Secondary *LC* Filter Analysis and Design Techniques for Current-Mode-Controlled Converters

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Abstract—**Small-signal characteristics of current-mode-controlled PWM converters with a second-stage *LC* filter are analyzed. A secondary filter can be designed to provide good attenuation of the switching ripple while maintaining adequate stability margins with capacitive loading. Design guidelines for the filter are given.**

I. Introduction

MANY SWITCHING power converters have applications that require very low high-frequency noise and ripple on the output. It is often desirable to add a second-stage *LC* filter to the output of these power supplies to reduce the noise and ripple [1]. A single-stage filter which meets the requirements can become very large and impractical, especially for boost and buck–boost converters which have pulsating output currents. Unfortunately, a second *LC* filter gives up to 180° additional phase delay in the control-to-output transfer function, and it can make the system unstable if improperly designed. Also, the output of a power supply often has a capacitive load which can affect second-filter resonance and make a power supply unstable.

This paper analyzes the buck converter with a second *LC* filter in detail. It is shown that a very effective second filter can be designed which is stable under a wide range of capacitive loading conditions. The resonant frequencies and damping coefficients of the second filter are derived, and design guidelines are given. It is shown that the current-loop gain [2], [3] of the buck converter is not affected by the addition of the second filter when a small-filter inductance is used.

A common approach to designing the second filter is to add a small inductor and filter after the main *LC* power stage to remove high-frequency noise and ripple. It is shown in this paper that this is not a good design approach.

Three design examples are presented to demonstrate the use of analysis results. Two filter examples are designed for a buck converter. One of the second filters shows the problems which arise with a poor design. A third example is the design of a second filter for a buck–boost converter. In each of the design examples, the small-signal analysis was performed using the EASY5 [4] software, and the circuits were simulated using the state-space simulation program COSMIR [5].

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II. Buck Converter with Second *LC* Filter

To show the effect of a second *LC* filter on the performance of a buck converter, the circuit of Fig. 1 was analyzed in detail. Duty-cycle-to-inductor-current and duty-cycle-to-output-voltage transfer functions were found and separated into transfer functions showing the original characteristics without the second filter and the effect of the second filter only. The control-to-output transfer function with the current-feedback loop closed was used to assess the stability margin of the system. This loop gain clearly shows the effect of a second filter [2]. By considering the voltage and current transfer functions, the effect of the second filter on the outer loop gain can be derived.

The state-space equations for the circuit of Fig. 1, after perturbation and averaging [6], are

$$\begin{bmatrix} \dot{i}_{L_f} \\ \dot{v}_{C_f} \\ \dot{i}_{L_o} \\ \dot{v}_{C_o} \end{bmatrix} = \begin{bmatrix} \frac{-R_{Cf}}{L_f} & \frac{-1}{L_f} & \frac{R_{Cf}}{L_f} & 0 \\ \frac{1}{C_f} & 0 & \frac{-1}{C_f} & 0 \\ \frac{-R_{Cf}}{L_o} & \frac{1}{L_o} & \frac{-R_{Ce}}{L_o} & \frac{-1}{L_o} \\ 0 & 0 & \frac{1}{C_o} & \frac{1}{C_o R_L} \end{bmatrix} \begin{bmatrix} i_{L_f} \\ v_{C_f} \\ i_{L_o} \\ v_{C_o} \end{bmatrix} + \begin{bmatrix} \frac{V_g}{L_f} \\ 0 \\ 0 \\ 0 \end{bmatrix} \hat{d} \tag{1}$$

where

$$R_{C_e} = R_{C_f} + R_{C_o}. \tag{2}$$

All the desired transfer functions can be found from this equation with some specific circuit assumptions discussed below.

III. Buck Converter Resonant Frequencies

The resonant frequencies of the system with the second filter are of interest to the designer. These can be found from the determinant of the A matrix. The parasitic resis-

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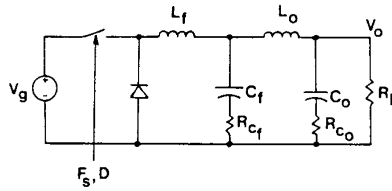


Fig. 1. Buck converter with second LC filter.

tive elements of the circuit do not significantly affect the resonant frequencies of the circuit, and the analysis can be performed more easily with these elements removed.

Assuming that the main power inductor L_f is much greater than the second-filter inductor L_o and setting $R_{C_f} = R_{C_o} = 0$ and $R_L = \infty$, the determinant of the A matrix is

$$\Delta(s) = s^4 + s^2\left[\frac{1}{L_oC_f} + \frac{1}{L_oC_o}\right] + \frac{1}{L_fL_oC_fC_o}$$

$$\simeq \left[s^2 + \frac{1}{L_f(C_f + C_o)}\right]\left[s^2 + \frac{1}{L_oC_f} + \frac{1}{L_oC_o}\right]. \tag{3}$$

This equation gives the resonant frequencies of the circuit by inspection. The first resonance is

$$\omega_p = \frac{1}{\sqrt{L_f(C_f + C_o)}} \tag{4}$$

and the resonance due to the second filter is

$$\omega_f = \frac{1}{\sqrt{L_oC_s}} \tag{5}$$

where C_s is the series combination of the capacitors:

$$C_s = \frac{C_fC_o}{C_f + C_o}. \tag{6}$$

The first resonance of the system is determined by the parallel combination of the two filter capacitors and by the large filter inductor L_f. The second resonance is determined by the series combination of the filter capacitors and by the smaller inductor L_o. The smaller capacitor, therefore, determines second-filter resonance, and the larger capacitor determines the first-filter resonance, regardless of the arrangement of the capacitors.

IV. Filter Analysis for $C_f >> C_o$

A. Transfer Function Analysis

When the parasitic resistances are included in the system, the determinant of the A matrix can be approximately factored if either $C_f >> C_o$ or $C_f << C_o$. For the case $C_f >> C_o$, the determinant is

$$\Delta(s) = \left[s^2L_fC_f + s\left[C_fR_{C_f} + \frac{L_f}{R_L}\right] + 1\right] \cdot \left[s^2L_oC_o + s\left[C_oR_{C_o} + \frac{L_o}{R_L}\right] + 1\right]. \tag{7}$$

the duty-cycle-to-inductor-current transfer function, after cancelling numerator terms with part of the determinant, can then be found:

$$\frac{\hat{i}_{L_o}}{\hat{d}} = V_g \frac{(sC_fR_L + 1)}{s^2L_fC_f + s\left[C_fR_{C_f} + \frac{L_f}{R_L}\right] + 1} = F_i(s). \tag{8}$$

$F_i(s)$ is the same as the duty-cycle-to-inductor-current transfer function obtained with a single-stage filter with an inductor L_f and capacitor C_f.

The duty-cycle-to-output-voltage transfer function when $C_f >> C_o$ is

$$\frac{\hat{v}_o}{\hat{d}} = \left[V_g \frac{(sC_fR_{C_f} + 1)}{s^2L_fC_f + s\left[C_fR_{C_f} + \frac{L_f}{R_L}\right] + 1}\right] \cdot \left[\frac{sC_oR_{C_o} + 1}{s^2L_oC_o + s\left[C_oR_{C_o} + \frac{L_o}{R_L}\right] + 1}\right] = F_v(s)F_1(s). \tag{9}$$

The transfer function $F_v(s)$ is equal to the duty-cycle-to-output-voltage transfer function for a single-stage filter with inductor L_f and capacitor C_f. Transfer function $F_1(s)$ shows the effect of the second filter on the voltage gain.

From $F_1(s)$, the quality factor of the second-filter resonance can be directly determined:

$$Q_f = \frac{1}{\omega_f\left[C_oR_{C_o} + \frac{L_o}{R_L}\right]} \tag{10}$$

where ω_f is given in (5).

B. Analysis of High-Frequency Output Impedance

The effect of the second LC filter on the output impedance is important. At higher frequencies, where the second filter affects the performance, the first inductor L_f can be considered an open circuit. The effect on the output impedance of the second LC filter can be shown by analyzing the resulting third-order system with L_f removed.

The high-frequency output impedance with $C_f >> C_o$ is given by

$$Z_o^h(s) \simeq \left[R_L \frac{sC_fR_{C_f} + 1}{sC_fR_L + 1}\right] \cdot \left[\frac{s^2L_oC_o + s\left[C_oR_{C_o} + \frac{L_o}{R_{C_o}}\right] + 1}{s^2L_oC_o + s\left[C_oR_{C_o} + \frac{L_o}{R_L}\right] + 1}\right] = Z_p^h(s)Z_f^h(s). \tag{11}$$

The function $Z_p^h(s)$ is the high-frequency value of the output impedance of the power stage without a second-stage filter and with capacitor C_f. The effect of the second filter on the output impedance is given by $Z_f^h(s)$. For a small value of output capacitor C_o, significant peaking of the output impedance can occur at the second-filter resonance. The value of the peaking Z_o' relative to the system with no filter Z_p' is

$$Z_o'(s) = Z_p'(s)\frac{\left[C_oR_{C_e} + \dfrac{L_o}{R_{C_o}}\right]}{\left[C_oR_{C_e} + \dfrac{L_o}{R_L}\right]}. \tag{12}$$

C. Conclusion

When the first capacitor C_f is made larger, the control-to-inductor-current transfer function is the same as that for a buck converter without the second-stage filter. The outer loop gain is, therefore, only affected by the control-to-output-voltage transfer function. The duty-cycle-to-output-voltage transfer function can be split into two parts: the first is identical to the voltage gain of the converter without the second-stage filter; the second shows the effect of the second-stage filter only.

The second-filter resonance is determined by the small inductor L_o and by the output capacitor C_o. The filter configuration with $C_f >> C_o$ is undesirable since the second-filter resonance is a sensitive function of any capacitive loading which may be applied to the system. Addition of a capacitive load will reduce the second resonance and can cause instability.

The expression for high-frequency output impedance shows another disadvantage of this configuration. Significant peaking of the output impedance can occur at the second-filter resonance. Both of the undesirable effects of this filter configuration are demonstrated later with a design example.

V. Filter Analysis for $C_o >> C_f$

A. Transfer Function Analysis

A second possible filter design is with the output capacitor C_o much larger than the filter capacitor C_f. The duty-cycle-to-inductor-current transfer function, after cancelling numerator terms with part of the determinant, can be found for this case:

$$\begin{aligned}\frac{\hat{i}_{Lf}}{\hat{d}} &= V_g\frac{sC_oR_L + 1}{s^2L_fC_o + s\left[C_oR_{C_o} + \dfrac{L_f}{R_L}\right] + 1} \\ &= F_i(s)\end{aligned} \tag{13}$$

where $F_i(s)$ is the same as the duty-cycle-to-inductor-current transfer function obtained with a single-stage filter with an inductor L_f and capacitor C_o.

The duty-cycle-to-output-voltage transfer function when $C_o >> C_f$ is

$$\begin{aligned}\frac{\hat{v}_o}{\hat{d}} &= \left[V_g\frac{sC_oR_{C_o} + 1}{s^2L_fC_o + s\left[C_oR_{C_o} + \dfrac{L_f}{R_L}\right] + 1}\right] \\ &\quad\cdot\left[\frac{sC_fR_{C_f} + 1}{s^2L_oC_o + sC_oR_{C_e} + 1}\right] \\ &= F_v(s)F_2(s).\end{aligned} \tag{14}$$

Transfer function $F_v(s)$ is the same as the duty-cycle-to-output-voltage transfer function for a single-stage filter with inductor L_f and capacitor C_o. Transfer function $F_2(s)$ shows the effect of the second filter on the voltage gain.

From $F_2(s)$, the quality factor of the second-filter resonance can be directly determined:

$$Q_f = \frac{\omega_f L_o}{R_{C_e}} \tag{15}$$

where ω_f is given in (5). Notice that the load resistance does not provide any damping for this case.

B. Analysis of High-Frequency Output Impedance

The high-frequency output impedance with $C_o >> C_f$ is given by

$$\begin{aligned}Z_o^h(s) &\simeq R_L\frac{[sC_oR_{C_o} + 1]}{[sC_oR_L + 1]} \\ &= Z_p^h(s).\end{aligned} \tag{16}$$

The second filter does not affect the high-frequency output impedance in this case.

C. Conclusion

The control-to-inductor-current transfer function is unaffected by the second-stage filter when the output capacitor is larger. The control-to-output-voltage can be split into two parts. One is equal to the transfer function obtained without the second filter, and the other shows only the effect of the second filter. In this case, the second-filter resonance is determined by the small inductor L_o and the filter capacitor C_f.

This is a much more desirable system, since the second-filter resonance is unaffected by capacitive loading. The first-filter resonance will shift with a capacitive load, but this does not cause any stability problems, as is demonstrated later in the design examples. There is no peaking of the output impedance with this filter configuration.

VI. Design Guidelines for a Second LC Filter

Based on the above analysis, some design guidelines can be given for the second LC filter.

1) Design the power supply and optimize the current-mode control without the second-stage filter. Do not increase the bandwidth of the outer loop gain unnecessarily

since this will constrain the design of the second filter. A wide-bandwidth outer loop gain is not usually required for excellent performance when current-mode control is used [2], [7].

2) Split the value of the output capacitor into two unequal parts. The smaller part should be the smallest value of capacitance which can carry the ripple current from the main filter inductor L_f. This small capacitance should be used for the filter capacitor C_f. The remaining capacitance is used for the output capacitor C_o.

3) Select the second-filter inductor L_o such that the second-filter resonant frequency, given by (5), is higher (at least three times) than the crossover frequency of the outer loop gain.

4) Use (15) to ensure that the second filter is adequately damped. If the filter is underdamped, a smaller value of L_o must be used. The required amount of damping will depend upon the placement of the second resonant frequency with respect to the first. If these frequencies are close together, Q_f from (15) should be less than one. In calculating the damping of the filter, it must also be considered that a low-impedance capacitive load could be placed at the output, reducing the value of the damping resistor R_{C_o}.

An effective second LC filter can be designed by following these guidelines. The system will be stable with capacitive loading and the output impedance will not be deteriorated by the second filter.

VII. Buck Converter Design Example with $C_o >> C_f$

A 500-W 5-V output buck converter was designed with the following circuit components (refer to circuit diagram of Fig. 1):

$$V_g = 12\ \text{V} \quad V_o = 5\ \text{V} \quad F_s = 100\ \text{kHz} \quad L_f = 3\ \mu\text{H}$$

$$C_o = 6 \times 1300\ \mu\text{F} = 7800\ \mu\text{F} \quad R_{C_o} = \tfrac{18}{6}\ \text{m}\Omega = 3\ \text{m}\Omega$$

$$R_L = 0.05\ \Omega.$$

An integral and lead–lag compensation circuit was designed to give a crossover frequency of the outer loop gain of 3.5 kHz and a phase margin of 85°, as shown by the loop gain of Fig. 2. Increasing the bandwidth beyond this point does not provide significant reduction in the peak value of either the output impedance or the audiosusceptibility [2], [7]. The peak-to-peak output-voltage ripple for this design, shown in Fig. 4(a), was 30 mV.

A second-stage LC filter was designed to eliminate switching spikes and to reduce the output-voltage ripple. As discussed earlier, it is desirable to make filter capacitor C_f smaller than output capacitor C_o. Using the same class of capacitors as for the original single-stage buck design, the following component values were selected:

$$L_f = 3\ \mu\text{H} \qquad L_o = 0.2\ \mu\text{H}$$

$$C_f = 2 \times 1300\ \mu\text{F} = 2600\ \mu\text{F} \qquad R_{C_f} = \tfrac{18}{2}\ \text{m}\Omega = 9\ \text{m}\Omega$$

$$C_o = 4 \times 1300\ \mu\text{F} = 5200\ \mu\text{F} \qquad R_{C_o} = \tfrac{18}{4}\ \text{m}\Omega = 4.5\ \text{m}\Omega.$$

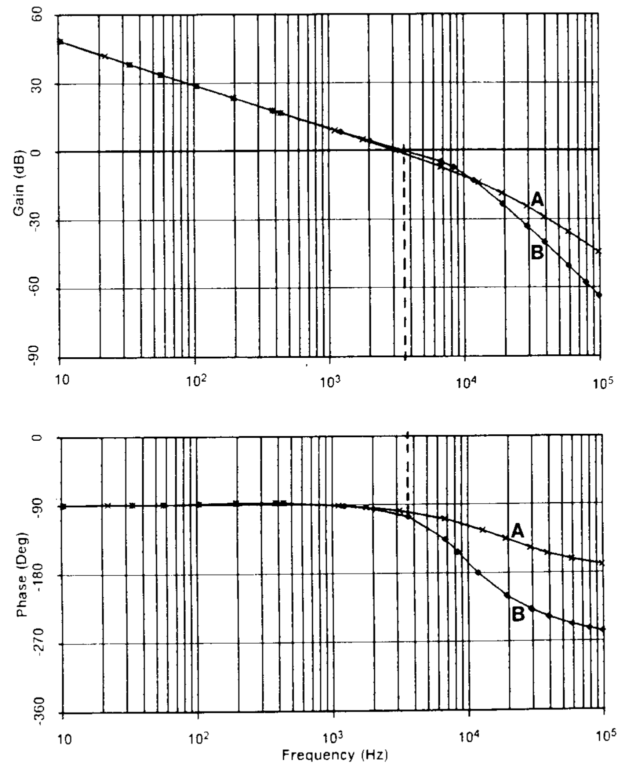


Fig. 2. Loop gain of buck converter. A: Single-stage filter. B: Second LC filter.

The smallest number of capacitors that could carry the ripple current in C_f was used. The remaining capacitors were used for the output capacitor. With this choice of components, the second resonance from (5) is

$$\begin{aligned} \omega_f &= \frac{1}{\sqrt{L_0 C_s}} \\ &= 5.37 \times 10^4 \quad \text{rad/s} \\ &= 8.55 \quad \text{kHz}. \end{aligned} \tag{17}$$

The quality factor of the second-filter resonance is given by (15):

$$\begin{aligned} Q_f &= \frac{\omega_f L_o}{R_{C_e}} \\ &= 0.79. \end{aligned} \tag{18}$$

The second LC-filter design meets the requirements for stability. The second resonant frequency is above the crossover frequency of the outer loop gain, and the second filter is sufficiently damped to prevent peaking of the gain.

Fig. 2 shows the loop gain for the buck converter with and without the second filter. The crossover with the second filter is 3.8 kHz, and the phase margin is 70°. The gain margin with the second filter is 15 dB.

The improvement in the high-frequency audio susceptibility is shown in Fig. 3. The second filter gives 20 dB more attenuation at the switching frequency. A small

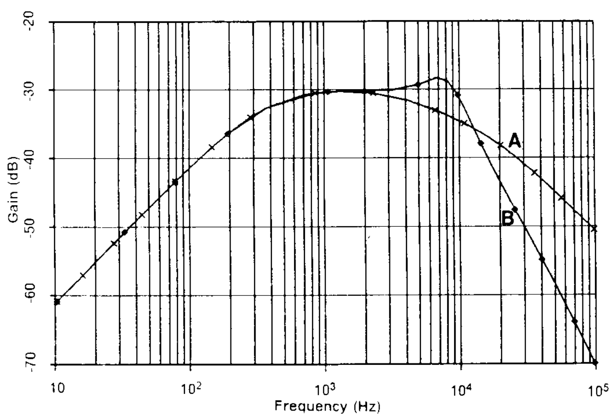


Fig. 3. Audio susceptibility of buck converter. A: Single-stage filter. B: Second LC filter.

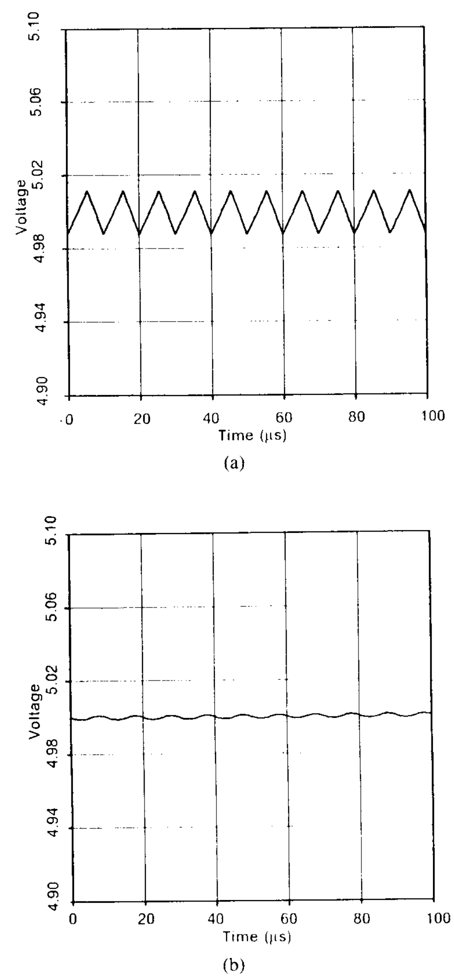


Fig. 4. Output-voltage ripple of buck converter. (a) Single-stage filter. (b) Second LC filter.

peaking effect in the audio susceptibility can be observed at the second-filter resonance. This occurs because the second resonant frequency was placed close to the crossover frequency. The analysis of the second-filter transfer functions does not show this peaking since open-loop conditions were assumed.

The dramatic effect of the addition of the second filter on the output-voltage ripple is clearly shown in Fig. 4. With the addition of just the small inductor L_o, the output ripple was reduced to 3 mV.

Fig. 5 shows the output impedance of the converter with and without the second filter. The curves are different at high frequencies because the value of capacitor C_o was changed from 7800 to 5200 μF. As predicted by (12), however, there is no second-order peaking due to the second-stage filter.

Fig. 6 shows the responses of the system to a 10-A step load, with and without the second-stage filter. Both responses are very similar, except for a small amount of ringing with the second filter. This shows that the control circuit designed for the converter without the second filter could still be used. No additional feedback loops are required to control the system with the second filter.

Although the second-filter resonance was placed close to the loop-gain crossover, the system maintains its stability with capacitive loading. Fig. 7 shows the loop gain of the system with an increasing capacitive load. As the load capacitance is increased from zero to 10 000 μF (R_{C_o} was correspondingly reduced to 2 mΩ), there is only a very small change in the second-filter resonance, from 8.5 to 7.5 kHz. The larger capacitor has very little effect on the series combination of C_o and C_f. There is also very little effect on the high-frequency phase characteristic. The added loaded capacitance does have the effect of reducing the gain of the outer voltage loop, thereby increasing the stability margin of the system.

As the load capacitance is increased further to 20 800 μF, there is no perceptible shift in the second-filter resonance, and the high-frequency phase characteristic remains unchanged. The loop gain is again reduced by the added capacitance, and the system is very stable.

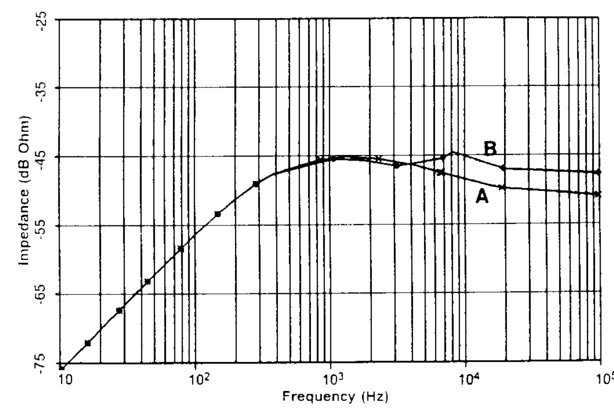


Fig. 5. Output impedance of buck converter. A: Single-stage filter. B: Second LC filter.

VIII. Buck Converter Design Example with $C_f >> C_o$

To demonstrate the problems with a poorly designed filter, a second filter was designed for the above circuit choosing C_f much larger than the output capacitor C_o. The

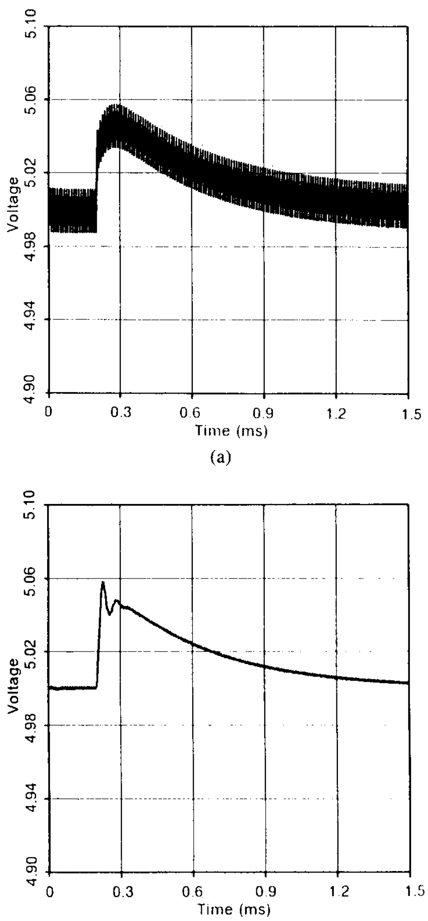


Fig. 6. 10-A step-load response of buck converter. (a) Single-stage filter. (b) Second LC filter.

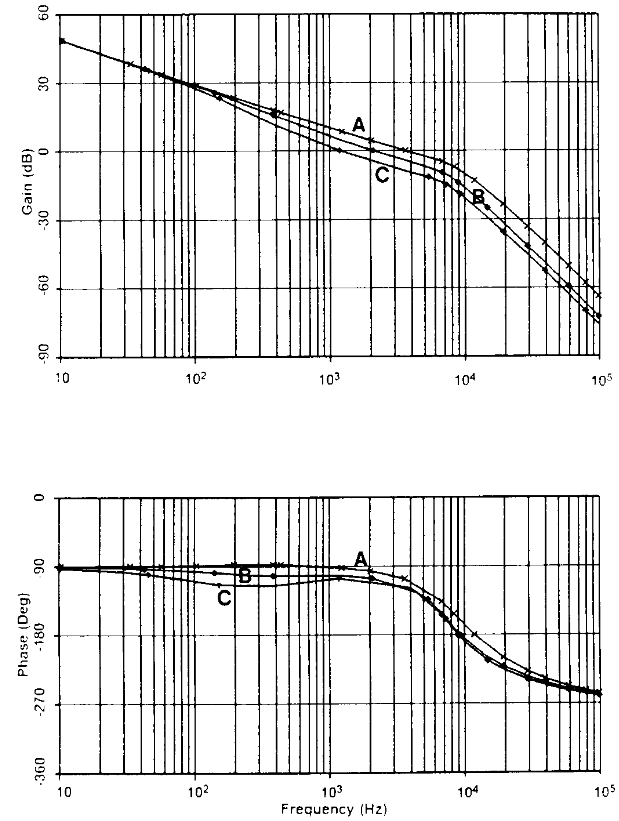


Fig. 7. Loop gain of buck converter with capacitive loading. A: $C_L = 0$. B: $C_L = 10\,400\ \mu$F. C: $C_L = 20\,800\ \mu$F.

following circuit parameters were selected:

$L_f = 3\ \mu$H $\qquad L_o = 0.2\ \mu$H

$C_f = 6 \times 1300\ \mu\text{F} = 7800\ \mu\text{F} \qquad R_{C_f} = \frac{18}{6}\ \text{m}\Omega = 3\ \text{m}\Omega$

$C_o = 300\ \mu$F $\qquad R_{C_o} = 5\ \text{m}\Omega$.

For this design, the second resonance can be found from (5):

$$\omega_f = \frac{1}{\sqrt{L_o C_s}} = 1.31 \times 10^5 \quad \text{rad/s} = 20.9 \quad \text{kHz}. \tag{19}$$

The quality factor of the second-filter resonance is given by (10):

$$Q_f = \frac{1}{\omega_f\left[C_o R_{C_o} + \dfrac{L_o}{R_L}\right]} = 1.39. \tag{20}$$

The second filter is a little underdamped but placed at a sufficiently high frequency to prevent stability problems. However, this filter configuration is very susceptible to capacitive loading. Fig. 8 shows the effect of capacitive loading on the loop gain of this system. The first plot shows that the system with no capacitive load has a crossover frequency of 3.5 kHz with a phase margin of 80° and a gain margin of 13 dB. The second plot shows that a 7800-μF load shifts the second resonance from 20.9 to 5.5 kHz. The high-frequency phase characteristic shows a considerable change with the capacitive loading, and the system is close to instability.

Fig. 9 shows the output impedance of the converter with a single-stage filter and with the poorly designed second-stage filter. The severe peaking of the output impedance at the second-filter resonance, predicted by (12), is apparent.

IX. Buck–Boost Converter Design Example

Although the analysis presented is for the buck converter, the results can be applied to other types of converters with a second LC filter. Second-stage filters are commonly required for buck–boost and boost converters to suppress the high switching ripple resulting from pulsating output currents.

A buck–boost converter was designed with the follow-

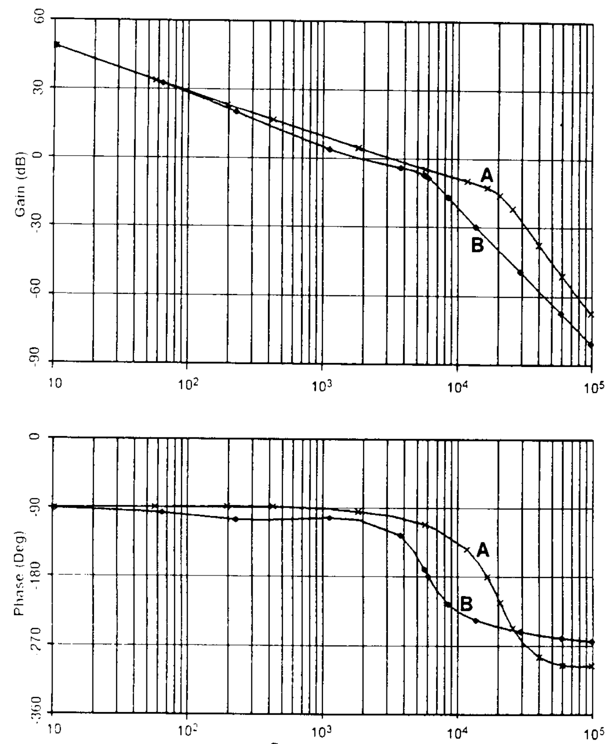


Fig. 8. Loop gain of buck converter with capacitive loading and poorly designed second filter. A: $C_L = 0$. B: $C_L = 7800\ \mu F$.

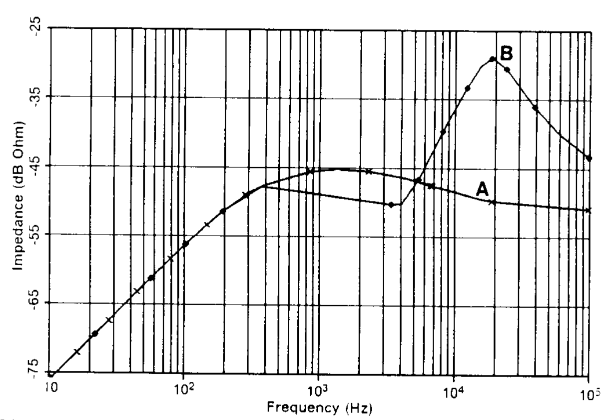


Fig. 9. Output impedance of buck converter with poorly designed second filter. A: Single-stage filter. B: Second *LC* filter.

ing components:

$V_g = 12$ V $V_o = 5$ V $F_s = 100$ kHz $L_f = 5\ \mu H$

$C_o = 5 \times 1300\ \mu F = 6500\ \mu F$ $R_{C_o} = \frac{18}{5}\ m\Omega = 3.6\ m\Omega$

$R_L = 0.25\ \Omega$.

An integral and lead-lag compensation circuit was designed to give a crossover frequency of the outer loop gain of 3 kHz and a phase margin of 85°, as shown by the loop gain of Fig. 10. The buck-boost converter had a right-half-plane zero at 13.3 kHz. The peak-to-peak output-voltage ripple for this design was 120 mV.

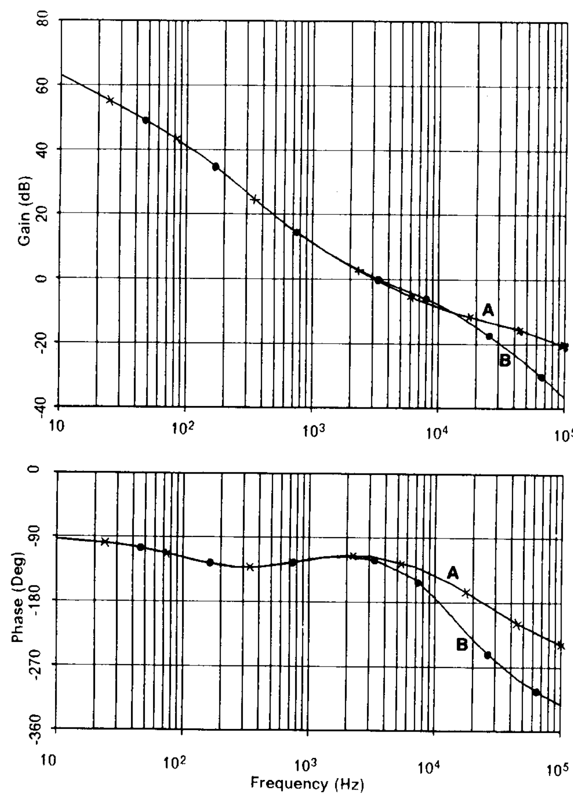


Fig. 10. Loop gain of buck-boost converter. A: Single-stage filter. B: Second *LC* filter.

A second-stage *LC* filter was designed to eliminate switching spikes and to reduce the output-voltage ripple. Using the same class of capacitors as for the original single-stage buck-boost design, the following component values were selected:

$L_f = 3\ \mu H$ $L_o = 0.15\ \mu H$

$C_f = 2 \times 1300\ \mu F = 2600\ \mu F$ $R_{C_f} = \frac{18}{2}\ m\Omega = 9\ m\Omega$

$C_o = 3 \times 1300\ \mu F = 3900\ \mu F$ $R_{C_o} = \frac{18}{3}\ m\Omega = 6\ m\Omega$.

The smallest number of capacitors that could carry the ripple current in C_f was used. The remaining capacitors were used for the output capacitor. With this choice of components, the second resonance from (5) is

$$\omega_f = \frac{1}{\sqrt{L_o C_s}} = 6.53 \times 10^4 \ \text{rad/s} = 10.4 \ \text{kHz}. \tag{21}$$

The quality factor of the second-filter resonance is given by (15):

$$Q_f = \frac{\omega_f L_o}{R_{C_e}} = 0.65. \tag{22}$$

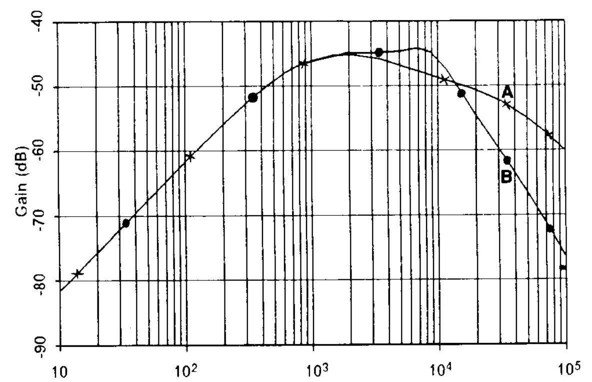
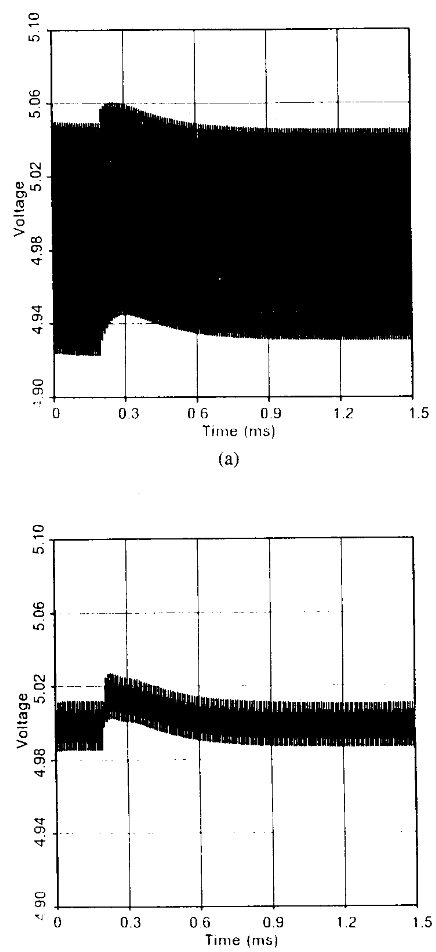


Fig. 11. Audio susceptibility of buck-boost converter. A: Single-stage filter. B: Second LC filter.



(a)

(b)

Fig. 12. 2-A step-load response of buck-boost converter. (a) Single-stage filter. (b) Second LC filter.

This filter meets the requirement for a stable design as the first capacitor was smaller, the second resonance was above the loop-gain crossover, and the second filter was well damped. Fig. 10 shows the loop gain of this converter with and without the second-stage filter. The phase

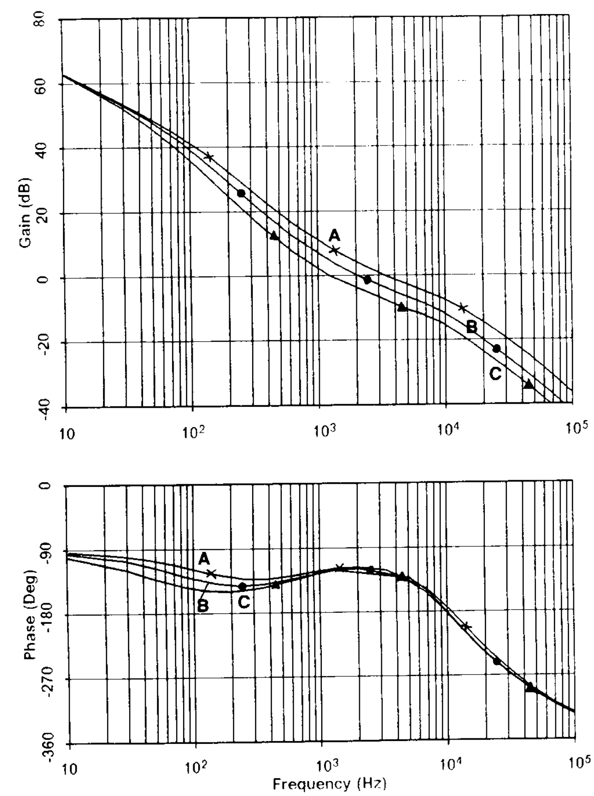


Fig. 13. Loop gain of buck-boost converter with capacitive loading. A: $C_L = 0$. B: $C_L = 10\,400\ \mu F$. C: $C_L = 20\,800\ \mu F$.

margin with the second filter was 60°, and the gain margin was 10 dB.

The audio susceptibility of this converter is shown in Fig. 11. The second-stage filter gave an extra 15 dB of attenuation at the switching frequency. Fig. 12 shows the output voltage ripple and a 2-A step-load response of the converter with and without the second filter. The output ripple was reduced from 120 to 25 mV. The same control network was effective for both systems, giving a 20-mV peak overshoot for the load transient.

The effect of capacitive loading on the buck-boost converter with the second filter is shown in Fig. 13. Like the buck converter, the system remained very stable under all capacitive loading conditions. The high-frequency phase characteristic was unaffected by the capacitive load, and the crossover frequency was reduced with additional capacitance.

X. Conclusion

The buck converter with a second LC filter has been analyzed in detail. It was shown that the current loop gain of a current-mode-controlled converter is unaffected by the addition of a second LC filter when the second-filter inductance is much smaller than the large filter inductance. The duty-cycle-to-output-voltage transfer function was separated into two parts, clearly showing the effect of the second filter on this gain and hence on the outer-loop transfer function. It was shown that a well-designed filter does not degrade the output impedance of a power converter.

Guidelines were formulated for the design of an effective second *LC* filter that maintains a good stability margin even under severe capacitive loading conditions. These design guidelines can be applied to other power converters with second-stage filters. It was demonstrated that a poorly-designed second-stage filter can be made unstable with capacitive loading, and it can cause severe peaking in the output impedance characteristic of the power supply at the second-filter resonance.

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