A SIMPLE POWER DIODE MODEL WITH FORWARD AND REVERSE RECOVERY

Cliff L. Ma and Peter O. Lauritzen
Department of Electrical Engineering, FT-10
University of Washington
Seattle, WA 98195

Abstract

The basic diode charge-control model used in SPICE is extended by employing the lumped charge concept of Linvill to derive a set of model equations from simplified device physics. Both forward and reverse recovery phenomena are included as well as emitter recombination. The complete model requires only 7 relative simple equations and three additional device parameters beyond the generic SPICE diode model.

1. Introduction

Modeling of the transient behavior of power diodes is extremely important for power electronic circuit simulation. Yet, the diode models widely used in analog circuit simulators such as SPICE do not contain the equations for forward and reverse recovery[1]. Inclusion of these phenomena is critical for the simulation of switching power loss, conducted EMI or any other waveform sensitive effects.

Existing power diode models can be classified as either macro-circuit models [2] or detailed physical models [3]. The macro-circuit models are composed of electrical equivalent circuits which are not directly related to internal physical processes in the device. Thus, they are normally valid for only a narrow range of circuit operating conditions. In contrast, most physical models include equations for drift and diffusion of electrons and holes. They usually contain many mathematical equations and parameters, and are complicated to incorporate into circuit simulators.

The diode model presented here is a simplified physical model for a high voltage *p-i-n* structure operating in high level injection as is typical for most power diodes. The model is an extension of the basic charge-control diode model using the lumped charge concept of Linvill [4]. The equations for both forward and reverse recovery as well as the emitter recombination are derived from simplified semiconductor charge transport equations.

2. Model Description and Derivation

2.1 Reverse Recovery

Reverse recovery occurs when a forward conducting diode is turned off rapidly, and the internally stored charges cause transient reverse current to flow at high reverse voltage. Figure 1 shows a typical reverse recovery current waveform with an inductive load.

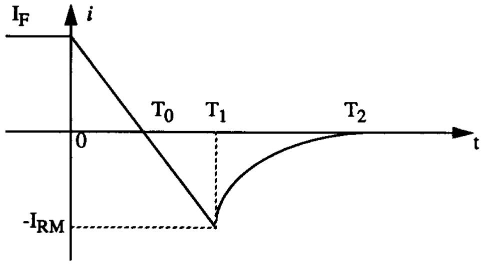

Figure 1. *Turn-off* current waveform with an inductive load

Figure 2 shows the charge distribution in a conducting *p-i-n* diode under forward conduction corresponding to $t \leq 0$ on the curve in Figure 1. The charge distribution in the *i*-region is assigned to four charge storage nodes following the lumped model approach developed by Linvill [4]. Here for simplicity, equal hole and electron mobilities are assumed which makes the charge distributions symmetric and allows the analysis to be simplified to only half of the structure.

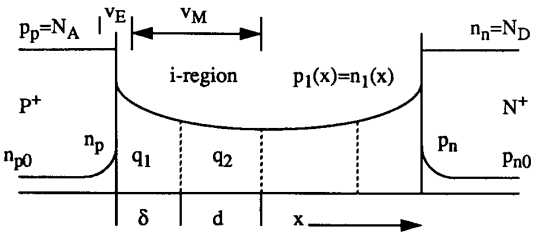

Figure 2. Charge distribution profile in diode forward conduction

The total charge in the left node is $q_1 = qA\delta p_1$, and in the adjacent node is $q_2 = qAdp_2$, where q is the unit electron charge, A the junction area, δ and d the widths of the respective regions, and p_1 and p_2 the average hole concentrations corresponding to q_1 and q_2. According to the ambipolar diffusion equation, the current between the two nodes can be expressed as

$$i(t) = -2qAD_a\frac{dp}{dx} = \frac{2qAD_a(p_1 - p_2)}{\frac{\delta}{2} + \frac{d}{2}} \qquad (1)$$

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where D_a is the ambipolar diffusion constant. When the diode becomes reverse biased, the charge q_1 is exhausted first. To prevent an instantaneous reverse current jump at $t = T_1$, we minimize the charge q_1 by setting $\delta \rightarrow 0$ which makes $q_1 \rightarrow 0$. In equation (2) $q_0 = qAdp_1$ represents the variable remaining after $\delta \rightarrow 0$.

$$i(t) = \frac{q_0 - q_2}{T_{12}} \quad \text{where} \quad T_{12} = \frac{d^2}{4D_a} \tag{2}$$

Here the term T_{12} represents the approximate diffusion transit time across the region q_2. The charge control continuity equation for q_2 is

$$0 = \frac{dq_2}{dt} + \frac{q_2}{\tau} - \frac{q_0 - q_2}{2T_{12}} \tag{3}$$

The first term represents charge storage, the second, recombination with lifetime τ, and the third, diffusion of charge from the p^+i junction into q_2. Finally, the variable q_0 which represents the injected charge level at the junction is related to the junction voltage v_E developed from the standard Law of Junction as follow:

$$q_0 = I_s \tau \left[exp\left(\frac{v_E}{V_T}\right) - 1 \right] \tag{4}$$

Here I_s represents the diode saturation current constant. Equations (2), (3) and (4) are the complete set of equations for this lumped charge model which includes reverse recovery. Further information on this reverse recovery model is given in reference [5].

2.2 Forward Recovery

Forward recovery occurs when a diode switches rapidly from its *off-state* to its *on-state*. During the transient, a high forward voltage builds up across the diode because of initial low conductivity in the *i*-region. As the injected carrier concentration increases, the voltage across the *i*-region soon decreases to the normal steady state diode forward drop. In the model, the injected carrier concentration is determined by the magnitude of q_2 according to the following equation for carrier conduction by drift.

$$i = qA\mu (p_2 + p_{M0}) \frac{v_M}{d} = \mu (q_2 + q_{M0}) \frac{v_M}{d^2} = \frac{(q_2 + q_{M0}) v_M}{4T_{12}V_T} \tag{5}$$

Here v_M is the voltage across half of the *i*-region, μ is the ambipolar mobility, and $q_{M0} = qdAp_{M0}$ is the conduction charge due to *i*-region background doping p_{M0}. To make equation (5) more meaningful for application engineers, q_{M0} is replaced by the initial resistance R_{M0} in the *i*-region, using $q_{M0} = 2V_T T_{12}/R_{M0}$. Thus equation (5) becomes

$$v_M = \frac{2V_T T_{12} R_{M0} i}{q_2 R_{M0} + 2V_T T_{12}} \tag{6}$$

2.3 Emitter Recombination

Under very high current level, the carrier recombination due to injection into the heavily doped p^+ and n^+ emitter regions (end regions in Figure 2) becomes significant and must also be included [6]. The following expression is added to include this effect.

$$i_E = \frac{(n_p - n_{p0}) qAL_p}{\tau_p} = I_{SE}\left[exp\left(\frac{2v_E}{V_T}\right) - 1 \right] \tag{7}$$

Here, i_E is the end region recombination current, n_p the injected electron concentration in the p^+-region, n_{p0} the initial electron concentration, L_p the electron diffusion length, and τ_p is the lifetime which is much smaller than that in the i-region. The constant I_{SE} is derived in the following manner. During high level injection the electron and hole concentrations are equal ($n_1 = p_1$) in the i-region. The end region carrier concentrations n_p, p_p in Figure 2 can be

$$n_p = n_1 exp\left(-\frac{\phi_B - v_E}{V_T}\right) \tag{8}$$

expressed as

$$p_p = p_1 exp\left(\frac{\phi_B - v_E}{V_T}\right) = N_A \tag{9}$$

Where, ϕ_B is the built-in potential. Replace n_1 in equation (8) by the p_1 expression acquired from equation (9) and we obtain equation (10) for n_p. Insert n_p into equation (7) to obtain i_E as a function of v_E. The remaining constants can be represented by I_{SE} as given in equation (11).

$$n_p = N_A exp\left(\frac{-2(\phi_B - v_E)}{V_T}\right) = N_A exp\left(\frac{-2\phi_B}{V_T}\right) exp\left(\frac{2v_E}{V_T}\right) \tag{10}$$

$$I_{SE} = \frac{qAL_p N_A}{\tau_p} exp\left(\frac{2\phi_B}{V_T}\right) \tag{11}$$

Normally, I_{SE} is many orders of magnitude smaller than I_s, because i_E becomes a major part of the total current component only at very high currents, To make $i_E = 0$ in (7) when $v_E = 0$, the "-1" term is added in the equation.

2.4 Complete Model

To complete the model equations, contact resistance and junction capacitance also must be included. This resistance is represented by an internal resistor Rs placed in series as shown in Figure 3.

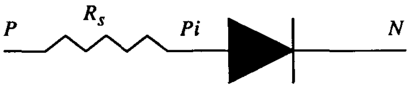

Figure 3. Contact series resistor of the diode

The conventional SPICE junction capacitance model [1] also needs to be added. For v_E less than $\phi_B/4$, equation (12) is used:

$$C_j = \frac{C_j(0)}{\left(1 - \frac{v_E}{\phi_B}\right)^m} \tag{12}$$

Here, $C_j(0)$ is the zero-bias junction capacitance. When the junction voltage v_E is greater than $\phi_B/4$, the capacitance increases linearly with slope equal to that at $v_E = \phi_B/4$, for which equation (13) is employed.

$$C_j = \frac{m}{\phi_B}\frac{C_j(0)\,v_E}{(\frac{1}{2})^{m+1}} - (m-1)\frac{C_j(0)}{(\frac{1}{2})^m} \tag{13}$$

The total stored charge q_j in the capacitor is

$$q_j = \int C_j dv_E \tag{14}$$

Finally, three substitutions enable the complete set of model equations to be collected together in a simple and consistent form. The substitutions are: $q_E=2q_0$, $q_M=2q_2$ *and* $T_M=2T_{12}$.

$$i_M = \frac{q_E - q_M}{T_M} \tag{15}$$

$$0 = \frac{dq_M}{dt} + \frac{q_M}{\tau} - \frac{q_E - q_M}{T_M} \tag{16}$$

$$q_E = I_s\tau\left[exp\left(\frac{v_E}{V_T}\right) - 1\right] \tag{17}$$

$$v_M = \frac{V_T T_M R_{M0} i}{q_M R_{M0} + V_T T_M} \tag{18}$$

$$i_E = I_{SE}\left[exp\left(\frac{2v_E}{V_T}\right) - 1\right] \tag{19}$$

$$v = 2(v_E + v_M) + R_s i \tag{20}$$

$$i = i_E + i_M + \frac{dq_j}{dt} \tag{21}$$

3. Model Demonstration and Discussion

Equations (15) to (21) were installed on the Saber simulator using the MAST modeling language. For simplicity, the parasitic elements are not included in the program(template) which is listed in the Appendix. Some sample simulation results are presented here. The model should also work in other analog circuit simulators which provide for insertion of mathematical equations.

Figure 3 shows the reverse recovery waveforms with the same lifetime τ but two different values for the transit time T_M. The larger T_M, the more slowly the stored charges are removed from the base region. This usually corresponds to a wider base, higher breakdown voltage diode.

A sample forward recovery voltage waveform is shown in Figure 4. The diode's initial low conductivity causes the high forward voltage drop at the beginning of the transient. The peak voltage depends on T_M, τ, *and* R_{MO}.

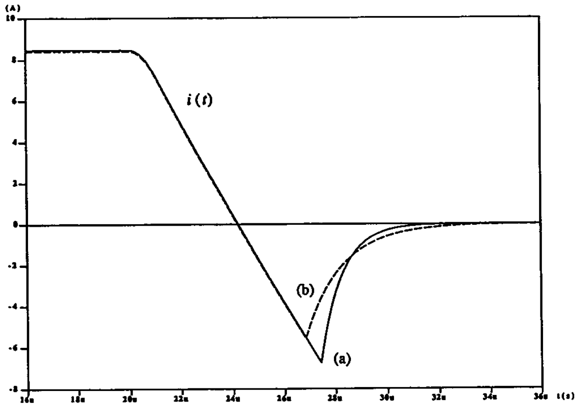

Figure 3. Two reverse recovery waveforms with $\tau=5\mu s$, and (a) $T_M=1\mu s$ (b) $T_M=2\mu s$

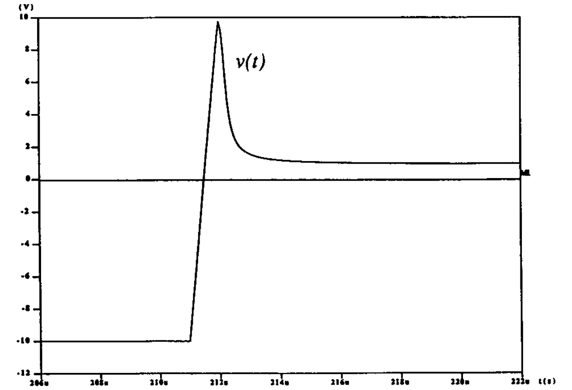

Figure 4. Forward recovery voltage waveform with $R_{M0}=1.3K$, $T_M=5\mu s$ and $\tau=5\mu s$

An examination of the forward bias static *i-v* plot allows one to see more detailed information on the current components, i_E and i_M, and the total current *i*. Figure 5 is a semi-log plot of these currents vs. forward voltage where the model parameters are chosen to make $i_E = i_M = 10A$ (at $i = 20A$). Note, that at low current levels, i_M dominates the total current *i*; but at current levels above *30A*, the end region emitter recombination current i_E becomes the dominant current component.

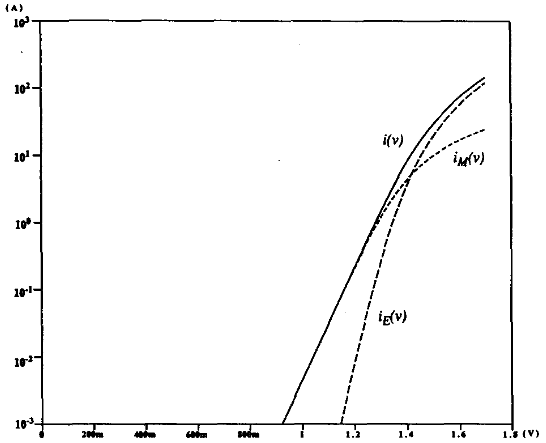

Figure 5. Diode forward *i-v* characteristic showing *i(v)* compared with internal current components $i_E(v)$ and $i_M(v)$. ($I_s=10^{-10}$, $I_{SE}=5\times10^{-22}$, $R_{M0}=0$ and $T_M=\tau=2\mu s$)

Figure 6 is again a static *i-v* plot under the same conditions as Figure 5, but here the internal voltage components v_E, v_M are compared with half of the diode voltage $v/2 = v_E + v_M$. With the current less than *10A*, the *i*-v_E curve is a straight line since v_M is a small constant and $ln(i) \propto v_E$. When the current is greater than *10A*, the slope increases and $ln(i) \propto 2v_E$. However, v_M which is a constant at low currents from *10µA to 10A, now* starts increasing dramatically above 10A due to the emitter recombination effect. Here, the injected carrier concentration in the *i*-region fails to increase as fast as the total current, which causes v_M to rapidly increase. The rapid increase in the external terminal voltage *v* above $i = 10\ A$ is due to the increase in v_M.

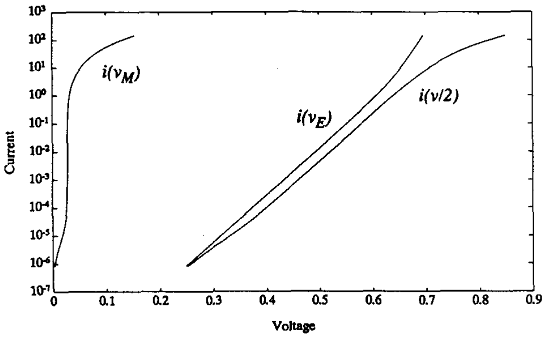

Figure 6. Diode forward *i-v* characteristic showing *i(v/2)* compared with $i(v_E)$ and $i(v_M)$. The same parameters are used as in Figure 5.

4. Determination of Model Parameters

Nine model parameters τ, T_M, I_s, R_{M0}, I_{SE}, R_s, $C_j(0)$, ϕ_B, and *m* need to be determined in the following order. Only three parameters are new: T_M, R_{MO} and I_{SE}; the others are identical to the standard parameters used in the generic SPICE diode model.

1) The parameters τ and T_M can be determined from a diode *turn-off* current waveform such as the typical inductive load waveform shown in Figure 1. Three measurements are needed: the slope of the linear ramp for $0 < t < T_1$, the maximum reverse current I_{RM}, and the reverse recovery time constant τ_{rr} from the exponential curve for $t > T_1$. From these measurements, one can easily calculate τ and T_M using the parameter determination procedure provided in reference [5].

2) The parameter I_s is similar to the diode saturation current I_s used in the SPICE model. Under the static low current condition, $i_M >> i_E$ and $q_M >> V_T T_M/R_{M0}$, from equations (15), (16), (18) and (20), we can derive the following equation to determine I_s from measured *i* and *v*.

$$I_s = \frac{\left(1 + \frac{T_M}{\tau}\right) i}{exp\left(\frac{v}{2V_T} - \frac{T_M}{\tau}\right)} \tag{22}$$

3) The forward recovery parameter R_{MO} can be extracted from the initial slope of the diode forward recovery *i-v* waveform using $R_{MO} = v(t)/i(t)$. If such a measurement is difficult, R_{MO} could be estimated from the *i*-region doping concentration and geometry using

$$R_{M0} = \frac{V_T T_M}{qdAp_{M0}} \tag{23}$$

The parameter R_{MO} should not be set to zero or the static high current *i-v* characteristic will be incorrect.

4) The parameters R_S and I_{SE} should be selected so as to best match the measured high current i-v characteristic.

5) The capacitance parameters $C_j(0)$, *m* and ϕ_B are determined from a standard capacitance-voltage plot in the conventional manner.

5. Conclusion

This *p-i-n* diode model includes complete forward and reverse recovery as well as the high current effects caused by emitter recombination. A major feature of the model is that the same equations are valid through all regions of operation. Except for the capacitance equation, no conditional statements are needed to define regions over which specific equations are valid. The model is simple and practical and should be considered as a new standard diode model for circuit simulation.

A similar modeling approach also can be applied to other conductivity modulated devices, including power devices such as SCRs, GTOs, BJTs and IGBTs.

6. Acknowledgments

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References

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[6] B. Jayant Baliga, *Modern Power Devices*, John Wiley & Sons, Inc., 1987, Chapter 5.

Appendix

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 ###################################################
 #      This is a power P-i-N diode model template #
 #      without parasitic elements.                #
 ###################################################

element template dpin0  p n =model
electrical p,n

struc {             # Default parameter
number Is=1e-12,    # Saturation current
       tau=5e-5,    # Carrier lifetime
       TM=5e-5,     # Transit time
       Ise=1e-22    # High current factor
       } model = ()

{
number k=1.381e-23,  # Boltzman's constant
       q=1.602e-19,  # Electron charge
       Rm0=1k,       # Initial resistance,
       temp=27       # device temperature

val v vd         # Diode voltage
val i ie         # Emitter recombination current
val i im         # Diffusion current from junction
var v ve         # Junction voltage
var v vm         # Base voltage drop
var q qe         # Concentration level at the edge of i-region
var q qm         # Charge at the center of the base region
number vt

# Sample points and Newton steps
struc {number point, inc;}\        # Sample points of ve,vm and qm
sve[*]=[(-100k,100),(-100,.1),(0,10m),(.3,.1m),(1.5,.1),(10,0)],
svm[*]=[(-10000k,100),(-100,.1),(0,.01),(20,0)],
nve[*]=[(0,.1),(2,0)],                #  Newton steps
nvm[*]=[(0,.01),(20,0)],
sqm[*]=[(-1e-7,1e-13),(0,1e-13),(1e-8,1e-10),(1e-6,1e-8),(10,0)]

parameters {
     vt=k*(temp+273)/q
          }

values {
       vd=v(p)-v(n)
       ie=model->Ise*(limexp(2*ve/vt)-1)    # Eqn(19) in the paper
       im=qe/model->TM-qm/model->TM         # Eqn(15) in the paper
        }

control_section {
       sample_points(ve,sve)
       sample_points(vm,svm)
       sample_points(qm,sqm)
       newton_step(ve,nve)
       newton_step(vm,nvm)
       }

equations {
   ve: vd=2*ve+2*vm                                 # Eqn(20)
   qe: qe=model->tau*model->Is*(limexp(ve/vt)-1)    # Eqn(17)
   qm: 0=d_by_dt(qm)+qm/model->tau-qe/model->TM+qm/model->TM # Eqn(16)
   vm: ie+im=qm*vm/vt/model->TM+vm/Rm0              # Eqn(18)
       i(p->n)+=ie+im  # Total current              # Eqn(21)
            }
     }
```