Appendix 1

RMS Values of Commonly Observed Converter Waveforms

The waveforms encountered in power electronics converters can be quite complex, containing modulation at the switching frequency and often also at the ac line frequency. During converter design, it is often necessary to compute the rms values of such waveforms. In this appendix, several useful formulas and tables are developed that allow these rms values to be quickly determined.

RMS values of the doubly modulated waveforms encountered in PWM rectifier circuits are discussed in section 18.1.

A1.1 SOME COMMON WAVEFORMS

DC, Fig. A1.1:

$$rms = I \tag{A1.1}$$

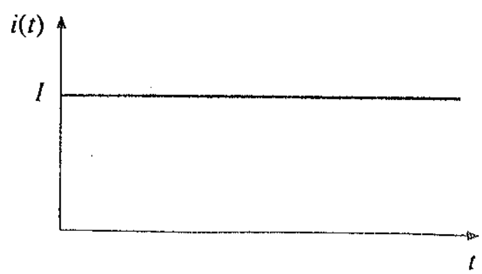

Fig. A1.1.

DC plus linear ripple, Fig. A1.2:

$$rms = I\sqrt{1 + \frac{1}{3}\left(\frac{\Delta i}{I}\right)^2} \tag{A1.2}$$

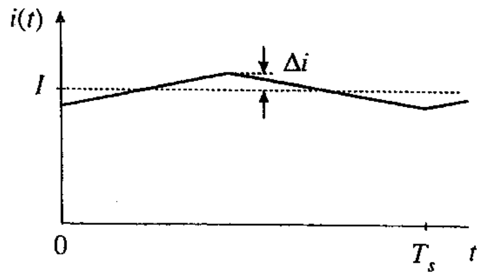

Fig. A1.2.

Square wave, Fig. A1.3:

$$rms = I_{pk} \tag{A1.3}$$

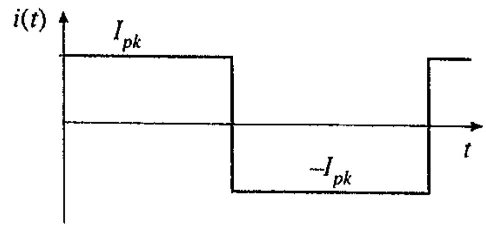

Fig. A1.3.

Sine wave, Fig. A1.4:

$$rms = \frac{I_{pk}}{\sqrt{2}} \tag{A1.4}$$

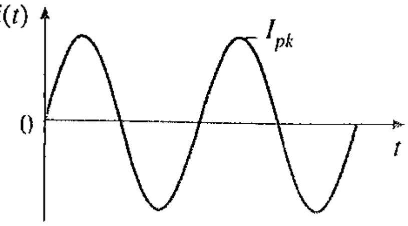

Fig. A1.4.

Pulsating waveform, Fig. A1.5:

$$rms = I_{pk}\sqrt{D} \tag{A1.5}$$

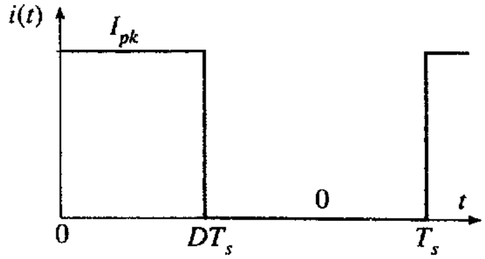

Fig. A1.5.

Pulsating waveform with linear ripple, Fig. A1.6:

$$rms = I\sqrt{D}\sqrt{1+\frac{1}{3}\left(\frac{\Delta i}{I}\right)^2} \tag{A1.6}$$

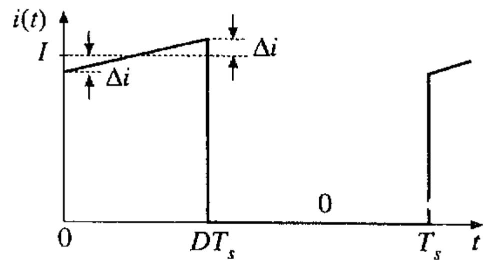

Fig. A1.6.

Triangular waveform, Fig. A1.7:

$$rms = I_{pk}\sqrt{\frac{D_1+D_2}{3}} \tag{A1.7}$$

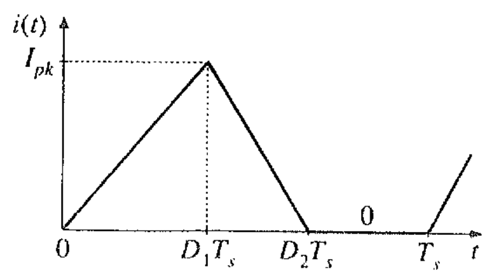

Fig. A1.7.

Triangular waveform, Fig. A1.8:

$$rms = I_{pk}\sqrt{\frac{D_1}{3}} \tag{A1.8}$$

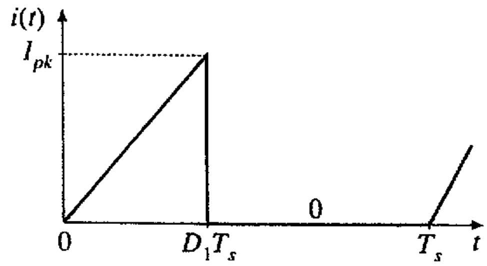

Fig. A1.8.

Triangular waveform, no dc component, Fig. A1.9:

$$rms = \frac{\Delta i}{\sqrt{3}} \tag{A1.9}$$

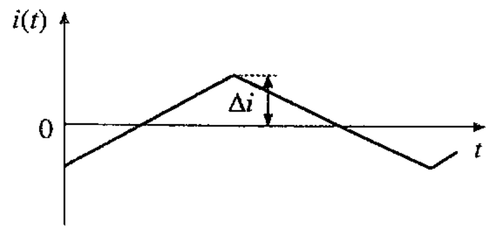

Fig. A1.9.

Center-tapped bridge winding waveform, Fig. A1.10:

$$rms = \frac{1}{2} I_{pk}\sqrt{1+D} \tag{A1.10}$$

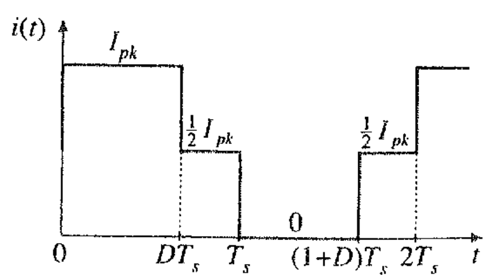

Fig. A1.10.

General stepped waveform, Fig. A1.11:

$$rms = \sqrt{D_1 I_1^2 + D_2 I_2^2 + \cdots} \qquad \text{(A1.11)}$$

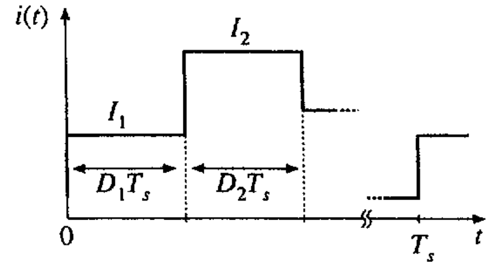

Fig. A1.11.

A1.2 GENERAL PIECEWISE WAVEFORM

For a periodic waveform composed of n piecewise segments as in Fig. A1.12, the rms value is

$$rms = \sqrt{\sum_{k=1}^{n} D_k u_k} \qquad \text{(A1.12)}$$

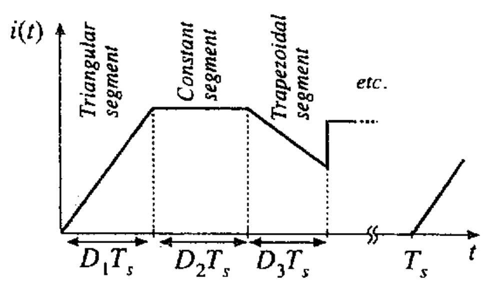

Fig. A1.12 General piecewise waveform.

where D_k is the duty cycle of segment k, and u_k is the contribution of segment k. The u_ks depend on the shape of the segments —several common segment shapes are listed below:

Constant segment, Fig. A1.13:

$$u_k = I_1^2 \qquad \text{(A1.13)}$$

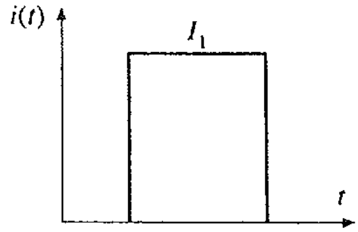

Fig. A1.13.

Triangular segment, Fig. A1.14:

$$u_k = \frac{1}{3} I_1^2 \tag{A1.14}$$

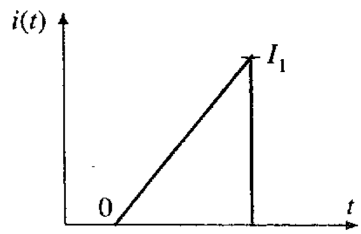

Fig. A1.14.

Trapezoidal segment, Fig. A1.15:

$$u_k = \frac{1}{3}\left(I_1^2 + I_1 I_2 + I_2^2\right) \tag{A1.15}$$

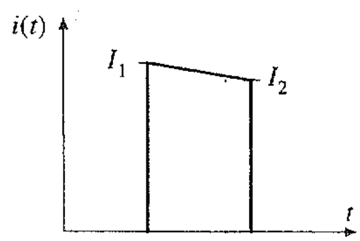

Fig. A1.15.

Sinusoidal segment, half or full period, Fig. A1.16:

$$u_k = \frac{1}{2} I_{pk}^2 \tag{A1.16}$$

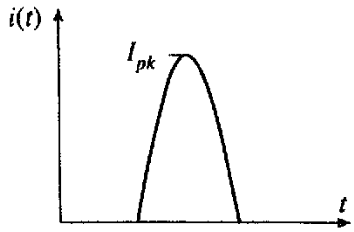

Fig. A1.16.

Sinusoidal segment, partial period: as in Fig. A1.17, a sinusoidal segment of less than one half-period, which begins at angle θ_1 and ends at angle θ_2. The angles θ_1 and θ_2 are expressed in radians:

$$u_k = \frac{1}{2} I_{pk}^2 \left(1 - \frac{\sin(\theta_2 - \theta_1)\cos(\theta_2 + \theta_1)}{(\theta_2 - \theta_1)} \right) \tag{A1.17}$$

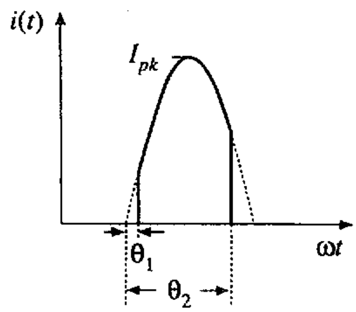

Fig. A1.17.

Example

A transistor current waveform contains a current spike due to the stored charge of a freewheeling diode. The observed waveform can be approximated as shown in Fig. A1.18. Estimate the rms current.

The waveform can be divided into six approximately linear segments, as shown. The D_k and u_k for each segment are

1. Triangular segment:

$$D_1 = (0.2\ \mu s)/(10\ \mu s) = 0.02$$

$$u_1 = I_1^2/3 = (20\ A)^2/3 = 133\ A^2$$

2. Constant segment:

$$D_2 = (0.2\ \mu s)/(10\ \mu s) = 0.02$$

$$u_2 = I_1^2 = (20\ A)^2 = 400\ A^2$$

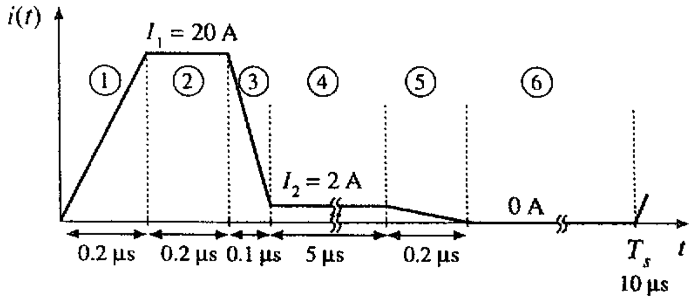

Fig. A1.18 Example: an approximate transistor current waveform, including estimated current spike due to diode stored charge.

3. Trapezoidal segment:

$$D_3 = (0.1\ \mu s)/(10\ \mu s) = 0.01$$

$$u_3 = (I_1^2 + I_1 I_2 + I_2^2)/3 = 148\ A^2$$

4. Constant segment:

$$D_4 = (5\ \mu s)/(10\ \mu s) = 0.5$$

$$u_4 = I_2^2 = (2\ A)^2 = 4\ A^2$$

5. Triangular segment:

$$D_5 = (0.2\ \mu s)/(10\ \mu s) = 0.02$$

$$u_5 = I_2^2/3 = (2\ A)^2/3 = 1.3\ A^2$$

6. Zero segment:

$$u_6 = 0$$

The rms value is

$$rms = \sqrt{\sum_{k=1}^{6} D_k u_k} = 3.76\ A \qquad (A1.18)$$

Even though its duration is very short, the current spike has a significant impact on the rms value of the current —without the current spike, the rms current is approximately 2.0 A.