To develop a simple analytical method for the analysis of parallel and series-parallel converters with capacitive output filter (Fig. 1);
2. To obtain easy-to-use formulas and to apply them to develop design procedure for this class of converters.

II. MAIN EQUATIONS DESCRIBING STEADY-STATE PROCESSES

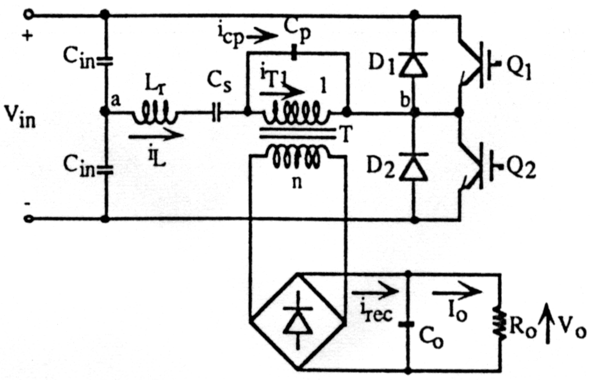
The analysis was carried out under the following basic assumptions:

1. The converter's elements (switches, transformer, inductor, capacitors) are ideal.
2. The reflected capacitance of the output filter C_o and the capacitance of the input capacitors C_{in} (in the case of half bridge configuration) are much larger than the capacitance of series and parallel capacitors (C_s and C_p).
3. The converter operates in continuous current mode.
4. The current i_L of the resonant inductor L_r (Fig. 2) is approximated by:

$$i_L = I_{Lm} \sin \vartheta \quad (1)$$



(a)



(b)

Fig. 1. Half-bridge configuration of parallel (a) and series-parallel (b) resonant DC-DC converters with capacitive output filter.

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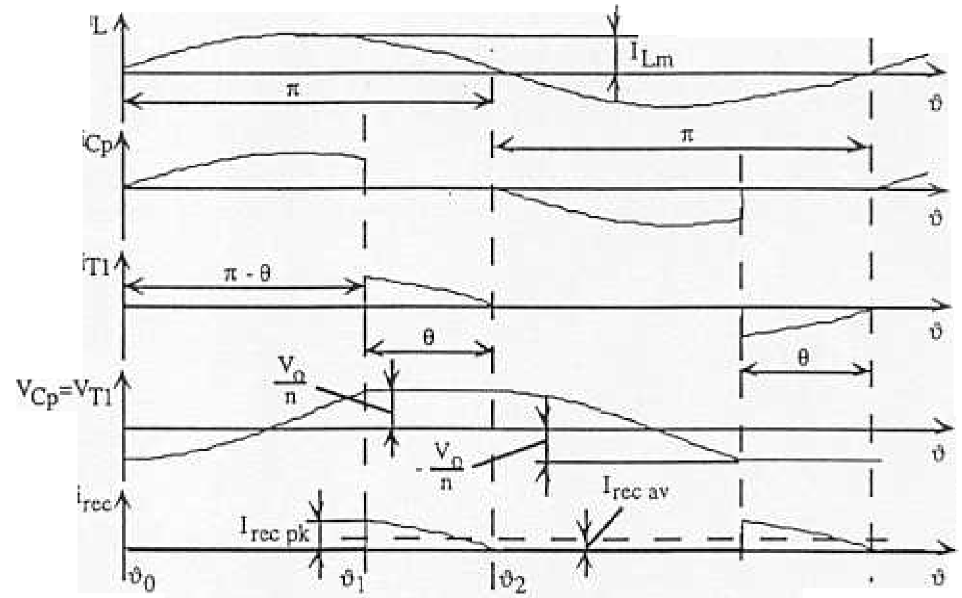

Fig. 2. Current and voltage waveforms (see Fig. 1 for notation)

where I_{Lm} is the peak value and $\vartheta = 2\pi ft$ is normalized time in radians with zero value at ϑ_0 when the rectifier ceases to conduct, f is the switching frequency and t is time.

The inductor current i_L flows through the capacitor C_p during the non conducting interval of the rectifier ϑ_0-ϑ_1 and through the primary of the transformer T during the conducting interval of the rectifier ϑ_1-ϑ_2. The rectifier's conduction angle ϑ_2-ϑ_1 is defined as θ (Fig. 2).

The voltage across the capacitor C_p can be derived by applying the following initial conditions: at $\vartheta_0 = 0$ when the output rectifier ceases to conduct $v_{Cp}=-V_o/n$. At $\vartheta_1=\pi-\theta$, when the output rectifier begins to conduct again, $v_{Cp} = V_o/n$, where V_o is the output voltage and n is the transformer's turns ratio (secondary to primary). Applying (1) along with the above boundary conditions we get:

$$v_{Cp} = \frac{V_o}{n(1+\cos\theta)}\left[(1-\cos\theta)-2\cos\vartheta\right] \qquad (2)$$

$$I_{Lm} = \frac{2V_o\omega C_p}{n(1+\cos\theta)} \qquad (3)$$

where $\omega = 2\pi f$.

The peak inductor voltage is found to be:

$$V_{Lm} = I_{Lm}\,\omega L_r = \frac{2V_o}{n(1+\cos\theta)}\left(\frac{\omega}{\omega_p}\right)^2 \qquad (4)$$

where

$$\omega_p = \frac{1}{\sqrt{L_r C_p}} \qquad (5)$$

The peak voltage across the series capacitor C_s (in series-parallel converter) is:

$$V_{Csm} = \frac{I_{Lm}}{\omega C_s} = \frac{2V_o}{n(1+\cos\theta)}\frac{C_p}{C_s} \qquad (6)$$

The rectifier's peak current $I_{rec\ pk}$ at ϑ_1 (Fig. 2) corresponds to the reflected current of capacitor C_p just prior to the transition:

$$I_{rec\ pk} = \frac{I_{Lm}}{n}\sin(\pi-\theta) = \frac{I_{Lm}}{n}\sin\theta \qquad (7)$$

The output current of the converter I_o is equal to the average rectifier's current $I_{rec\ av}$:

$$I_o = I_{rec\ av} = \frac{1}{\pi n}\int_{\pi-\theta}^{\pi} I_{Lm}\sin\vartheta\, d\vartheta = \frac{2}{\pi n} I_{Lm}\sin^2\left(\frac{\theta}{2}\right) \qquad (8)$$

or applying (3):

$$I_o = I_{rec\ av} = \frac{2}{\pi n^2} V_o\omega C_p\tan^2\left(\frac{\theta}{2}\right) \qquad (9)$$

On the other hand:

$$I_o = I_{rec\ av} = \frac{V_o}{R_o} \qquad (10)$$

where R_o is the load resistance.

From (9) and (10) we find the rectifier's conduction angle:

$$\theta = 2\tan^{-1}\sqrt{\frac{\pi}{2}\frac{\omega_p}{\omega}\frac{Z_p n^2}{R_o}} = 2\tan^{-1}\sqrt{\frac{\pi}{2}\frac{n^2}{\omega C_p R_o}} \qquad (11)$$

where

$$Z_p = \sqrt{\frac{L_r}{C_p}}$$

Relationship (11) is depicted in Fig. 3.

Now we find the first harmonics voltage and phase angle at the transformer's primary (Fig. 4). The primary transformer voltage v_{T1} is the voltage v_{Cp} across the capacitor C_p.

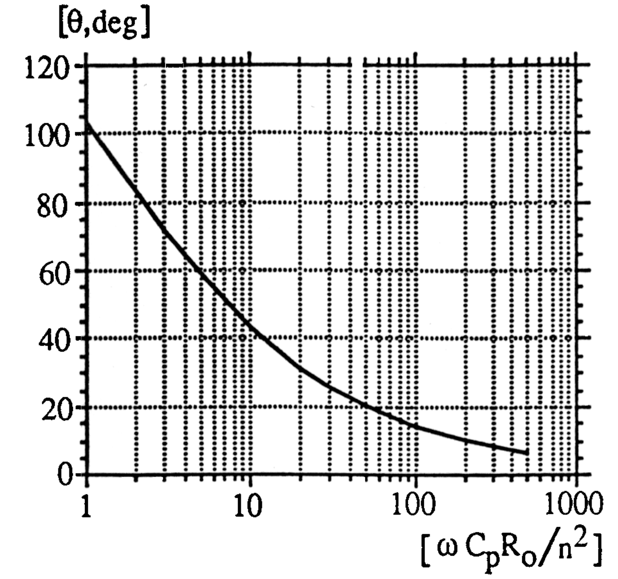

Fig. 3. Rectifier conduction angle θ as a function on the load coefficient $\omega C_p R_o/n^2$.

Applying (2) and the condition that v_{Cp} is equal to V_o/n during the conduction interval of the rectifier, we find the peak value of the first harmonics of the primary transformer voltage:

$$V_{T(1)m} = V_{Cp(1)m} = \frac{V_o}{n} k_v \quad (13)$$

where k_v is the voltage waveform coefficient:

$$k_v = \sqrt{a_{v(1)}^2 + b_{v(1)}^2} \quad (14)$$

and

$$a_{v(1)} = \frac{2}{\pi}\left(\frac{1}{1+\cos\theta}\left[(1-\cos\theta)\sin\theta - \left(\pi-\theta-\frac{1}{2}\sin 2\theta\right)\right] - \sin\theta\right) \quad (15)$$

$$b_{v(1)} = \frac{2}{\pi}(1-\cos\theta) \quad (16)$$

The relationship k_v as a function of θ is depicted in Fig. 5 (solid curve). This can be approximated by:

$$k_v = 1 + 0.27\sin\left(\frac{\theta}{2}\right) \quad (17)$$

(Fig. 5, dashed curve).

The phase angle of the first harmonics component of the transformer's voltage (referred to the instant ϑ_o, Fig. 4) will be:

$$\xi_{v(1)} = \tan^{-1}\left(\frac{a_{v(1)}}{b_{v(1)}}\right) \quad (18)$$

Note that $\xi_{v(1)}<0$ because $a_{v(1)}<0$.

The primary transformer current i_{T1} is the current of the resonant inductor L_r during the conduction interval of the rectifier and is zero when the rectifier does not conduct. Applying (1) we find the phase angle of the first harmonics transformer primary current (referred to the instant ϑ_o, Fig. 4):

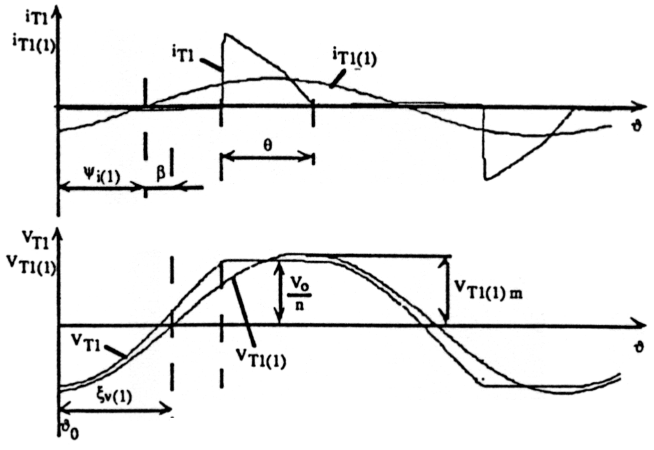

Fig. 4. The first harmonics of the current and voltage at the primary of the transformer.

$$\psi_{i(1)} = \tan^{-1}\left(-\frac{(1-\cos 2\theta)}{(2\theta-\sin 2\theta)}\right)$$

The difference

$$\beta = \xi_{v(1)} - \psi_{i(1)} \quad (20)$$

is the phase angle between the first harmonics of the primary transformer voltage and current (Fig. 4). The relationship β as a function of θ calculated from (15), (16), (18), (19) is depicted in Fig. 5 (solid curve). The graph can be described approximately by:

$$\beta = -25\sin\theta \text{ [deg]} \quad (21)$$

(Fig. 5, dashed curve).

The fact that the angle β is negative implies that the first harmonics of the primary transformer current $i_{T1(1)}$ leads the first harmonics of the primary transformer voltage $v_{T1(1)}$. Hence, this analysis implies that the rectifier and capacitor filter load can be approximately represented by an R-C equivalent circuit such as Fig. 6a. In the case of a parallel connected R-C circuit (Fig. 6a), the equivalent resistor R_e can be found from the relationship:

$$\frac{V_o^2}{R_o} = \frac{V_{T(1)m}^2}{2R_e} \quad (22)$$

Applying (13) we obtain:

$$R_e = R_o\frac{k_v^2}{2n^2} \quad (23)$$

The equivalent capacitance C_e is found from the equation (Fig. 6b):

$$\tan|\beta| = \omega C_e R_e \quad (24)$$

Applying (23) & (24) we obtain:

$$C_e = \frac{2n^2}{\omega R_o k_v^2}\tan|\beta| \quad (25)$$

The primary transformer rms current can therefore be represented by:

$$I_{T1\,rms} = \frac{V_{T(1)m}}{\sqrt{2}R_e\cos\beta}$$

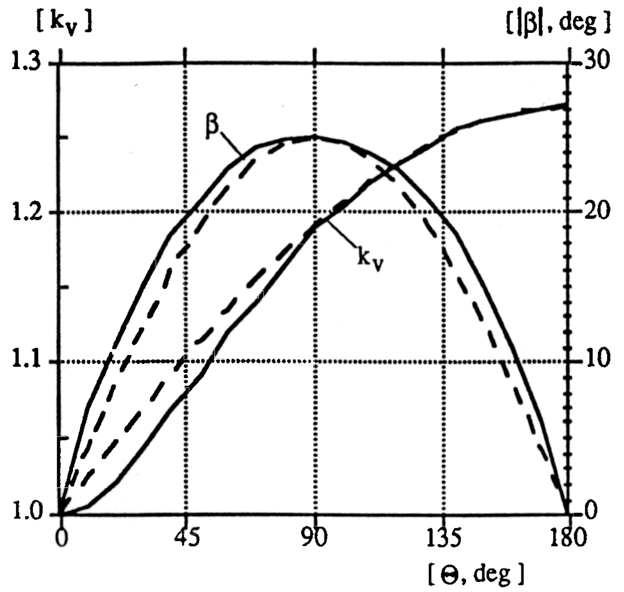

Fig. 5. The voltage waveform coefficient k_v and phase angle β versus the rectifier's conduction angle θ; solid curves obtained by Fourier analysis, dashed curves - by approximate eqs. (17) & (21).

or applying (10), (13) and (23)

$$I_{T1\ rms} = I_O \frac{\sqrt{2}n}{k_v \cos\beta} \tag{27}$$

The secondary transformer rms current will thus be:

$$I_{T2\ rms} = \frac{I_{T1\ rms}}{n} = I_O \frac{\sqrt{2}}{k_v \cos\beta} \tag{28}$$

Applying the equivalent resistance R_e and the equivalent capacitance C_e, the parallel and series-parallel converter with the capacitive output filter can be represented by the equivalent circuits of Fig. 7a and 7b respectively. v_{ab} is the voltage between the points a and b of the original converters (Fig. 1). This voltage has a rectangular waveform with an amplitude of $\pm gV_{in}$, where V_{in} is the converter input voltage and g is topology constant (g=1 in full bridge and g=0.5 in half bridge). Hence, the peak value of the first harmonics of the voltage v_{ab} can be found from equation:

$$V_{ab(1)m} = \frac{4}{\pi} gV_{in} \tag{29}$$

The expressions of the AC voltage ratio

$$k_{2-1} = \frac{V_{T(1)m}}{V_{ab(1)m}} \tag{30}$$

can be found by analyzing the equivalent circuits of Fig. 7.

For the parallel converter (Figs. 1a and 7a):

$$k_{2-1} = \frac{1}{\sqrt{[1-\omega^2 L_r(C_p+C_e)]^2 + (\frac{\omega L_r}{R_e})^2}} \tag{31}$$

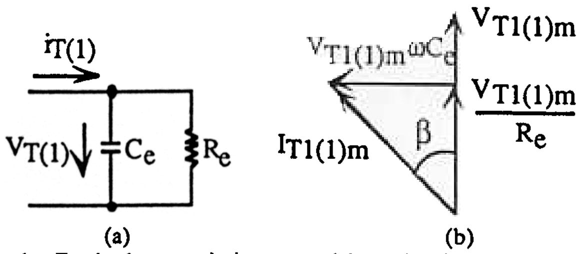

Fig. 6. Equivalent resistive-capacitive circuit replacing the loaded transformer (a) and vector diagram of its voltage and currents (b).

or applying (5), (23) and (25)

$$k_{2-1} = \frac{1}{\sqrt{[1-(\frac{\omega}{\omega_p})^2(1+\frac{\tan|\beta|}{\omega C_p R_e})]^2 + [(\frac{\omega}{\omega_p})^2 \frac{1}{\omega C_p R_e}]^2}} \tag{32}$$

For the series-parallel converter (Figs. 1b and 7b):

$$k_{2-1} = \frac{1}{\sqrt{[1+\frac{C_p+C_e}{C_s}-\omega^2 L_r(C_p+C_e)]^2 + (\omega L_r - \frac{1}{\omega C_s})^2 \frac{1}{R_e^2}}} \tag{33}$$

or applying (23) and (25)

$$k_{2-1} = \frac{1}{\sqrt{\{1-\frac{C_p}{C_s}[(\frac{\omega}{\omega_s})^2-1](1+\frac{\tan|\beta|}{\omega C_p R_e})\}^2 + \{\frac{C_p}{C_s}[(\frac{\omega}{\omega_s})^2-1]\frac{1}{\omega C_p R_e}\}^2}} \tag{34}$$

where

$$\omega_s = \frac{1}{\sqrt{L_r C_s}} \tag{35}$$

The DC transformer ratio of the converter

$$V_o^* = \frac{V_o}{ngV_{in}} = \frac{V_o}{nV_{T(1)m}} \frac{V_{T(1)m}}{V_{ab(1)m}} \frac{V_{ab(1)m}}{gV_{in}} \tag{36}$$

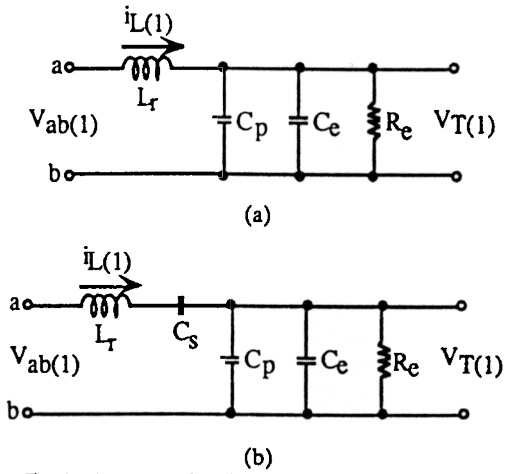

Fig. 7. Equivalent ac circuits of the parallel (a) and series-parallel (b) converter with capacitive output filter.

or applying (13), (29) and (30)

$$V_o^* = \frac{4}{\pi}\frac{k_2-1}{k_v} \qquad (37)$$

Now we define the input phase angle $\varphi_{(1)}$, i.e. the angle between the voltage v_{ab} and the first harmonics of the inductor L_r current $i_{L(1)}$ (Fig. 7).

In the parallel converter (Figs. 1a and 7a)

$$\tan\varphi_{(1)} = \left(\frac{\omega}{\omega_p}\right)^2 \frac{1}{\omega C_p R_e}\left[1+(\omega C_p R_e + \tan|\beta|)^2\right] - \left[\omega C_p R_e + \tan|\beta|\right] \qquad (38)$$

where R_e is equivalent load resistance (eq. (23)) and β is the phase angle between the first harmonics of the primary transformer voltage and current (Fig. 4, eq. (21)).

In the series-parallel converter (Figs. 1b and 7b)

$$\tan\varphi_{(1)} = \frac{1}{\omega C_p R_e}\frac{C_p}{C_s}\left\{\left(\frac{\omega}{\omega_s}\right)^2\left[1+(\omega C_p R_e + \tan|\beta|)^2\right] - 1\right\} - \left[\omega C_p R_e + \tan|\beta|\right]\left[1 + \frac{C_p}{C_s}\left(1 + \frac{\tan|\beta|}{\omega C_p R_e}\right)\right] \qquad (39)$$

For a sinusoidal inductor current (assumption 4) we obtain the following expressions for the average input current $I_{in\ av}$ and for the average currents of the transistors and anti parallel diodes $I_{Q\ av}$ and $I_{D\ av}$:

$$I_{in\ av} = \frac{2}{\pi} g I_{Lm} \cos\varphi_{(1)} \qquad (40)$$

$$I_{Q\ av} = \frac{1}{\pi} I_{Lm} \cos^2\left(\frac{\varphi_{(1)}}{2}\right) \qquad (41)$$

$$I_{D\ av} = \frac{1}{\pi} I_{Lm} \sin^2\left(\frac{\varphi_{(1)}}{2}\right) \qquad (42)$$

or applying (40)

$$I_{Q\ av} = \frac{I_{in\ av}}{4g}\left(\frac{1}{\cos\varphi_{(1)}} + 1\right) \qquad (43)$$

$$I_{D\ av} = \frac{I_{in\ av}}{4g}\left(\frac{1}{\cos\varphi_{(1)}} - 1\right) \qquad (44)$$

Applying (8) and (40) we obtain:

$$I_{in\ av} = ngI_o \frac{\cos\varphi_{(1)}}{\sin^2\left(\frac{\theta}{2}\right)} \qquad (45)$$

Neglecting losses (assumption 1):

$$\frac{I_{in\ av}}{I_o} = \frac{V_o}{V_{in}} \qquad (46)$$

Taking into account (45) and (46) we obtain an important expression of the output to input voltage ratio:

$$V_o^* = \frac{V_o}{ngV_{in}} = \frac{\cos\varphi_{(1)}}{\sin^2\left(\frac{\theta}{2}\right)} \qquad (47)$$

III. OUTPUT CHARACTERISTICS INCLUDING NO LOAD AND SHORT CIRCUIT CONDITIONS

Under no load condition ($R_o=\infty$) $k_v=1$; applying (32) and (34) we obtain:

For parallel converter (Figs. 1a and 7a)

$$V_{o\ NL}^* = \frac{4}{\pi}\frac{1}{\left|1-\left(\frac{\omega}{\omega_p}\right)^2\right|}$$

For series-parallel converter (Figs. 1b and 7b)

$$V_{o\ NL}^* = \frac{4}{\pi}\frac{1}{\left|1-\frac{C_p}{C_s}\left[\left(\frac{\omega}{\omega_s}\right)^2-1\right]\right|} = \frac{4}{\pi}\frac{1}{\left|1-\left(\frac{\omega}{\omega_p}\right)^2+\frac{C_p}{C_s}\right|}$$

Applying per unit system we choose the base current unit to be:

in parallel converter (Figs. 1a and 7a)

$$I_{bas\ p} = \frac{gV_{in}}{n\sqrt{\frac{L_r}{C_p}}} \qquad (50)$$

in series-parallel converter (Figs. 1b and 7b)

$$I_{bas\ s} = \frac{gV_{in}}{n\sqrt{\frac{L_r}{C_s}}} \qquad (51)$$

The average output current under short circuit condition ($R_o=0$) is found to be:

for parallel converter (Figs. 1a and 7a) (under assumption 4):

$$I_{o\ sh} = \frac{4}{\pi}\frac{gV_{in}}{n\omega L_r}\frac{2}{\pi} = \frac{8}{\pi^2}\frac{gV_{in}}{n\omega L_r} \qquad (52)$$

or in per unit system:

$$I_{o\ sh}^* = \frac{I_{o\ sh}}{I_{bas\ p}} = \frac{8}{\pi^2}\frac{\omega_p}{\omega} = 0.811\frac{\omega_p}{\omega} \qquad (53)$$

For parallel converter (Figs. 1a and 7a) a more exact equation can be derived without the restriction of assumption 4:

$$I_{o\ sh} = \frac{\pi}{4}\frac{gV_{in}}{n\omega L_r} \qquad (54)$$

or in per unit system:

$$I_{o\ sh}^* = \frac{I_{o\ sh}}{I_{bas\ p}} = \frac{\pi}{4}\frac{\omega_p}{\omega} = 0.785\frac{\omega_p}{\omega} \qquad (55)$$

For series-parallel converter (Figs. 1b and 7b)

$$I_{o\ sh} = \frac{4}{\pi}\frac{gV_{in}}{n\left|\omega L_r - \frac{1}{\omega C_s}\right|}\frac{2}{\pi} = \frac{8}{\pi^2}\frac{gV_{in}\omega C_s}{n\left|\left(\frac{\omega}{\omega_s}\right)^2-1\right|} \qquad (56)$$

or in per unit system:

$$I_{o\ sh}^* = \frac{I_{o\ sh}}{I_{bas\ s}} = \frac{8}{\pi^2}\frac{1}{\left|\frac{\omega}{\omega_s} - \frac{\omega_s}{\omega}\right|}$$

The equations derived above were applied to elucidate the output characteristics of the converters (Figs. 8 & 9, solid curves). These characteristics are in excellent agreement with simulation results obtained by our group (Figs. 8 & 9, crosses and circles). They also agree very well with the results of complex analysis of Johnson and Erickson [2] and Bhat [3]. The output to input voltage ratio of the parallel converter calculated by (17), (21), (32) and (37) agree with the results of Steigerwald [1] to within 10.6% (see Appendix).

Note that the determination of output characteristics by the proposed method is much simpler than by more exact analyses [1-3]. This benefit is most important for parameter optimization for which the numerical data given in previous studies [1-3] may not suffice. For example, in [3] the data for series-parallel converter is given only for the case $C_S=C_P$.

IV. CURRENT SOURCE AND VOLTAGE SOURCE CONDITIONS

The slope of the converter output characteristics (Figs. 8 & 9) is a function of the frequency ratio ω/ω_p and ω/ω_s. As can be seen, both converters can operate as a current source and as a voltage source.

Analysis of equations (32) and (34) shows the condition when the converters are operating as a current source. In this case terms which don't include R_o must be equal zero.

The parallel converter (Figs. 1a and 7a, eq. (32)) operates as a current source when

$$1-\left(\frac{\omega}{\omega_p}\right)^2 \approx 0 \qquad (58)$$

The series- parallel converter (Figs. 1b and 7b, eq. (34)) operates as a current source when

$$1-\frac{C_p}{C_s}\left[\left(\frac{\omega}{\omega_s}\right)^2-1\right] \approx 0 \qquad (59)$$

from where

$$\frac{\omega}{\omega_s} \approx \sqrt{\frac{C_s}{C_p}+1} \qquad (60)$$

The parallel converter (Figs. 1a and 7a, eq. (32)) functions as a voltage source if the terms in (32) which include R_o are negligible small as compared to the other terms in this equation; applying (23) we obtain:

$$\frac{2n^2}{k_v^2\omega C_p R_o} << \left|\frac{1}{\left(\frac{\omega}{\omega_p}\right)^2}-1\right| \qquad (61)$$

It is easy to see that this condition can be satisfied only in the cases when the switching frequency ω is far from the resonant frequency ω_p or when the normalized load resistance $(\omega C_p R_o)/n^2$ is very high.

The series-parallel converter (Figs. 1b and 7b, eq. (34)) operates as a voltage source when the switching frequency ω is near to the resonant frequency ω_s of the series network $L_r C_s$. Equation (34) reduces in this case to:

$$k_{2\text{-}1} \approx 1 \qquad (62)$$

and therefore equation (37) reduces to

$$V_o^* \approx \frac{4}{\pi k_v} \qquad (63)$$

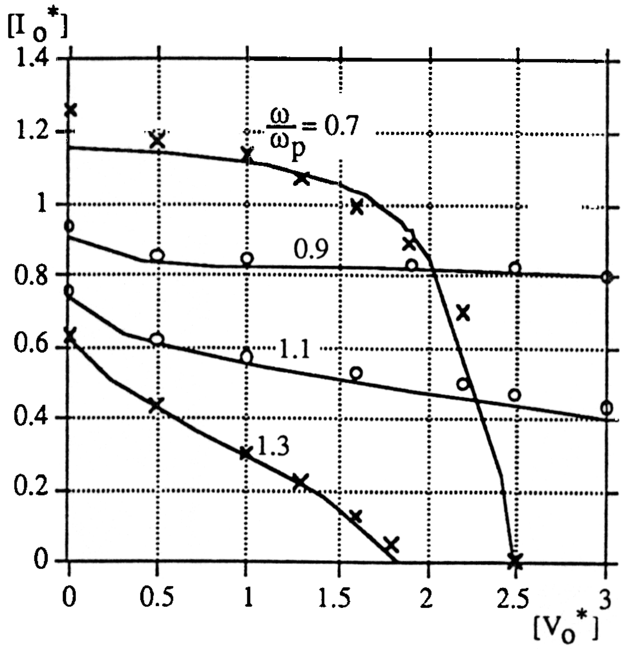

Fig. 8. The output characteristics of the parallel converter; solid curves obtained by calculation using the proposed method, points (x) and (o) obtained by simulation.

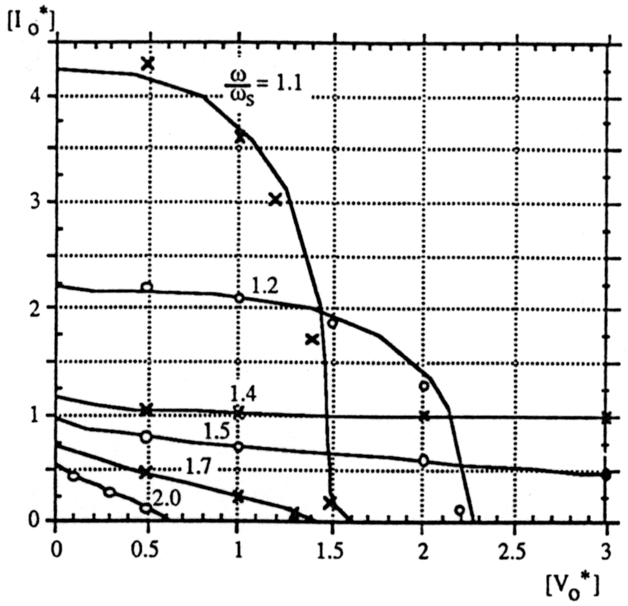

Fig. 9. The output characteristics of the series-parallel converter when $C_S=C_P$; solid curves obtained by calculation using the proposed method, points (x) and (o) obtained by simulation.

V. DESIGN PROCEDURE (parallel converter)

The following parameters are assumed to be given: output and input voltages (V_o and $V_{in\ min} \div V_{in\ max}$), load resistance ($R_{o\ min} \div R_{o\ max}$), nominal switching frequency f.

The design begins from the case $R_{o\ min}$, $V_{in\ min}$.

1. Set the values of the angles $\varphi_{(1)}$ and θ taking in account following considerations:

a) high values of $\varphi_{(1)}$ (for a given constant input current $I_{in\ av}$) correspond to high values of the peak inductor current I_{Lm} and high values of the average currents of the transistors and anti parallel diodes $I_{Q\ av}$, $I_{D\ av}$ (eqs. (40), (43), (44)); hence, the losses in the converter are also high;

b) low values of θ correspond to high ratios of $\frac{I_{rec\ pk}}{I_{rec\ av}}$ (bad utilization of rectifier diodes, eqs. (7) and (8));

c) the output to input voltage ratio V_o^* is a function of $\varphi_{(1)}$ and θ (eq. (47)), therefore for a given V_o and V_{in} the angles $\varphi_{(1)}$ and θ determinate the transformer's turn ratio n.

2. Calculate the following dimensionless parameters for the chosen combination of $\varphi_{(1)}$ and θ from step 1: $\omega C_p R_o/n^2$ - eq. (11), k_v - eq. (17), β - eq. (21), $\omega C_p R_e$ - eqs. (11) and (23), $\frac{\omega}{\omega_p}$ - eq. (38), $k_{2\text{-}1}$ - eq. (32), V_o^* - eq. (37) or (47), n - eq. (36).

3. Calculate the following dimensioned parameters for combination of $\varphi_{(1)}$ and θ from step 1 (using the data from step 2): capacitance C_p from $\omega C_p R_o/n^2$, inductance L_r - eq. (5), the peak of the inductor current I_{Lm} - eq. (3), average currents through the transistors and anti parallel diodes $I_{Q\ av}$ and $I_{D\ av}$ - eqs. (41) and (42), the peak current of the rectifier diodes $I_{rec\ pk}$ - eq. (7), the peak voltage across the inductor L_r - eq. (4), the transformer's rms currents $I_{T1\ rms}$ and $I_{T2\ rms}$ - eqs. (27) and (28).

4. Repeat the calculations of steps 1-3 for different $\varphi_{(1)}$ and θ to sort out inferior selections, for example, low capacitance C_p which is in the range of the parasitics.

5. Examine the case of $V_{in\ max}$, $R_{o\ min}$. Calculate new values of the switching frequency f, currents of all elements and voltages across them corresponding to the new value of the output to input voltage ratio V_o^*.

6. Last two cases ($V_{in\ min}$, $R_{o\ max}$ and $V_{in\ max}$, $R_{o\ max}$) are calculated in the same manner. Obtained values of the switching frequency, currents and voltages correspond here to the new value of the load resistance R_o.

7. Choose (by iteration) optimal values of the angles $\varphi_{(1)}$ and θ using results of calculation (steps 3 ÷ 6). Criteria of optimization can be different: highest efficiency, lowest cost, lowest weight and volume, minimal range in which the switching frequency must be changed etc.

Series-parallel converter can be designed and optimized in the same manner. But three parameters (instead of two) must be set in the step 1 of the design: $\varphi_{(1)}$, θ and C_p/C_s. Note that the third parameter C_p/C_s influence in particular the dependence between the peak of the inductor current I_{Lm} and the load resistance R_o. Due to action of C_s the peak current I_{Lm} becomes smaller when R_o increases. Hence, series-parallel converter can have higher efficiency at reduced output power.

VI. CONCLUSIONS

The proposed equivalent circuit method simplifies the analysis and design of parallel and series-parallel resonant DC-DC converters with capacitive output filter. In contrast to previous more exact numerical derivations, which were presented by tables and plots, the analytical relationships derived by the proposed method are more convenient as design tools and in particular for parameter optimization of these types of converters.

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APPENDIX

Output to input voltage ratio of the parallel DC-DC converter (V_o^*) computed by Steigerwald [1] and by proposed method

ω/ω_p	$\omega_p C_p R_o$	V_o^*	
		Steigerwald	Proposed method
0.699956	2.27678	2	2.008
1.15515	2.03744	1	0.996
0.939992	6.76539	5	5.084
0.939992	3.85285	3	2.971
0.939992	1.29127	1	0.992
0.939992	0.708309	0.5	0.553