

# ***Understanding Boost Power Stages in Switchmode Power Supplies***

*Application  
Report*

## IMPORTANT NOTICE

Texas Instruments and its subsidiaries (TI) reserve the right to make changes to their products or to discontinue any product or service without notice, and advise customers to obtain the latest version of relevant information to verify, before placing orders, that information being relied on is current and complete. All products are sold subject to the terms and conditions of sale supplied at the time of order acknowledgement, including those pertaining to warranty, patent infringement, and limitation of liability.

TI warrants performance of its semiconductor products to the specifications applicable at the time of sale in accordance with TI's standard warranty. Testing and other quality control techniques are utilized to the extent TI deems necessary to support this warranty. Specific testing of all parameters of each device is not necessarily performed, except those mandated by government requirements.

CERTAIN APPLICATIONS USING SEMICONDUCTOR PRODUCTS MAY INVOLVE POTENTIAL RISKS OF DEATH, PERSONAL INJURY, OR SEVERE PROPERTY OR ENVIRONMENTAL DAMAGE ("CRITICAL APPLICATIONS"). TI SEMICONDUCTOR PRODUCTS ARE NOT DESIGNED, AUTHORIZED, OR WARRANTED TO BE SUITABLE FOR USE IN LIFE-SUPPORT DEVICES OR SYSTEMS OR OTHER CRITICAL APPLICATIONS. INCLUSION OF TI PRODUCTS IN SUCH APPLICATIONS IS UNDERSTOOD TO BE FULLY AT THE CUSTOMER'S RISK.

In order to minimize risks associated with the customer's applications, adequate design and operating safeguards must be provided by the customer to minimize inherent or procedural hazards.

TI assumes no liability for applications assistance or customer product design. TI does not warrant or represent that any license, either express or implied, is granted under any patent right, copyright, mask work right, or other intellectual property right of TI covering or relating to any combination, machine, or process in which such semiconductor products or services might be or are used. TI's publication of information regarding any third party's products or services does not constitute TI's approval, warranty or endorsement thereof.

---

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Boost Power Stage Steady-State Analysis</b>	<b>2</b>
2.1	Boost Steady-State Continuous Conduction Mode Analysis	2
2.2	Boost Steady-State Discontinuous Conduction Mode Analysis	7
2.3	Critical Inductance	10
<b>3</b>	<b>Boost Power Stage Modeling</b>	<b>12</b>
3.1	Boost Continuous Conduction Mode Small-Signal Analysis	13
3.2	Boost Discontinuous Conduction Mode Small-Signal Analysis	16
<b>4</b>	<b>Component Selection</b>	<b>19</b>
4.1	Output Capacitance	19
4.2	Output Inductance	20
4.3	Power Switch	21
4.4	Output Diode	22
<b>5</b>	<b>Example Design</b>	<b>24</b>
<b>6</b>	<b>Summary</b>	<b>25</b>
<b>7</b>	<b>References</b>	<b>27</b>

### List of Figures

1 Boost Power Stage Schematic .....	2
2 Boost Power Stage States .....	3
3 Continuous Mode Boost Power Stage Waveforms .....	4
4 Boundary Between Continuous and Discontinuous Mode .....	7
5 Discontinuous Current Mode .....	8
6 Discontinuous Mode Boost Power Stage Waveforms .....	10
7 Power Supply Control Loop Components .....	12
8 Boost Nonlinear Power Stage Gain vs. Duty Cycle .....	13
9 DC and Small Signal CCM PWM Switch Model .....	14
10 CCM Boost Power Stage Model .....	14
11 Averaged (Nonlinear) DCM PWM Switch Model .....	16
12 DCM Boost Power Stage DC Model .....	17
13 Small Signal DCM Boost Power Stage Model .....	18

---

# ***Understanding Boost Power Stages in Switchmode Power Supplies***

*Everett Rogers*

---

## **ABSTRACT**

A switching power supply consists of the power stage and the control circuit. The power stage performs the basic power conversion from the input voltage to the output voltage and includes switches and the output filter. This report addresses the boost power stage only and does not cover control circuits. The report includes detailed steady-state and small-signal analysis of the boost power stage operating in continuous and discontinuous modes. A discussion of power stage component requirements is included.

---

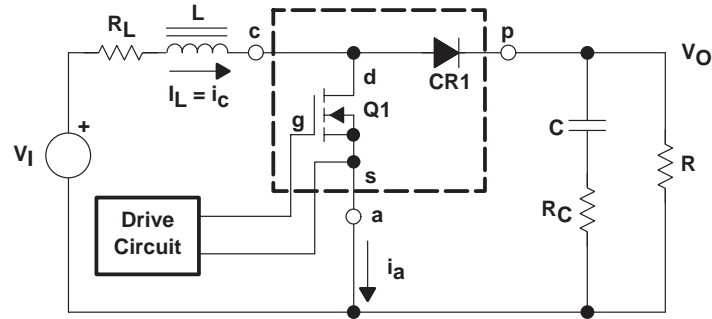
## **1 Introduction**

The three basic switching power supply topologies in common use are the buck, boost, and buck-boost. These topologies are non-isolated, i.e., the input and output voltages share a common ground. There are however, isolated derivations of these non-isolated topologies. The power supply topology refers to how the switches, output inductor, and output capacitor are connected. Each topology has unique properties, including the steady-state voltage conversion ratios, the nature of the input and output currents, and the character of the output voltage ripple. Another important property is the frequency response of the duty-cycle-to-output-voltage transfer function.

The boost is a popular non-isolated power stage topology, sometimes called a step-up power stage. Power supply designers choose the boost power stage because the required output voltage is always higher than the input voltage, is the same polarity, and is not isolated from the input. The input current for a boost power stage is continuous, or non-pulsating, because the input current is the same as the inductor current. The output current for a boost power stage is discontinuous, or pulsating, because the output diode conducts only during a portion of the switching cycle. The output capacitor supplies the entire load current for the rest of the switching cycle.

This application report describes steady-state operation and gives ideal waveforms for the boost converter in continuous and discontinuous modes. The duty-cycle-to-output-voltage transfer function is given using the PWM switch model.

Figure 1 shows a simplified schematic of the boost power stage with a drive circuit block included. Power switch Q1 is an n-channel MOSFET. The output diode is CR1. Inductor L and capacitor C make up the effective output filter. The capacitor equivalent series resistance (ESR),  $R_C$ , and the inductor dc resistance,  $R_L$ , are included in the analysis. Resistor R represents the load seen by the power supply output.



**Figure 1. Boost Power Stage Schematic**

During normal operation of the boost power stage, Q1 is repeatedly switched on and off with the on and off times governed by the control circuit. This switching action creates a train of pulses at the junction of Q1, CR1, and L. Although inductor L is connected to output capacitor C only when CR1 conducts, an effective L/C output filter is formed. It filters the train of pulses to produce a dc output voltage,  $V_O$ . The following sections give a more detailed quantitative analysis.

## 2 Boost Power Stage Steady-State Analysis

A power stage can operate in continuous or discontinuous inductor current mode. In continuous inductor current mode, current flows continuously in the inductor during the entire switching cycle in steady-state operation. In discontinuous inductor current mode, inductor current is zero for a portion of the switching cycle. It starts at zero, reaches a peak value, and returns to zero during each switching cycle. The two modes are discussed in greater detail later, and design guidelines are given for the inductor value to maintain a chosen mode of operation as a function of rated load. It is desirable for a power stage to stay in only one mode over its expected operating conditions because the power stage frequency response changes significantly between the two modes of operation.

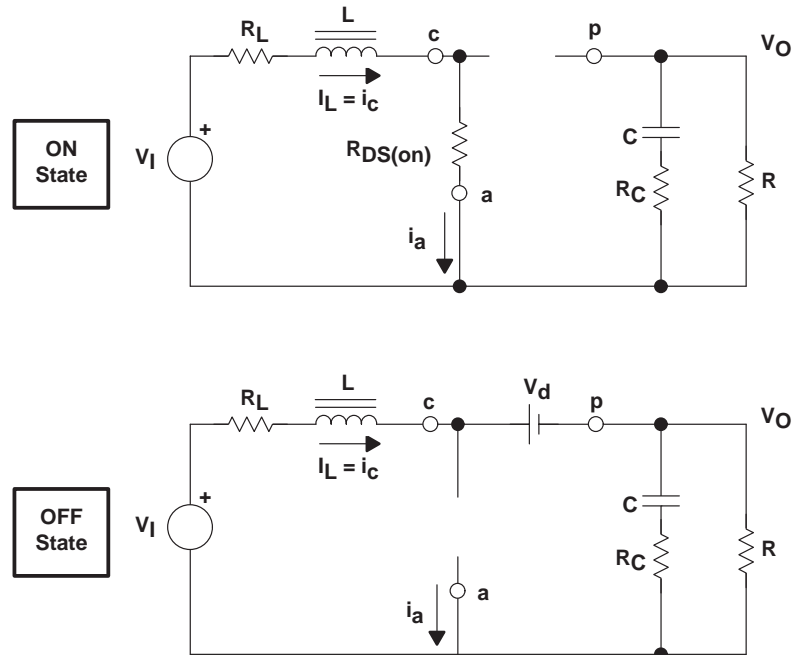
For this analysis, an n-channel power MOSFET is used, and a positive voltage,  $V_{GS(on)}$ , is applied from the gate to the source terminals of Q1 by the drive circuit to turn on the MOSFET. The advantages of using an n-channel MOSFET are its lower  $R_{DS(on)}$  (compared to a p-channel MOSFET), and the ease of driving it in a boost power stage configuration.

Transistor Q1 and diode CR1 are drawn inside a dashed-line box with terminals labeled a, p, and c. This is explained in the *Boost Power Stage Modeling* section.

### 2.1 Boost Steady-State Continuous Conduction Mode Analysis

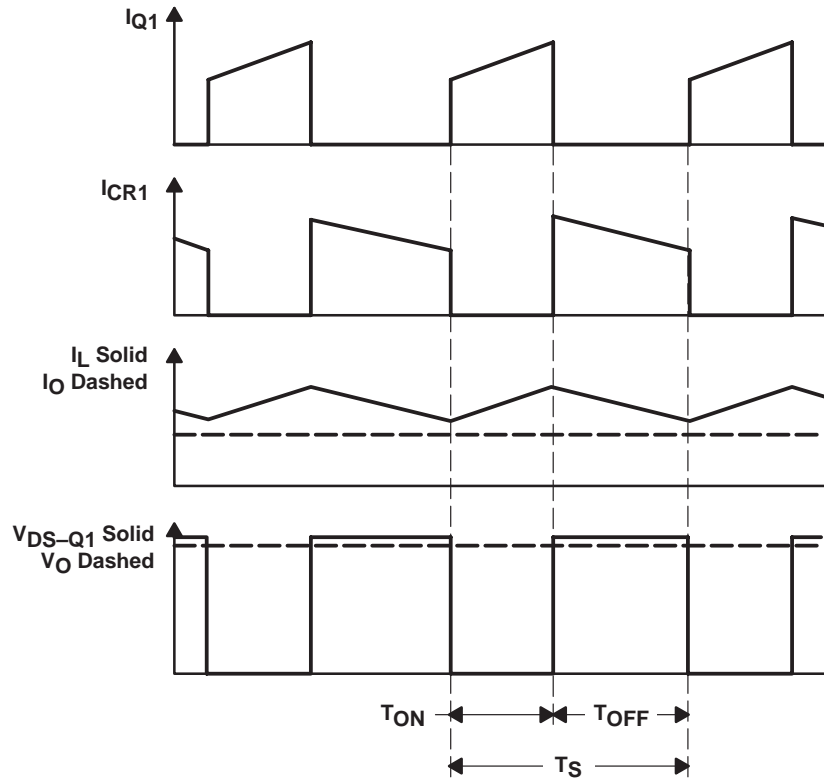
The following is a description of steady-state operation in continuous conduction mode. The main result of this section is a derivation of the voltage conversion relationship for the continuous conduction mode boost power stage. This result is important because it shows how the output voltage depends on duty cycle and input voltage, or how, conversely, the duty cycle can be calculated based on input and output voltages. Steady state implies that the input voltage, output voltage, output load current, and duty-cycle are fixed and not varying. Capital letters are generally given to variable names to indicate a steady-state quantity.

In continuous conduction mode, the boost power stage assumes two states per switching cycle. In the on state, Q1 is on and CR1 is off. In the off state, Q1 is off and CR1 is on. A simple linear circuit can represent each of the two states where the switches in the circuit are replaced by their equivalent circuit during each state. Figure 2 shows the linear circuit diagram for each of the two states.



**Figure 2. Boost Power Stage States**

The duration of the on state is  $D \times T_S = T_{ON}$ , where  $D$  is the duty cycle set by the control circuit, expressed as a ratio of the switch on time to the time of one complete switching cycle,  $T_S$ . The duration of the off state is  $T_{OFF}$ . Since there are only two states per switching cycle for continuous conduction mode,  $T_{OFF}$  is equal to  $(1-D) \times T_S$ . The quantity  $(1-D)$  is sometimes called  $D'$ . These times are shown along with the waveforms in Figure 3.



**Figure 3. Continuous Mode Boost Power Stage Waveforms**

Refer to Figures 1 and 2. During the on state, Q1, which presents a low drain-to-source resistance,  $R_{DS(on)}$ , has a small voltage drop of  $V_{DS}$ . There is also a small voltage drop across the dc resistance of the inductor equal to  $I_L \times R_L$ . Thus the input voltage,  $V_I$ , minus losses,  $(V_{DS} + I_L \times R_L)$ , is applied across inductor L. Diode CR1 is off during this time because it is reverse biased. The voltage applied to the right side of L is the MOSFET on voltage,  $V_{DS}$ . The inductor current,  $I_L$ , flows from the input source,  $V_I$ , through Q1 to ground. During the on state, the voltage across the inductor is constant and equal to  $V_I - (V_{DS} + I_L \times R_L)$ . Adopting the polarity convention for  $I_L$  shown in Figure 2, the inductor current increases as a result of the applied voltage. Also, since the applied voltage is essentially constant, the inductor current increases linearly.

The inductor-current increase can be calculated by using a version of the familiar relationship:

$$V_L = L \times \frac{di_L}{dt} \Rightarrow \Delta I_L = \frac{V_L}{L} \times \Delta T$$

The inductor current increase during the on state is given by:

$$\Delta I_L (+) = \frac{V_I - (V_{DS} + I_L \times R_L)}{L} \times T_{ON}$$

The quantity  $\Delta I_L (+)$  is the inductor ripple current. During this period, all of the output load current is supplied by output capacitor C.



Refer to Figure 1 and Figure 2. When Q1 is off, it presents a high drain-to-source impedance. Therefore, since the current flowing in inductor L cannot change instantaneously, the current shifts from Q1 to CR1. Due to the decreasing inductor current, the voltage across the inductor reverses polarity until rectifier CR1 becomes forward biased and turns on. The voltage applied to the left side of L remains the same as before at  $V_I - I_L \times R_L$ . The voltage applied to the right side of L is now the output voltage,  $V_O$ , plus the diode forward voltage,  $V_d$ . The inductor current,  $I_L$ , now flows from the input source,  $V_I$ , through CR1 to the output capacitor and load resistor combination. During the off state, the voltage across the inductor is constant and equal to  $(V_O + V_d + I_L \times R_L) - V_I$ . Maintaining the same polarity convention, this applied voltage is negative (or opposite in polarity from the applied voltage during the on time). Hence, the inductor current decreases during the off time. Also, since the applied voltage is essentially constant, the inductor current decreases linearly.

The inductor current decrease during the off state is given by:

$$\Delta I_L (-) = \frac{(V_O + V_d + I_L \times R_L) - V_I}{L} \times T_{OFF}$$

The quantity  $\Delta I_L (-)$  is also the inductor ripple current.

In steady-state conditions, the current increase,  $\Delta I_L (+)$ , during the on time and the current decrease,  $\Delta I_L (-)$ , during the off time are equal. Otherwise the inductor current would have a net increase or decrease from cycle to cycle which would not be a steady state condition. Therefore, these two equations can be equated and solved for  $V_O$  to obtain the continuous conduction mode boost voltage conversion relationship:

The steady-state equation for  $V_O$  is:

$$V_O = (V_I - I_L \times R_L) \times \left(1 + \frac{T_{ON}}{T_{OFF}}\right) - V_d - V_{DS} \times \left(\frac{T_{ON}}{T_{OFF}}\right)$$

And, using  $T_S$  for  $T_{ON} + T_{OFF}$ , and using  $D = T_{ON} / T_S$  and  $(1-D) = T_{OFF} / T_S$ , the steady-state equation for  $V_O$  is:

$$V_O = \frac{V_I - I_L \times R_L}{1-D} - V_d - V_{DS} \times \frac{D}{1-D}$$

Notice that in simplifying the above,  $T_{ON} + T_{OFF}$  is assumed to be equal to  $T_S$ . This is true only for continuous conduction mode, as the discontinuous conduction mode analysis will show.

**NOTE:** An important observation: Setting the two values of  $\Delta I_L$  equal to each other is equivalent to balancing the volt-seconds on the inductor. The volt-seconds applied to the inductor is the product of the voltage applied and the time that it is applied. This is the best way to calculate unknown values such as  $V_O$  or  $D$  in terms of known circuit parameters, and this method will be used repeatedly in this report. Volt-second balance on the inductor is a physical necessity, and should be understood at least as well as Ohms Law.

In the above equations for  $\Delta I_L (+)$  and  $\Delta I_L (-)$ , the output voltage was implicitly assumed to be constant with no ac ripple voltage during the on and off times. This is a common simplification and involves two separate effects. First, the output capacitor is assumed to be large enough so that its voltage change is negligible. Second, the voltage due to the capacitor ESR is assumed to be negligible. These assumptions are valid because the ac ripple voltage is designed to be much less than the dc part of the output voltage.

The above voltage conversion relationship for  $V_O$  illustrates that  $V_O$  can be adjusted by adjusting the duty cycle,  $D$ , and is always greater than the input because  $D$  is a number between 0 and 1. This relationship approaches one as  $D$  approaches zero and increases without bound as  $D$  approaches one. A common simplification is to assume  $V_{DS}$ ,  $V_d$ , and  $R_L$  are small enough to ignore. Setting  $V_{DS}$ ,  $V_d$ , and  $R_L$  to zero, the above equation simplifies considerably to:

$$V_O = \frac{V_I}{1-D}$$

A simplified, qualitative way to visualize the circuit operation is to consider the inductor as an energy storage element. When Q1 is on, energy is added to the inductor. When Q1 is off, the inductor and the input voltage source deliver energy to the output capacitor and load. The output voltage is controlled by setting the on time of Q1. For example, by increasing the on time of Q1, the amount of energy delivered to the inductor is increased. More energy is then delivered to the output during the off time of Q1 resulting in an increase in the output voltage.

Unlike the buck power stage, the average of the inductor current is not equal to the output current. To relate the inductor current to the output current, refer to Figure 2 and Figure 3. Note that the inductor delivers current to the output only during the off state of the power stage. This current averaged over a complete switching cycle is equal to the output current because the average current in the output capacitor must be equal to zero.

The relationship between the average inductor current and the output current for the continuous mode boost power stage is given by:

$$I_{L(Avg)} \times \frac{T_{OFF}}{T_S} = I_{L(Avg)} \times (1-D) = I_O$$

or,

$$I_{L(Avg)} = \left( \frac{I_O}{1-D} \right)$$

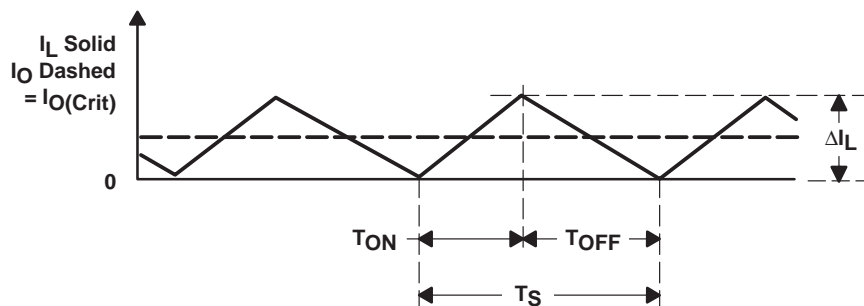
Another important observation is that the average inductor current is proportional to the output current, and since the inductor ripple current,  $\Delta I_L$ , is independent of output load current, the minimum and the maximum values of the inductor current track the average inductor current exactly. For example, if the average inductor current decreases by 2 A due to a load current decrease, then the minimum and maximum values of the inductor current decrease by 2 A (assuming continuous conduction mode is maintained).

The foregoing analysis was for the boost power stage operation in continuous inductor current mode. The next section is a description of steady-state operation in discontinuous conduction mode. The main result is a derivation of the voltage conversion relationship for the discontinuous conduction mode boost power stage.

## 2.2 Boost Steady-State Discontinuous Conduction Mode Analysis

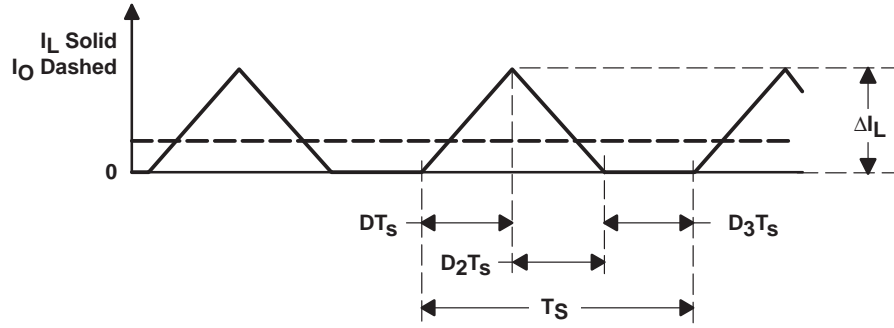
Now consider what happens when the load current is decreased and the conduction mode changes from continuous to discontinuous. Recall that for continuous conduction mode, the average inductor current tracks the output current, i.e., if the output current decreases, then so does the average inductor current. In addition, the minimum and maximum peaks of the inductor current follow the average inductor current exactly.

If the output load current is reduced below the critical current level, the inductor current will be zero for a portion of the switching cycle. This is evident from the waveforms shown in Figure 3, since the peak-to-peak amplitude of the ripple current does not change with output load current. In a boost power stage, if the inductor current attempts to fall below zero, it just stops at zero (due to the unidirectional current flow in CR1) and remains there until the beginning of the next switching cycle. This operating mode is discontinuous current mode. A power stage operating in discontinuous mode has three unique states during each switching cycle as opposed to two states for continuous mode. Figure 4 shows the inductor current condition where the power stage is at the boundary between continuous and discontinuous mode. This is where the inductor current just falls to zero and the next switching cycle begins immediately after the current reaches zero.



**Figure 4. Boundary Between Continuous and Discontinuous Mode**

Further reduction in output load current puts the power stage into discontinuous current conduction mode. Figure 5 shows this condition. The discontinuous mode power stage frequency response is quite different from the continuous mode frequency response and is given in the *Boost Power Stage Modeling* section. Also, the input-to-output relationship is quite different, as the following derivation shows.



**Figure 5. Discontinuous Current Mode**

To begin the derivation of the discontinuous current mode boost power stage voltage conversion ratio, recall that there are three unique states that the power stage assumes during discontinuous current mode operation. The on state is when Q1 is on and CR1 is off. The off state is when Q1 is off and CR1 is on. The idle state is when both Q1 and CR1 are off. The first two states are identical to the continuous mode case, and the circuits of Figure 9 are applicable except that  $T_{OFF} \neq (1-D) \times T_S$ . The remainder of the switching cycle is the idle state. In addition, the dc resistance of the output inductor, the output diode forward voltage drop, and the power MOSFET on-state voltage drop are all assumed to be small enough to omit.

The duration of the on state is  $T_{ON} = D \times T_S$ , where  $D$  is the duty cycle set by the control circuit, expressed as a ratio of the switch on time to the time of one complete switching cycle,  $T_S$ . The duration of the off state is  $T_{OFF} = D_2 \times T_S$ . The idle time is the remainder of the switching cycle and is given as  $T_S - T_{ON} - T_{OFF} = D_3 \times T_S$ . These times are shown with the waveforms in Figure 6.

Without going through the detailed explanation as before, the equations for the inductor current increase and decrease are given below.

The inductor current increase during the on state is given by:

$$\Delta I_L (+) = \frac{V_I}{L} \times T_{ON} = \frac{V_I}{L} \times D \times T_S = I_{PK}$$

The ripple current magnitude,  $\Delta I_L (+)$ , is also the peak inductor current,  $I_{pk}$  because in discontinuous mode, the current starts at zero each cycle.

The inductor current decrease during the off state is given by:

$$\Delta I_L (-) = \frac{V_O - V_I}{L} \times T_{OFF} = \frac{V_O - V_I}{L} \times D_2 \times T_S$$

As in the continuous conduction mode case, the current increase,  $\Delta I_L (+)$ , during the on time and the current decrease during the off time,  $\Delta I_L (-)$ , are equal. Therefore, these two equations can be equated and solved for  $V_O$  to obtain the first of two equations to be used to solve for the voltage conversion ratio:

$$V_O = V_I \times \frac{T_{ON} + T_{OFF}}{T_{OFF}} = V_I \times \frac{D + D2}{D2}$$

Now calculate the output current (the output voltage  $V_O$  divided by the output load  $R$ ). It is the average over the complete switching cycle of the inductor current during the  $D2$  interval.

$$I_O = \frac{V_O}{R} = \frac{1}{T_S} \times \left[ \frac{1}{2} \times I_{PK} \times D2 \times T_S \right]$$

Now, substitute the relationship for  $I_{PK}$  ( $\Delta I_L$ ) into the above equation to obtain:

$$I_O = \frac{V_O}{R} = \frac{1}{T_S} \times \left[ \frac{1}{2} \times \left( \frac{V_I}{L} \times D \times T_S \right) \times D2 \times T_S \right]$$

$$I_O = \frac{V_O}{R} = \frac{V_I \times D \times D2 \times T_S}{2 \times L}$$

We now have two equations, one for the output current just derived, and one for the output voltage, both in terms of  $V_I$ ,  $D$ , and  $D2$ . Now solve each equation for  $D2$  and set the two equations equal to each other. The resulting equation can be used to derive an expression for the output voltage,  $V_O$ .

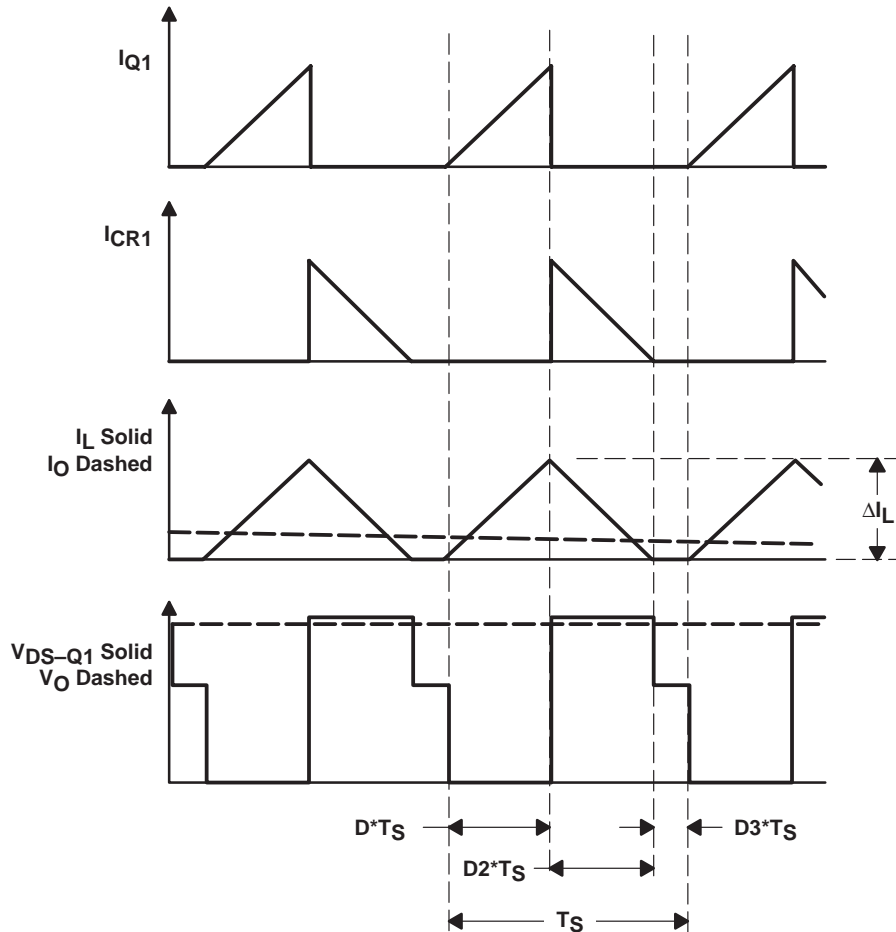
The discontinuous conduction mode boost voltage conversion relationship is given by:

$$V_O = V_I \times \frac{1 + \sqrt{1 + \frac{4 \times D^2}{K}}}{2}$$

Where  $K$  is defined as:

$$K = \frac{2 \times L}{R \times T_S}$$

The above relationship shows one of the major differences between the two conduction modes. For discontinuous conduction mode, the voltage conversion relationship is a function of the input voltage, duty cycle, power stage inductance, switching frequency, and output load resistance; for continuous conduction mode, the voltage conversion relationship is only dependent on the input voltage and duty cycle.



**Figure 6. Discontinuous Mode Boost Power Stage Waveforms**

In typical applications, the boost power stage is operated in either continuous conduction mode or discontinuous conduction mode. For a particular application, one conduction mode is chosen and the power stage is designed to maintain the same mode. The next section gives relationships for the power stage inductance to allow it to operate in only one conduction mode, given ranges for input and output voltage and output load current.

### 2.3 Critical Inductance

The previous analyses for the boost power stage have been for continuous and discontinuous conduction modes of steady-state operation. The conduction mode of a power stage is a function of input voltage, output voltage, output current, and the value of the inductor. A boost power stage can be designed to operate in continuous mode for load currents above a certain level usually 5 to 10% of full load. Usually, the input voltage range, output voltage, and load current are defined by the power stage specification. This leaves the inductor value as the design parameter to maintain continuous conduction mode.

The minimum value of inductor to maintain continuous conduction mode can be determined by the following procedure.

First, define  $I_{O(Crit)}$  as the minimum output current to maintain continuous conduction mode, normally referred to as the critical current. This value is shown in Figure 4. Since we are working toward a minimum value for the inductor, it is more straightforward to perform the derivation using the inductor current. The minimum average inductor current to maintain continuous conduction mode is given by:

$$I_{(min-avg)} = \frac{\Delta I_L}{2}$$

Second, calculate L such that the above relationship is satisfied. To solve the above equation, either relationship,  $\Delta I_L (+)$  or  $\Delta I_L (-)$ , may be used for  $\Delta I_L$ . Note also that either relationship for  $\Delta I_L$  is independent of the output current level. Here,  $\Delta I_L (+)$  is used. The worst case condition for the boost power stage (giving the largest  $L_{min}$ ) is at an input voltage equal to one-half of the output voltage because this gives the maximum  $\Delta I_L$ .

Now, substituting and solving for  $L_{min}$ :

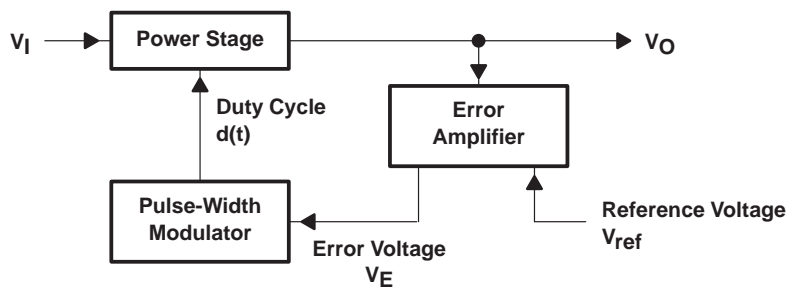
$$L_{min} \geq \frac{1}{2} \times (V_I + V_{DS} - I_L \times R_L) \times \frac{T_{ON}}{I_{L(min)}}$$

The above equation can be simplified and put in a form that is easier to apply as shown:

$$L_{min} \geq \frac{V_O \times T_S}{16 \times I_{O(crit)}}$$

Using the inductor value just calculated ensures continuous conduction mode operation for output load currents above the critical current level,  $I_{O(crit)}$ .

### 3 Boost Power Stage Modeling



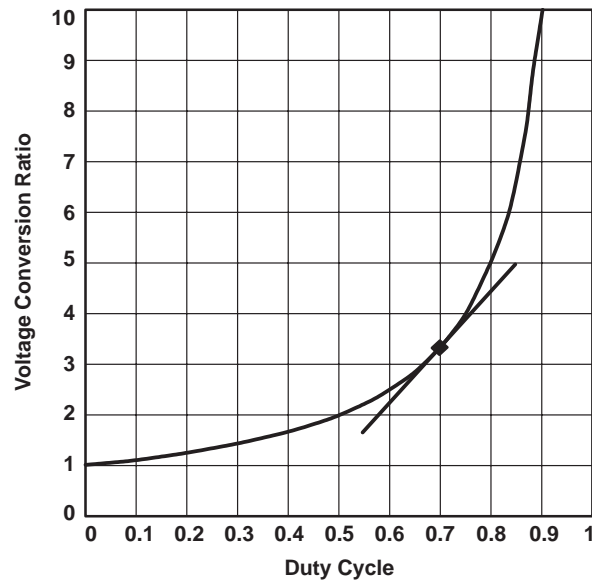
**Figure 7. Power Supply Control Loop Components**

Modeling the power stage presents one of the main challenges to the power supply designer. A popular technique involves modeling only the switching elements of the power stage. An equivalent circuit for these elements is derived and is called the PWM switch model where PWM is the abbreviation for pulse width modulated. This approach is presented here.

As shown in Figure 7, the power stage has two inputs: the input voltage and the duty cycle. The duty cycle is the control input, i.e., this input is a logic signal which controls the switching action of the power stage and hence the output voltage. Most power stages have a nonlinear voltage conversion ratio versus duty cycle. Figure 8 illustrates this nonlinearity with a graph of the steady-state voltage conversion ratio for a boost power stage operating in continuous conduction mode as a function of steady-state duty cycle,  $D$ .

The nonlinear characteristics are a result of the switching action of the power stage switching components, Q1 and CR1. Vorperian [5] observed that the only nonlinear components in a power stage are the switching devices; the remainder of the circuit consists of linear elements. It was also shown in reference [5] that a linear model of only the nonlinear components could be derived by averaging the voltages and currents associated with these nonlinear components over one switching cycle. The model is then substituted into the original circuit for analysis of the complete power stage. Thus, a model of the switching devices is given and is called the PWM switch model.





**Figure 8. Boost Nonlinear Power Stage Gain vs. Duty Cycle**

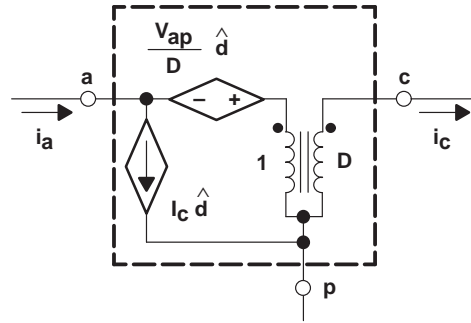
The basic objective of modeling power stages is to represent the ac behavior at a given operating point and to be linear around the operating point. Linearity allows the use of the many analysis tools available for linear systems. Referring again to Figure 8, choosing the operating point at  $D = 0.7$ , allows a straight line to be constructed that is tangent to the original curve at the point where  $D = 0.7$ . This is an illustration of linearization about an operating point, a technique used in deriving the PWM switch model. Qualitatively, if the variations in duty cycle are kept small, a linear model accurately represents the nonlinear behavior of the power stage being analyzed.

Since power stages can operate in continuous conduction mode (CCM) or discontinuous conduction mode (DCM), the PWM switch models for the two conduction modes are explained below.

### 3.1 Boost Continuous Conduction Mode Small-Signal Analysis

To model the boost power stage operation in CCM, use the CCM PWM switch model derived in the application report *Understanding Buck Power Stages in Switchmode Power Supplies*, TI Literature Number SLVA057. The PWM switch model is inserted into the power stage circuit by replacing the switching elements. The CCM PWM switch model is shown in Figure 9. This model is useful for determining the DC operating point of a power stage and for finding ac transfer functions of a power stage.

In Figure 1, the power transistor, Q1, and the catch diode, CR1, are drawn inside a dashed-line box. These are the components that are replaced by the PWM switch equivalent circuit. Terminals a, p, and c are terminal labels of the PWM switch model.



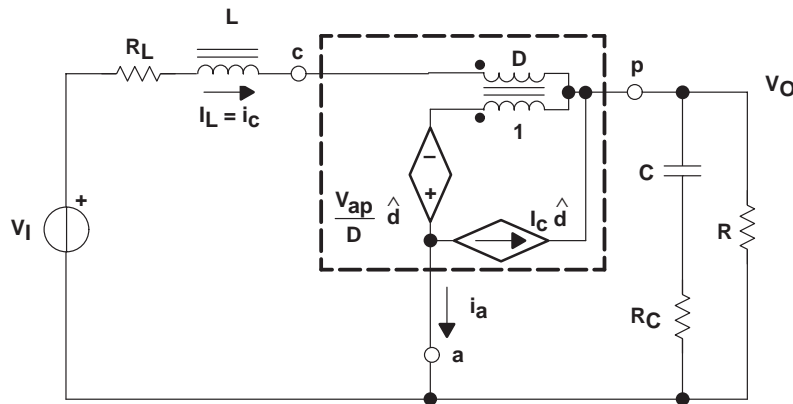
**Figure 9. DC and Small Signal CCM PWM Switch Model**

Terminal a (active) connects to the active switch. Similarly, terminal p (passive) connects to the passive switch. Lastly, terminal c (common) is common to both the active and passive switches. All three commonly used power stage topologies contain active and passive switches and the above terminal definitions can be applied. In addition, substituting the PWM switch model into other power stage topologies also produces a valid model for that particular power stage. To use the PWM switch model in other power stages, just substitute the model shown in Figure 9 into the power stage in the appropriate orientation.

In the PWM switch model of Figure 9 and subsequent occurrences of the model, the capital letters indicate steady-state (or dc) quantities dependent on the operating point of the circuit under study. The lowercase letters indicate time-variable quantities that can have dc and ac components. The lowercase letters with a caret (hat) indicate the small ac variations of that particular variable.

For example,  $D$  represents the steady-state duty cycle,  $\hat{d}$  represents small ac variations of the duty cycle, and  $d$  or  $d(t)$  represents the complete duty cycle including any dc component and ac variations.

The PWM switch model of Figure 9 is inserted into the boost power stage schematic of Figure 1 by replacing transistor Q1 and output diode CR1 with the model. Figure 10 shows the resulting model for the boost power stage. Examples of dc analysis and ac small-signal analysis are given.



**Figure 10. CCM Boost Power Stage Model**

An example dc analysis follows to illustrate how simple power stage analysis becomes with the PWM switch model. For dc analysis,  $\hat{d}$  is zero, L is a short, and C is open. Then a simple loop equation gives:

$$-V_I + I_C \times R_L + V_{cp} + V_O = 0$$

but

$$V_{cp} = V_{ap} \times D = -V_O \times D$$

and

$$I_O = \frac{V_O}{R} = I_C - I_a = I_C - I_C \times D \Rightarrow I_C = \frac{V_O}{R} \times \frac{1}{1-D}$$

Substituting the above relationships for  $V_{cp}$  and  $I_C$  into the first equation and solving for  $V_O$ , gives:

$$V_O = V_I \times \frac{1}{1-D} \times \frac{1}{1 + \frac{R_L}{R \cdot (1-D)^2}}$$

The above equation is usually expressed as a ratio of the output voltage,  $V_O$ , to the input voltage,  $V_I$ , and is usually called  $M$  as shown:

$$M = \frac{V_O}{V_I} = \frac{1}{1-D} \times \frac{1}{1 + \frac{R_L}{R \cdot (1-D)^2}}$$

An ac analysis can not be performed until after the dc analysis is completed because PWM switch parameters  $V_{ap}$  and  $I_C$  must be determined from the dc analysis. For ac analysis, the following transfer functions can be calculated: open-loop line-to-output, open-loop input impedance, open-loop output impedance, and open-loop control-to-output. The control-to-output, or duty-cycle-to-output, is the transfer function most used for control loop analysis. To determine this transfer function, first use the results from the dc analysis for operating point information. This information determines the parameter values of the dependent sources; for example:

$$V_{ap} = -V_O$$

and

$$I_C = I_L = \frac{I_O}{1-D} = \frac{V_O}{R \times (1-D)} = \frac{M \times V_I}{R \times (1-D)}$$

These two equations are then used with loop equations to derive the duty-cycle-to-output-voltage transfer function from the circuit shown in Figure 10. Then set the input voltage equal to zero to get only the ac component of the transfer function. Without going through all the details, it can be shown that the transfer function can be expressed as:

$$\frac{\hat{v}_O(s)}{\hat{d}} = G_{do} \times \frac{\left(1 + \frac{s}{\omega_{z1}}\right) \times \left(1 - \frac{s}{\omega_{z2}}\right)}{1 + \frac{s}{\omega_o \times Q} + \frac{s^2}{\omega_o^2}}$$

where,

$$G_{do} \approx \frac{V_I}{(1-D)^2}$$

$$\omega_{z1} = \frac{1}{R_C \times C}$$

$$\omega_{z2} \approx \frac{(1-D)^2 \times R - R_L}{L}$$

$$\omega_o \approx \frac{1}{\sqrt{L \times C}} \times \sqrt{\frac{R_L + (1-D)^2 \times R}{R}}$$

$$Q \approx \frac{\omega_o}{\frac{R_L}{L} + \frac{1}{C \times (R + R_C)}}$$

### 3.2 Boost Discontinuous Conduction Mode Small-Signal Analysis

To model the boost power stage operation in discontinuous conduction mode (DCM), follow a similar procedure as above for CCM. A PWM switch model is inserted into the power stage circuit by replacing the switching elements. The derivation for the DCM PWM switch model is given in the application report *Understanding Buck-Boost Power Stages in Switchmode Power Supplies*, TI Literature Number SLVA059. This derivation can also be found in *Fundamentals of Power Electronics*. [4] Figure 11 shows the large signal nonlinear version of the DCM PWM switch model. This model is useful for determining the dc operating point of a power supply. The input port is simply modeled with a resistor,  $R_e$ . The value of  $R_e$  is given by:

$$R_e = \frac{2 \times L}{D^2 \times T_S}$$

The output port is modeled as a dependent power source. This power source delivers power equal to that dissipated by the input resistor,  $R_e$ .

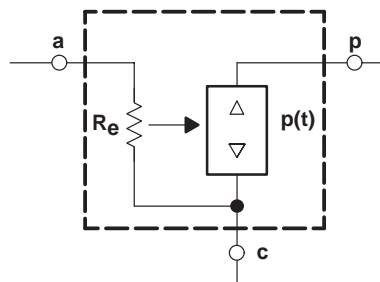
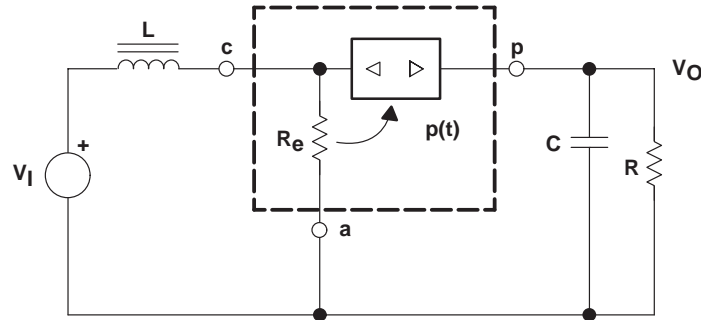


Figure 11. Averaged (Nonlinear) DCM PWM Switch Model

To illustrate discontinuous conduction mode power supply steady-state analysis using this model, examine the boost power stage. The analysis proceeds like the CCM case. The equivalent circuit is substituted into the original circuit. The inductor is treated as a short circuit and the capacitor is treated as an open circuit. Figure 12 shows the DCM boost power stage model schematic.



**Figure 12. DCM Boost Power Stage DC Model**

To illustrate using the model to determine the dc operating point, simply write the equations for the above circuit. This circuit can be described by the network equations shown. First, set the power dissipated in  $R_e$  equal to the power delivered by the dependent power source: The power dissipated by  $R_e$  is:

$$\frac{V_I^2}{R_e}$$

The power delivered by the dependent power source is:

$$V_{p(t)} \times I_{p(t)} = (V_O - V_I) \times \frac{V_O}{R}$$

Setting the two power expressions equal to each other, after rearranging, gives:

$$V_O^2 \times \left(\frac{1}{R}\right) - V_O \times \left(\frac{V_I}{R}\right) = \left(\frac{V_I^2}{R_e}\right) \Rightarrow V_O^2 \times \left(\frac{1}{V_I}\right) - V_O \times (1) - \left(V_I \times \frac{R}{R_e}\right) = 0$$

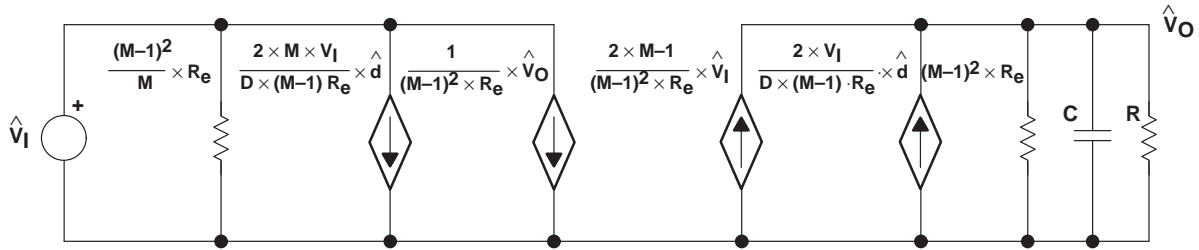
Solving the quadratic for  $V_O$ , taking the larger of the two roots and simplifying, gives the voltage conversion relationship as before.

$$V_O = V_I \times \frac{1 + \sqrt{1 + \frac{4 \times D^2}{K}}}{2}$$

Where

$$K = \frac{2 \times L}{R \times T_S}$$

To derive the small signal model, the circuit of Figure 12 is perturbed and linearized following a procedure similar to the one in the CCM derivation. See reference [4] for the details of the derivation. Figure 13 shows the resulting small signal model for the boost power stage operating in DCM.



**Figure 13. Small Signal DCM Boost Power Stage Model**

The duty-cycle-to-output transfer function for the buck-boost power stage operating in DCM is given by:

$$\frac{\hat{V}_O}{\hat{d}} = G_{do} \times \frac{1}{1 + \frac{s}{\omega_p}}$$

Where

$$G_{do} = 2 \times \frac{V_O}{D} \times \frac{M-1}{2 \times M-1}$$

$$M = \frac{V_O}{V_I}$$

and

$$\omega_p = \frac{2 \times M-1}{(M-1) \times R \times C}$$

## 4 Component Selection

This section discusses the function of the main components of the buck-boost power stage, and gives electrical requirements and applied stresses for each component.

The completed power supply, made up of a power stage and a control circuit, usually must meet a set of minimum performance requirements. This set of requirements is the power supply specification. Many times, the power supply specification determines individual component requirements.

### 4.1 Output Capacitance

In switching power supply power stages, the function of output capacitance is to store energy. The energy is stored in its electric field due to the voltage applied. Thus, qualitatively, the function of a capacitor is to attempt to maintain a constant voltage.

The output capacitance for a boost power stage is generally selected to limit output voltage ripple to the level required by the specification. The series impedance of the capacitor and the power stage output current determine the output voltage ripple. The three elements of the capacitor that contribute to its impedance (and output voltage ripple) are equivalent series resistance (ESR), equivalent series inductance (ESL), and capacitance (C). The following discussion gives guidelines for output capacitor selection.

For continuous inductor current mode operation, to determine the amount of capacitance needed as a function of output load current,  $I_O$ , switching frequency,  $f_S$ , and desired output voltage ripple,  $\Delta V_O$ , the following equation is used assuming all the output voltage ripple is due to the capacitor's capacitance. This is because the output capacitor supplies the entire output load current during the power stage on-state.

$$C \geq \frac{I_{O(Max)} \times D_{Max}}{f_s \times \Delta V_O}$$

where,  $I_{O(Max)}$  is the maximum output current

and  $D_{Max}$  is the maximum duty cycle.

For discontinuous inductor current mode operation, to determine the amount of capacitance needed, the following equation is used assuming all the output voltage ripple is due to the capacitor's capacitance.

$$C \geq \frac{I_{O(Max)} \times \left( 1 - \sqrt{\frac{2 \times L}{R \times T_S}} \right)}{f_s \times \Delta V_O}$$

In many practical designs, to get the required ESR, a capacitor with much more capacitance than is needed must be selected.

For continuous inductor current mode operation and assuming there is enough capacitance such that the ripple due to the capacitance can be ignored, the ESR needed to limit the ripple to  $\Delta V_O$  V peak-to-peak is:

$$ESR \leq \frac{\Delta V_O}{\left( \frac{I_{O(Max)}}{1-D_{Max}} + \frac{\Delta I_L}{2} \right)}$$

For discontinuous inductor current mode operation, and assuming there is enough capacitance such that the ripple due to the capacitance can be ignored, the ESR needed to limit the ripple to  $\Delta V_O$  V peak-to-peak is simply:

$$ESR \leq \frac{\Delta V_O}{\Delta I_L}$$

Ripple current flowing through a capacitor's ESR causes power dissipation in the capacitor. This power dissipation causes a temperature increase internal to the capacitor. Excessive temperature can seriously shorten the expected life of a capacitor. Capacitors have ripple current ratings that are dependent on ambient temperature and should not be exceeded. Referring to Figure 3, the output capacitor ripple current is the output diode current,  $I_{CR1}$ , minus the output current,  $I_O$ . The RMS value of the ripple current flowing in the output capacitance (continuous inductor current mode operation) is given by:

$$I_{CRMS} = I_O \times \sqrt{\frac{D}{1-D}}$$

ESL can be a problem by causing ringing in the low megahertz region but can be controlled by choosing low ESL capacitors, limiting lead length (PCB and capacitor), and replacing one large device with several smaller ones connected in parallel.

Three capacitor technologies: low-impedance aluminum, organic semiconductor, and solid tantalum are suitable for low-cost commercial applications. Low-impedance aluminum electrolytics are the lowest cost and offer high capacitance in small packages, but ESR is higher than the other two. Organic semiconductor electrolytics, such as the Sanyo OS-CON series, have become very popular for the power-supply industry in recent years. These capacitors offer the best of both worlds—a low ESR that is stable over the temperature range, and high capacitance in a small package. Most of the OS-CON units are supplied in lead-mounted radial packages; surface-mount devices are available, but much of the size and performance advantage is sacrificed. Solid-tantalum chip capacitors are probably the best choice if a surface-mounted device is an absolute must. Products such as the AVX TPS family and the Sprague 593D family were developed for power-supply applications. These products offer a low ESR that is relatively stable over the temperature range, high ripple-current capability, low ESL, surge-current testing, and a high ratio of capacitance to volume.

## 4.2 Output Inductance

In switching power supply power stages, the function of inductors is to store energy. The energy is stored in their magnetic field due to the flow of current. Thus, qualitatively, the function of an inductor is usually to attempt to maintain a constant current or, equivalently, to limit the rate of change of current flow.



The value of output inductance of a boost power stage is generally selected to limit the peak-to-peak ripple current flowing in it. In doing so, the power stage's mode of operation, continuous or discontinuous, is determined. The inductor ripple current is directly proportional to the applied voltage and the time that the voltage is applied, and it is inversely proportional to its inductance. This was explained in detail previously.

Many designers prefer to design the inductor themselves but that topic is beyond the scope of this report. However, the following discusses the considerations necessary for selecting the appropriate inductor.

In addition to the inductance, other important factors to be considered when selecting the inductor are its maximum dc or peak current and maximum operating frequency. Using the inductor within its dc current rating is important to insure that it does not overheat or saturate. Operating the inductor at less than its maximum frequency rating insures that the maximum core loss is not exceeded, resulting in overheating or saturation.

Magnetic component manufacturers offer a wide range of off-the-shelf inductors suitable for dc/dc converters, some of which are surface mountable. There are many types of inductors available; the most popular core materials are ferrites and powdered iron. Bobbin or rod-core inductors are readily available and inexpensive, but care must be exercised in using them because they are more likely to cause noise problems than are other shapes. Custom designs are also feasible, provided the volumes are sufficiently high.

Current flowing through an inductor causes power dissipation in the inductor due to its dc resistance; the power dissipation is easily calculated. Power is also dissipated in the inductor's core due to the flux swing caused by the ac voltage applied across it, but this information is rarely directly given in manufacturer's data sheets. Occasionally, the inductor's maximum operating frequency and/or applied volt-seconds ratings give the designer some guidance regarding core loss. The power dissipation causes a temperature increase in the inductor. Excessive temperature can cause degradation in the insulation of the winding and cause increased core loss. Care should be exercised to insure all the inductor's maximum ratings are not exceeded.

The loss in the inductor is given by:

$$P_{\text{Inductor}} = \left( \frac{I_O}{1-D} \right)^2 \times R_{Cu} + P_{\text{Core}}$$

where,  $R_{Cu}$  is the winding resistance.

### 4.3 Power Switch

In switching power supply power stages, the function of the power switch is to control the flow of energy from the input power source to the output voltage. In a boost power stage, the power switch (Q1 in Figure 1) connects the input to the output filter when the switch is turned on and disconnects when the switch is off. The power switch must conduct the current in the inductor while on and block the full output voltage when off. Also, the power switch must change from one state to the other quickly in order to avoid excessive power dissipation during the switching transition.

The type of power switch considered in this report is a power MOSFET. Other power devices are available but in most instances, the MOSFET is the best choice in terms of cost and performance (when the drive circuits are considered). The two types of MOSFET available for use are the n-channel and the p-channel. n-channel MOSFETs are popular for use in boost power stages because driving the gate is simpler than the gate drive required for a p-channel MOSFET.

The power dissipated by the power switch, Q1, is given by:

$$P_{D(Q1)} = \left( \frac{I_o}{1-D} \right)^2 \times R_{DS(on)} \times D + \frac{1}{2} \times (V_o) \times \left( \frac{I_o}{1-D} \right)^2 (t_r + t_f) \times f_s + Q_{Gate} \times V_{GS} \times f_s$$

where  $t_r$  and  $t_f$  are the MOSFET turn-on and turn-off switching times

$Q_{Gate}$  is the MOSFET gate-to-source capacitance

Other than selecting p-channel versus n-channel, other parameters to consider while selecting the appropriate MOSFET are the maximum drain-to-source breakdown voltage,  $V_{(BR)DSS}$ , and the maximum drain current,  $I_{D(Max)}$ .

The MOSFET selected should have a  $V_{(BR)DSS}$  rating greater than the maximum output voltage, and some margin should be added for transients and spikes. The MOSFET selected should also have an  $I_{D(Max)}$  rating of at least two times the maximum inductor current. However, many times the junction temperature is the limiting factor, so the MOSFET junction temperature should also be calculated to make sure that it is not exceeded. The junction temperature can be estimated as follows:

$$T_J = T_A + P_D \times R_{\theta JA}$$

where  $T_A$  is the ambient or heatsink temperature

$R_{\theta JA}$  is the thermal resistance from the MOSFET chip to the ambient air or heatsink.

#### 4.4 Output Diode

The output diode conducts when the power switch turns off and provides a path for the inductor current. Important criteria for selecting the rectifier include: fast switching, breakdown voltage, current rating, low forward-voltage drop to minimize power dissipation, and appropriate packaging. Unless the application justifies the expense and complexity of a synchronous rectifier, the best solution for low-voltage outputs is usually a Schottky rectifier. The breakdown voltage must be greater than the maximum output voltage, and some margin should be added for transients and spikes. The current rating should be at least two times the maximum power stage output current (normally the current rating will be much higher than the output current because power and junction temperature limitations dominate the device selection).

The voltage drop across the diode in a conducting state is primarily responsible for the losses in the diode. The power dissipated by the diode can be calculated as the product of the forward voltage and the output load current. The switching losses which occur at the transitions from conducting to non conducting states are very small compared to conduction losses and are usually ignored.

The power dissipated by the output rectifier is given by:

$$P_{D(Diode)} = V_D \times I_O$$

where  $V_D$  is the forward voltage drop of the output rectifier.

The junction temperature can be estimated as follows:

$$T_J = T_A + P_D \times R_{\Theta JA}$$

## 5 Example Design

An example design using a boost power stage and the TL5001 controller is given in *SLVP088 20 V to 40 V Adjustable Boost Converter Evaluation Module User's Guide*, Texas Instruments Literature Number SLVU004.

## 6 Summary

This application report described and analyzed the operation of the boost power stage. The two modes of operation, continuous conduction mode and discontinuous conduction mode, were examined. Steady-state and small-signal analyses were performed on the boost power stage.

The main results of the steady-state analyses are summarized below.

The voltage conversion relationship for CCM is:

$$V_O = \frac{V_I - I_L \times R_L - V_d - V_{DS}}{1-D} \times \frac{D}{1-D}$$

or, a slightly simpler version:

$$V_O = V_I \times \frac{1}{1-D} \times \frac{1}{1 + \frac{R_L}{R \times (1-D)^2}}$$

which can be simplified to:

$$V_O = V_I \times \frac{1}{1-D}$$

The relationship between the average inductor current and the output current for the continuous mode boost power stage is given by:

$$I_{L(Avg)} = \frac{I_O}{(1-D)}$$

The discontinuous conduction mode boost voltage conversion relationship is given by:

$$V_O = V_I \times \frac{1 + \sqrt{1 + \frac{4 \times D^2}{K}}}{2}$$

Where  $K$  is defined as:

$$K = \frac{2 \times L}{R \times T_S}$$

The major results of the small-signal analyses are summarized below.

The small-signal duty-cycle-to-output transfer function for the boost power stage operating in CCM is given by:

$$\frac{\hat{V}_O(s)}{\hat{d}} = G_{do} \times \frac{\left(1 + \frac{s}{\omega_{z1}}\right) \times \left(1 - \frac{s}{\omega_{z2}}\right)}{1 + \frac{s}{\omega_o \times Q} + \frac{s^2}{\omega_o^2}}$$

where,

$$G_{do} \approx \frac{V_I}{(1-D)^2}$$

$$\omega_{z1} = \frac{1}{R_C \times C}$$

$$\omega_{z2} \approx \frac{(1-D)^2 \times R - R_L}{L}$$

$$\omega_o \approx \frac{1}{\sqrt{L \times C}} \times \sqrt{\frac{R_L + (1-D)^2 \times R}{R}}$$

$$Q \approx \frac{\omega_o}{\frac{R_L}{L} + \frac{1}{C \times (R + R_C)}}$$

The small-signal duty-cycle-to-output transfer function for the boost power stage operating in DCM is given by:

$$\frac{\hat{V}_O}{\hat{d}} = G_{do} \times \frac{1}{1 + \frac{s}{\omega_p}}$$

where

$$G_{do} = 2 \times \frac{V_O}{D} \times \frac{M-1}{2 \times M-1}$$

$$M = \frac{V_O}{V_I}$$

and

$$\omega_p = \frac{2 \times M-1}{(M-1) \times R \times C}$$

Also presented are requirements for the boost power stage components based on voltage and current stresses applied during the operation of the boost power stage.

For further study, an example design and several references are given.

## 7 References

1. Application Report *Designing With The TL5001 PWM Controller*, TI Literature Number SLVA034A.
2. Application Report *Designing Fast Response Synchronous Buck Regulators Using the TPS5210*, TI Literature Number SLVA044.
3. V. Vorperian, R. Tymerski, and F. C. Lee, "Equivalent Circuit Models for Resonant and PWM Switches," *IEEE Transactions on Power Electronics*, Vol. 4, No. 2, pp. 205-214, April 1989.
4. R. W. Erickson, *Fundamentals of Power Electronics*, New York: Chapman and Hall, 1997.
5. V. Vorperian, "Simplified Analysis of PWM Converters Using the Model of the PWM Switch: Parts I and II," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-26, pp. 490-505, May 1990.
6. E. van Dijk, *et al.*, "PWM-Switch Modeling of DC-DC Converters," *IEEE Transactions on Power Electronics*, Vol. 10, No. 6, pp. 659-665, November 1995.
7. G. W. Wester and R. D. Middlebrook, "Low-Frequency Characterization of Switched Dc-Dc Converters," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-9, pp. 376-385, May 1973.
8. R. D. Middlebrook and S. Cuk, "A General Unified Approach to Modeling Switching-Converter Power Stages," *International Journal of Electronics*, Vol. 42, No. 6, pp. 521-550, June 1977.
9. E. Rogers, "Control Loop Modeling of Switching Power Supplies," Proceedings of EETimes Analog and Mixed-Signal Applications Conference, July 13-14, 1998, San Jose, CA.
10. S. Cuk, "Modelling, analysis and design of switching converters," Ph.D. dissertation, California Institute of Technology, Pasadena, CA, Nov. 1976.

